



ENUMERATION OF NON PRINCIPAL CYCLIC CODES OF LENGTH 8 OVER $\mathbb{Z}_8$

Arpana Garg¹, Sudhir Pujara²

^{1,2} Associate Professor

^{1,2} Department of Mathematics

¹I.B. (P.G.) College, Panipat, India, ²D.B.G. Govt. College, Panipat, India.

Abstract : In this paper, all non principal cyclic codes of length 8 over $\mathbb{Z}_8$ have been enumerated on the basis of the structure, generators and degree of minimal degree polynomials belonging to cyclic codes of length 2^k over $\mathbb{Z}_8$. The rank of such codes has also been determined.

IndexTerms - Rings, Cyclic Codes, Minimal Degree Polynomial, Rank

I. INTRODUCTION

A linear code C of length n over a finite commutative ring R is defined as an R -submodule of R^n . A cyclic code C over R of length n is a linear code such that whenever $(c_0, c_1, c_2, \dots, c_{n-1})$ is in C , $(c_{n-1}, c_0, c_1, \dots, c_{n-2})$ also belongs to C . The rank of a code C (denoted by $\text{rank}(C)$) over a ring R is defined as the minimum number of generators of C as an R -module [1].

Cyclic codes over finite rings have been studied by many authors. For reference, see ([2], [3], [4],[5],[9],[10],[11]). In [6], the structure of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ as an ideal of the ring $\mathbb{Z}_8[x]/\langle x^n - 1 \rangle$ is obtained. Using this structure of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$, Garg et al. gave a distinguished set of generators and rank of such a cyclic code in [7] and it is proved that the rank of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ is $n - v$, where v is the degree of a minimal degree polynomial in the code. In the enumeration of cyclic codes of length 2^k over $\mathbb{Z}_8$, the minimal degree polynomial plays a vital role. However, for a given code with distinguished form of generators, the value of v is not always obvious. Abualrub et al. has found the degree of minimal degree polynomial in a cyclic code of length 2^k over $\mathbb{Z}_4$ in [1]. Garg et al. has found degree of minimal degree polynomial with leading coefficient 2, 4 or 6 in a cyclic code C of length $n = 2^k$ over $\mathbb{Z}_8$ in [8].

In this paper, using structure and degree of minimal degree polynomial of a cyclic code C of length $n = 2^k$ over $\mathbb{Z}_8$ given by [8], all non principal cyclic codes of length 8 over $\mathbb{Z}_8$ have been enumerated. The rank of such codes has also been determined and tabulated.

II. PRELIMINARIES

A distinguished set of generators of a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$ is obtained in [7]. Firstly, we recall those results for further reference.

Let C be cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$. Let $f(x) = q_0(x)$ be a minimal degree polynomial among all monic polynomials in C , $g(x) = 2q_1(x)$ be a minimal degree polynomial among all polynomials in C with leading coefficient 2 or 6 and $h(x) = 4q_2(x)$ be a minimal degree polynomial among all polynomials in C with leading coefficient 4. Let $\deg(f(x)) = r$, $\deg(g(x)) = s$ and $\deg(h(x)) = v$. It is obvious that $r \geq s \geq v$. A unique form for the polynomials $f(x)$, $g(x)$ and $h(x)$ is determined in Theorem 2.1 below. Note that the cyclic code C may or may not contain any monic polynomial or any polynomial with leading coefficient 2 or 6.

Theorem 2.1 [7] Let C be a cyclic code of length $n = 2^k$ over $\mathbb{Z}_8$. Let $f(x)$, $g(x)$ (if they exist) and $h(x)$ be polynomials in C as defined above. Then

1. $h(x) = 4q_2(x) = 4(x+1)^v$.
2. If C contains polynomials with leading coefficient 2 or 6, then there exists a unique polynomial in C of the type $2(x+1)^s + 4 \sum_{i=0}^{v-1} \alpha_i (x+1)^i$ of degree s , where $\alpha_i \in \mathbb{Z}_2$. Therefore, $g(x) = 2q_1(x)$ can be chosen as $2(x+1)^s + 4 \sum_{i=0}^{v-1} \alpha_i (x+1)^i$, where $\alpha_i \in \mathbb{Z}_2$.
3. If C contains monic polynomials, then there exists a unique polynomial in C of the type $(x+1)^r + 2 \sum_{i=0}^{s-1} \beta_i (x+1)^i + 4 \sum_{i=0}^{v-1} \gamma_i (x+1)^i$ of degree r , where $\beta_i, \gamma_i \in \mathbb{Z}_2$. Therefore, $f(x) = q_0(x)$ can be chosen as $(x+1)^r + 2 \sum_{i=0}^{s-1} \beta_i (x+1)^i + 4 \sum_{i=0}^{v-1} \gamma_i (x+1)^i$, where $\beta_i, \gamma_i \in \mathbb{Z}_2$.

Theorem 2.2 [7] A cyclic code C of length 2^k over $\mathbb{Z}_8$ is generated by one or more polynomials from the set $\{f(x), g(x), h(x)\}$ where

1. $f(x) = q_0(x) = (x+1)^r + 2 \sum_{i=0}^{s-1} \beta_i (x+1)^i + 4 \sum_{i=0}^{v-1} \gamma_i (x+1)^i$ with $\beta_i, \gamma_i \in \mathbb{Z}_2$
2. $g(x) = 2q_1(x) = 2(x+1)^s + 4 \sum_{i=0}^{v-1} \alpha_i (x+1)^i$ with $\alpha_i \in \mathbb{Z}_2$
3. $h(x) = 4q_2(x) = 4(x+1)^v$

Remark 2.3 Referring back to Theorem 2.2, the generators of a cyclic code of length 2^k over $\mathbb{Z}_8$ can further be written as

1. $f(x) = q_0(x) = (x+1)^r + 2(x+1)^c \beta(x) + 4(x+1)^d \gamma(x)$ where $\beta(x)$ and $\gamma(x)$ belong to $\mathbb{Z}_2[x]/\langle x^n - 1 \rangle$.
2. $g(x) = 2q_1(x) = 2(x+1)^s + 4(x+1)^e \alpha(x)$ where $\alpha(x) \in \mathbb{Z}_2[x]/\langle x^n - 1 \rangle$.

As stated in Theorem 2.2 above, a cyclic code of length 2^k over $\mathbb{Z}_8$ is generated by one or more polynomials from the set $\{f(x), g(x), h(x)\}$. Clearly, the minimal degree polynomial in any cyclic code of length 2^k over $\mathbb{Z}_8$ is $h(x) = 4(x+1)^v$. In case $h(x) = 4(x+1)^v$ is not a generator of the cyclic code of length 2^k over $\mathbb{Z}_8$, then the minimal degree polynomial $h(x) = 4(x+1)^v$ in the code is not obviously known. Therefore the minimal degree polynomial in the cyclic codes $\langle g(x) \rangle, \langle f(x), g(x) \rangle, \langle f(x) \rangle$ needs to be determined. In case, a cyclic code of length 2^k over $\mathbb{Z}_8$ is generated by $f(x)$ only, the degree of a minimal degree polynomial $g(x)$ with leading coefficient 2 or 6 in the code is also not obvious. The degree of minimal degree polynomials $g(x)$ and $h(x)$ in a cyclic code of length 2^k over $\mathbb{Z}_8$ has been obtained by Garg et al. in [8]. Using the results of [8], all non principal cyclic codes of length 8 over $\mathbb{Z}_8$ have been enumerated. The rank of such codes is also determined and tabulated in next section.

III. NON PRINCIPAL CYCLIC CODES OVER $\mathbb{Z}_8$ OF LENGTH 8

In this section, all non principal cyclic codes of length 8 over $\mathbb{Z}_8$ have been enumerated along with their rank and are tabulated in the following table. It has been found that there are 860 non principal cyclic codes of length 8 over $\mathbb{Z}_8$, out of which 59 are non principal cyclic codes with leading coefficient 2 or 6 and 801 are non principal monic cyclic codes.

Table 1: Non principal Cyclic Codes of Length 8 over $\mathbb{Z}_8$

Sr. No.	Code	Rank
1	$\langle 2(x+1), 4 \rangle$	8
2	$\langle 2(x+1)^2, 4 \rangle$	8
3	$\langle 2(x+1)^2, 4(x+1) \rangle$	7
4	$\langle 2(x+1)^2 + 4, 4(x+1) \rangle$	7
5	$\langle 2(x+1)^3, 4 \rangle$	8
6	$\langle 2(x+1)^3, 4(x+1) \rangle$	7
7	$\langle 2(x+1)^3, 4(x+1)^2 \rangle$	6
8	$\langle 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
9	$\langle 2(x+1)^3 + 4, 4(x+1) \rangle$	7
10	$\langle 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
11	$\langle 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
12	$\langle 2(x+1)^4, 4 \rangle$	8
13	$\langle 2(x+1)^4, 4(x+1) \rangle$	7
14	$\langle 2(x+1)^4, 4(x+1)^2 \rangle$	6
15	$\langle 2(x+1)^4, 4(x+1)^3 \rangle$	5
16	$\langle 2(x+1)^4 + 4, 4(x+1) \rangle$	7
17	$\langle 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
18	$\langle 2(x+1)^4 + 4, 4(x+1)^3 \rangle$	5
19	$\langle 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
20	$\langle 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
21	$\langle 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
22	$\langle 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
23	$\langle 2(x+1)^4 + 4(x+1), 4(x+1)^2 \rangle$	6
24	$\langle 2(x+1)^4 + 4(x+1), 4(x+1)^3 \rangle$	5
25	$\langle 2(x+1)^4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
26	$\langle 2(x+1)^4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
27	$\langle 2(x+1)^5, 4 \rangle$	8
28	$\langle 2(x+1)^5, 4(x+1) \rangle$	7
29	$\langle 2(x+1)^5, 4(x+1)^2 \rangle$	6
30	$\langle 2(x+1)^5, 4(x+1)^3 \rangle$	5
31	$\langle 2(x+1)^5 + 4, 4(x+1) \rangle$	7
32	$\langle 2(x+1)^5 + 4, 4(x+1)^2 \rangle$	6
33	$\langle 2(x+1)^5 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
34	$\langle 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6

35	$\langle 2(x+1)^5 + 4(x+1), 4(x+1)^3 \rangle$	5
36	$\langle 2(x+1)^5 + 4(x+1), 4(x+1)^4 \rangle$	4
37	$\langle 2(x+1)^5 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
38	$\langle 2(x+1)^5 + 4(x+1) + 4(x+1)^2, 4(x+1)^4 \rangle$	4
39	$\langle 2(x+1)^5 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
40	$\langle 2(x+1)^5 + 4(x+1) + 4(x+1)^3, 4(x+1)^4 \rangle$	4
41	$\langle 2(x+1)^5 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
42	$\langle 2(x+1)^6, 4 \rangle$	8
43	$\langle 2(x+1)^6, 4(x+1) \rangle$	7
44	$\langle 2(x+1)^6, 4(x+1)^2 \rangle$	6
45	$\langle 2(x+1)^6, 4(x+1)^3 \rangle$	5
46	$\langle 2(x+1)^6 + 4, 4(x+1) \rangle$	7
47	$\langle 2(x+1)^6 + 4(x+1), 4(x+1)^2 \rangle$	6
48	$\langle 2(x+1)^6 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
49	$\langle 2(x+1)^6 + 4(x+1)^2, 4(x+1)^4 \rangle$	4
50	$\langle 2(x+1)^6 + 4(x+1)^2, 4(x+1)^5 \rangle$	3
51	$\langle 2(x+1)^6 + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
52	$\langle 2(x+1)^6 + 4(x+1)^2 + 4(x+1)^4, 4(x+1)^5 \rangle$	3
53	$\langle 2(x+1)^7, 4 \rangle$	8
54	$\langle 2(x+1)^7, 4(x+1) \rangle$	7
55	$\langle 2(x+1)^7, 4(x+1)^2 \rangle$	6
56	$\langle 2(x+1)^7, 4(x+1)^3 \rangle$	5
57	$\langle 2(x+1)^7 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
58	$\langle 2(x+1)^7 + 4(x+1)^3, 4(x+1)^5 \rangle$	3
59	$\langle 2(x+1)^7 + 4(x+1)^3, 4(x+1)^6 \rangle$	2
60	$\langle (x+1), 2 \rangle$	8
61	$\langle (x+1), 4 \rangle$	8
62	$\langle (x+1) + 2, 4 \rangle$	8
63	$\langle (x+1)^2, 2 \rangle$	8
64	$\langle (x+1)^2, 2(x+1) \rangle$	7
65	$\langle (x+1)^2, 4 \rangle$	8
66	$\langle (x+1)^2, 4(x+1) \rangle$	7
67	$\langle (x+1)^2, 2(x+1), 4 \rangle$	8
68	$\langle (x+1)^2, 2(x+1) + 4 \rangle$	7
69	$\langle (x+1)^2 + 4, 4(x+1) \rangle$	6
70	$\langle (x+1)^2 + 4, 2(x+1) \rangle$	7
71	$\langle (x+1)^2 + 4, 2(x+1) + 4 \rangle$	7
72	$\langle (x+1)^2 + 2, 2(x+1) \rangle$	7
73	$\langle (x+1)^2 + 2 + 2(x+1), 4 \rangle$	8
74	$\langle (x+1)^2 + 2 + 2(x+1), 4(x+1) \rangle$	7
75	$\langle (x+1)^2 + 2, 4 \rangle$	8
76	$\langle (x+1)^2 + 2, 4(x+1) \rangle$	7
77	$\langle (x+1)^2 + 2(x+1), 4 \rangle$	8
78	$\langle (x+1)^2 + 2(x+1), 4(x+1) \rangle$	7
79	$\langle (x+1)^2 + 2, 2(x+1), 4 \rangle$	8
80	$\langle (x+1)^2 + 2 + 4, 4(x+1) \rangle$	7
81	$\langle (x+1)^2 + 2 + 2(x+1) + 4, 4(x+1) \rangle$	7
82	$\langle (x+1)^2 + 2(x+1) + 4, 4(x+1) \rangle$	7
83	$\langle (x+1)^3, 2 \rangle$	8
84	$\langle (x+1)^3, 2(x+1) \rangle$	7
85	$\langle (x+1)^3, 2(x+1)^2 \rangle$	6
86	$\langle (x+1)^3, 4 \rangle$	8
87	$\langle (x+1)^3, 4(x+1) \rangle$	7
88	$\langle (x+1)^3, 2(x+1) + 4 \rangle$	7
89	$\langle (x+1)^3, 2(x+1), 4 \rangle$	8
90	$\langle (x+1)^3, 2(x+1)^2, 4 \rangle$	8
91	$\langle (x+1)^3, 2(x+1)^2, 4(x+1) \rangle$	7
92	$\langle (x+1)^3, 2(x+1)^2 + 4 \rangle$	7
93	$\langle (x+1)^3, 2(x+1)^2 + 4(x+1) \rangle$	6
94	$\langle (x+1)^3 + 4, 4(x+1) \rangle$	7
95	$\langle (x+1)^3 + 4, 2(x+1) \rangle$	7
96	$\langle (x+1)^3 + 4, 2(x+1) + 4 \rangle$	7
97	$\langle (x+1)^3 + 4, 2(x+1)^2 \rangle$	6

98	$\langle (x+1)^3 + 4, 2(x+1)^2, 4(x+1) \rangle$	7
99	$\langle (x+1)^3 + 4, 2(x+1)^2 + 4 \rangle$	7
100	$\langle (x+1)^3 + 4 + 4(x+1), 2(x+1)^2 \rangle$	6
101	$\langle (x+1)^3 + 4 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
102	$\langle (x+1)^3 + 4(x+1), 2(x+1)^2 \rangle$	6
103	$\langle (x+1)^3 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
104	$\langle (x+1)^3 + 2, 2(x+1) \rangle$	8
105	$\langle (x+1)^3 + 2, 2(x+1)^2 \rangle$	8
106	$\langle (x+1)^3 + 2 + 2(x+1), 2(x+1)^2 \rangle$	8
107	$\langle (x+1)^3 + 2(x+1), 2(x+1)^2 \rangle$	7
108	$\langle (x+1)^3 + 2, 4 \rangle$	8
109	$\langle (x+1)^3 + 2 + 2(x+1), 4 \rangle$	8
110	$\langle (x+1)^3 + 2 + 2(x+1)^2, 4 \rangle$	8
111	$\langle (x+1)^3 + 2 + 2(x+1) + 2(x+1)^2, 4 \rangle$	8
112	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2, 4 \rangle$	8
113	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2, 4(x+1) \rangle$	7
114	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2, 4(x+1)^2 \rangle$	6
115	$\langle (x+1)^3 + 2(x+1), 4 \rangle$	8
116	$\langle (x+1)^3 + 2(x+1), 4(x+1) \rangle$	7
117	$\langle (x+1)^3 + 2(x+1), 4(x+1)^2 \rangle$	6
118	$\langle (x+1)^3 + 2(x+1)^2, 4 \rangle$	8
119	$\langle (x+1)^3 + 2(x+1)^2, 4(x+1) \rangle$	7
120	$\langle (x+1)^3 + 2(x+1), 2(x+1)^2 + 4 \rangle$	6
121	$\langle (x+1)^3 + 2(x+1), 2(x+1)^2, 4 \rangle$	8
122	$\langle (x+1)^3 + 2(x+1)^2, 4 \rangle$	8
123	$\langle (x+1)^3 + 2(x+1)^2, 4(x+1) \rangle$	7
124	$\langle (x+1)^3 + 2(x+1) + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
125	$\langle (x+1)^3 + 2(x+1) + 4, 4(x+1) \rangle$	7
126	$\langle (x+1)^3 + 2(x+1) + 4, 4(x+1)^2 \rangle$	6
127	$\langle (x+1)^3 + 2(x+1) + 4(x+1), 4(x+1)^2 \rangle$	6
128	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
129	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2 + 4, 4(x+1)^2 \rangle$	6
130	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2 + 4(x+1), 4(x+1)^2 \rangle$	6
131	$\langle (x+1)^3 + 2(x+1) + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
132	$\langle (x+1)^3 + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
133	$\langle (x+1)^3 + 2(x+1) + 4, 2(x+1)^2 \rangle$	7
134	$\langle (x+1)^3 + 2(x+1) + 4, 2(x+1)^2 + 4 \rangle$	6
135	$\langle (x+1)^3 + 2(x+1) + 4, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
136	$\langle (x+1)^3 + 2(x+1) + 4 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
137	$\langle (x+1)^3 + 2(x+1) + 4 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
138	$\langle (x+1)^3 + 2(x+1) + 4, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
139	$\langle (x+1)^4, 2 \rangle$	8
140	$\langle (x+1)^4, 2(x+1) \rangle$	7
141	$\langle (x+1)^4, 2(x+1), 4 \rangle$	8
142	$\langle (x+1)^4, 2(x+1) + 4 \rangle$	7
143	$\langle (x+1)^4, 2(x+1)^2 \rangle$	6
144	$\langle (x+1)^4, 2(x+1)^2, 4 \rangle$	8
145	$\langle (x+1)^4, 2(x+1)^2, 4(x+1) \rangle$	7
146	$\langle (x+1)^4, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
147	$\langle (x+1)^4, 2(x+1)^2 + 4(x+1) \rangle$	6
148	$\langle (x+1)^4, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
149	$\langle (x+1)^4, 2(x+1)^2 + 4 \rangle$	6
150	$\langle (x+1)^4, 2(x+1)^3 \rangle$	6
151	$\langle (x+1)^4, 2(x+1)^3, 4 \rangle$	8
152	$\langle (x+1)^4, 2(x+1)^3, 4(x+1) \rangle$	7
153	$\langle (x+1)^4, 2(x+1)^3 + 4 \rangle$	7
154	$\langle (x+1)^4, 2(x+1)^3 + 4(x+1) \rangle$	5
155	$\langle (x+1)^4, 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
156	$\langle (x+1)^4, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
157	$\langle (x+1)^4, 4 \rangle$	8
158	$\langle (x+1)^4, 4(x+1) \rangle$	7
159	$\langle (x+1)^4 + 4, 4(x+1) \rangle$	7
160	$\langle (x+1)^4 + 4, 2(x+1) \rangle$	7

161	$\langle (x+1)^4 + 4, 2(x+1) + 4 \rangle$	7
162	$\langle (x+1)^4 + 4, 2(x+1)^2 \rangle$	6
163	$\langle (x+1)^4 + 4, 2(x+1)^2, 4(x+1) \rangle$	7
164	$\langle (x+1)^4 + 4, 2(x+1)^2 + 4 \rangle$	6
165	$\langle (x+1)^4 + 4, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
166	$\langle (x+1)^4 + 4, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
167	$\langle (x+1)^4 + 4, 2(x+1)^2 + 4(x+1) \rangle$	6
168	$\langle (x+1)^4 + 4, 2(x+1)^3 \rangle$	6
169	$\langle (x+1)^4 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
170	$\langle (x+1)^4 + 4, 2(x+1)^3 + 4 \rangle$	7
171	$\langle (x+1)^4 + 4, 2(x+1)^3 + 4(x+1) \rangle$	6
172	$\langle (x+1)^4 + 4(x+1), 2(x+1)^2 \rangle$	6
173	$\langle (x+1)^4 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
174	$\langle (x+1)^4 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
175	$\langle (x+1)^4 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
176	$\langle (x+1)^4 + 4(x+1), 2(x+1)^3 \rangle$	6
177	$\langle (x+1)^4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
178	$\langle (x+1)^4 + 4 + 4(x+1), 2(x+1)^2 \rangle$	6
179	$\langle (x+1)^4 + 4 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
180	$\langle (x+1)^4 + 4 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
181	$\langle (x+1)^4 + 4 + 4(x+1), 2(x+1)^3 \rangle$	6
182	$\langle (x+1)^4 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
183	$\langle (x+1)^4 + 2, 2(x+1) \rangle$	7
184	$\langle (x+1)^4 + 2, 2(x+1)^2 \rangle$	8
185	$\langle (x+1)^4 + 2 + 2(x+1), 2(x+1)^2 \rangle$	8
186	$\langle (x+1)^4 + 2 + 2(x+1), 2(x+1)^3 \rangle$	8
187	$\langle (x+1)^4 + 2, 2(x+1)^3 \rangle$	8
188	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2, 2(x+1)^3 \rangle$	8
189	$\langle (x+1)^4 + 2 + 2(x+1)^2, 2(x+1)^3 \rangle$	8
190	$\langle (x+1)^4 + 2(x+1), 2(x+1)^2 \rangle$	7
191	$\langle (x+1)^4 + 2(x+1), 2(x+1)^3 \rangle$	7
192	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2, 2(x+1)^3 \rangle$	7
193	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3 \rangle$	6
194	$\langle (x+1)^4 + 2, 4 \rangle$	8
195	$\langle (x+1)^4 + 2, 4(x+1) \rangle$	7
196	$\langle (x+1)^4 + 2 + 4, 4(x+1) \rangle$	7
197	$\langle (x+1)^4 + 2 + 2(x+1), 4 \rangle$	8
198	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2, 4 \rangle$	7
199	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 + 4 \rangle$	7
200	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
201	$\langle (x+1)^4 + 2 + 2(x+1) + 2(x+1)^3, 4 \rangle$	8
202	$\langle (x+1)^4 + 2 + 2(x+1)^2, 4 \rangle$	8
203	$\langle (x+1)^4 + 2 + 2(x+1)^2, 4(x+1) \rangle$	7
204	$\langle (x+1)^4 + 2 + 2(x+1)^2, 4(x+1)^2 \rangle$	6
205	$\langle (x+1)^4 + 2 + 2(x+1)^2, 4(x+1)^3 \rangle$	5
206	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
207	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4, 4(x+1)^2 \rangle$	6
208	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4, 4(x+1)^3 \rangle$	5
209	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
210	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
211	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
212	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
213	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1), 4(x+1)^2 \rangle$	6
214	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1), 4(x+1)^3 \rangle$	6
215	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
216	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
217	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3, 4(x+1) \rangle$	7
218	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3, 4(x+1)^2 \rangle$	6
219	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
220	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
221	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
222	$\langle (x+1)^4 + 2 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
223	$\langle (x+1)^4 + 2 + 2(x+1)^3, 4 \rangle$	8

224	$\langle (x+1)^4 + 2 + 2(x+1)^3, 4(x+1) \rangle$	7
225	$\langle (x+1)^4 + 2 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
226	$\langle (x+1)^4 + 2(x+1), 4 \rangle$	8
227	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2, 4 \rangle$	8
228	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^3, 4 \rangle$	8
229	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
230	$\langle (x+1)^4 + 2(x+1), 2(x+1)^2, 4 \rangle$	8
231	$\langle (x+1)^4 + 2(x+1) + 4, 2(x+1)^2 \rangle$	7
232	$\langle (x+1)^4 + 2(x+1) + 4, 2(x+1)^2 + 4 \rangle$	7
233	$\langle (x+1)^4 + 2(x+1), 2(x+1)^2 + 4 \rangle$	7
234	$\langle (x+1)^4 + 2(x+1), 2(x+1)^3, 4 \rangle$	8
235	$\langle (x+1)^4 + 2(x+1), 2(x+1)^3 + 4 \rangle$	7
236	$\langle (x+1)^4 + 2(x+1) + 4, 2(x+1)^3 \rangle$	7
237	$\langle (x+1)^4 + 2(x+1) + 4, 2(x+1)^3 + 4 \rangle$	7
238	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2, 2(x+1)^3 + 4 \rangle$	7
239	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2, 2(x+1)^3, 4 \rangle$	8
240	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^3 + 4 \rangle$	7
241	$\langle (x+1)^4 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^3 \rangle$	7
242	$\langle (x+1)^4 + 2(x+1)^2, 4 \rangle$	8
243	$\langle (x+1)^4 + 2(x+1)^2, 4(x+1) \rangle$	7
244	$\langle (x+1)^4 + 2(x+1)^2, 4(x+1)^2 \rangle$	6
245	$\langle (x+1)^4 + 2(x+1)^2, 4(x+1)^3 \rangle$	5
246	$\langle (x+1)^4 + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
247	$\langle (x+1)^4 + 2(x+1)^2 + 4, 4(x+1)^2 \rangle$	6
248	$\langle (x+1)^4 + 2(x+1)^2 + 4, 4(x+1)^3 \rangle$	5
249	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
250	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
251	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
252	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
253	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1), 4(x+1)^2 \rangle$	6
254	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1), 4(x+1)^3 \rangle$	5
255	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
256	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
257	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
258	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3, 4(x+1) \rangle$	7
259	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3, 4(x+1)^2 \rangle$	6
260	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
261	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
262	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
263	$\langle (x+1)^4 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
264	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3 \rangle$	6
265	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3, 4 \rangle$	8
266	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3, 4(x+1) \rangle$	7
267	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3 + 4 \rangle$	7
268	$\langle (x+1)^4 + 2(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	6
269	$\langle (x+1)^4 + 2(x+1)^2 + 4, 2(x+1)^3 + 4 \rangle$	7
270	$\langle (x+1)^4 + 2(x+1)^2 + 4, 2(x+1)^3 \rangle$	6
271	$\langle (x+1)^4 + 2(x+1)^2 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
272	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 \rangle$	6
273	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 \rangle$	6
274	$\langle (x+1)^4 + 2(x+1)^2 + 4, 2(x+1)^3 + 4(x+1) \rangle$	6
275	$\langle (x+1)^4 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
276	$\langle (x+1)^4 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
277	$\langle (x+1)^5, 2 \rangle$	8
278	$\langle (x+1)^5, 2(x+1) \rangle$	7
279	$\langle (x+1)^5, 2(x+1), 4 \rangle$	8
280	$\langle (x+1)^5, 2(x+1) + 4 \rangle$	7
281	$\langle (x+1)^5, 2(x+1)^2 \rangle$	6
282	$\langle (x+1)^5, 2(x+1)^2, 4 \rangle$	8
283	$\langle (x+1)^5, 2(x+1)^2, 4(x+1) \rangle$	7
284	$\langle (x+1)^5, 2(x+1)^2 + 4 \rangle$	6
285	$\langle (x+1)^5, 2(x+1)^2 + 4(x+1) \rangle$	6
286	$\langle (x+1)^5, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6

287	$\langle (x+1)^5, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
288	$\langle (x+1)^5, 2(x+1)^3 \rangle$	6
289	$\langle (x+1)^5, 2(x+1)^3, 4 \rangle$	8
290	$\langle (x+1)^5, 2(x+1)^3, 4(x+1) \rangle$	7
291	$\langle (x+1)^5, 2(x+1)^3 + 4 \rangle$	7
292	$\langle (x+1)^5, 2(x+1)^3 + 4(x+1) \rangle$	5
293	$\langle (x+1)^5, 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
294	$\langle (x+1)^5, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
295	$\langle (x+1)^5, 4 \rangle$	8
296	$\langle (x+1)^5, 4(x+1) \rangle$	7
297	$\langle (x+1)^5 + 4, 4(x+1) \rangle$	7
298	$\langle (x+1)^5 + 4, 4(x+1)^2 \rangle$	6
299	$\langle (x+1)^5 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
300	$\langle (x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
301	$\langle (x+1)^5 + 4, 2(x+1) \rangle$	7
302	$\langle (x+1)^5 + 4, 2(x+1) + 4 \rangle$	7
303	$\langle (x+1)^5 + 4, 2(x+1)^2 \rangle$	6
304	$\langle (x+1)^5 + 4, 2(x+1)^2 + 4 \rangle$	6
305	$\langle (x+1)^5 + 4, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
306	$\langle (x+1)^5 + 4, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
307	$\langle (x+1)^5 + 4, 2(x+1)^2, 4(x+1) \rangle$	7
308	$\langle (x+1)^5 + 4, 2(x+1)^2 + 4(x+1) \rangle$	6
309	$\langle (x+1)^5 + 4, 2(x+1)^3 \rangle$	6
310	$\langle (x+1)^5 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
311	$\langle (x+1)^5 + 4, 2(x+1)^3 + 4 \rangle$	7
312	$\langle (x+1)^5 + 4(x+1), 2(x+1)^2 \rangle$	6
313	$\langle (x+1)^5 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
314	$\langle (x+1)^5 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
315	$\langle (x+1)^5 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
316	$\langle (x+1)^5 + 4(x+1), 2(x+1)^3 \rangle$	6
317	$\langle (x+1)^5 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
318	$\langle (x+1)^5 + 4(x+1), 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
319	$\langle (x+1)^5 + 4(x+1), 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
320	$\langle (x+1)^5 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
321	$\langle (x+1)^5 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
322	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^2 \rangle$	6
323	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
324	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
325	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
326	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3 \rangle$	6
327	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	5
328	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
329	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1)^2 \rangle$	5
330	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3, 4(x+1)^2 \rangle$	6
331	$\langle (x+1)^5 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
332	$\langle (x+1)^5 + 4 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
333	$\langle (x+1)^5 + 4 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
334	$\langle (x+1)^5 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
335	$\langle (x+1)^5 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
336	$\langle (x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 \rangle$	5
337	$\langle (x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
338	$\langle (x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
339	$\langle (x+1)^5 + 2, 2(x+1) \rangle$	8
340	$\langle (x+1)^5 + 2, 2(x+1)^2 \rangle$	8
341	$\langle (x+1)^5 + 2 + 2(x+1), 2(x+1)^2 \rangle$	8
342	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4 \rangle$	7
343	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^3 \rangle$	7
344	$\langle (x+1)^5 + 2(x+1), 2(x+1)^2 \rangle$	7
345	$\langle (x+1)^5 + 2(x+1), 2(x+1)^3 \rangle$	7
346	$\langle (x+1)^5 + 2(x+1), 2(x+1)^4 \rangle$	7
347	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^4 \rangle$	7
348	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4 \rangle$	7

349	$\langle (x+1)^5 + 2(x+1)^2, 2(x+1)^3 \rangle$	6
350	$\langle (x+1)^5 + 2(x+1), 4 \rangle$	8
351	$\langle (x+1)^5 + 2(x+1), 4(x+1) \rangle$	7
352	$\langle (x+1)^5 + 2(x+1) + 4, 4(x+1) \rangle$	7
353	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 4 \rangle$	8
354	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
355	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4, 4 \rangle$	8
356	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4, 4 \rangle$	8
357	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4 \rangle$	7
358	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4 + 4 \rangle$	7
359	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4 + 4 \rangle$	7
360	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^3, 4 \rangle$	8
361	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^3 + 4 \rangle$	7
362	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^3 \rangle$	7
363	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^3 + 4 \rangle$	7
364	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 4 \rangle$	8
365	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 4(x+1) \rangle$	7
366	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 4(x+1)^2 \rangle$	6
367	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 4(x+1)^3 \rangle$	5
368	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 4(x+1)^4 \rangle$	4
369	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
370	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
371	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
372	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
373	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1), 4(x+1)^3 \rangle$	5
374	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
375	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
376	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2, 4(x+1)^4 \rangle$	4
377	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
378	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
379	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4, 4 \rangle$	8
380	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4, 4(x+1) \rangle$	7
381	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4, 4(x+1)^2 \rangle$	6
382	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
383	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4(x+1), 4(x+1)^2 \rangle$	6
384	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4, 4(x+1) \rangle$	7
385	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
386	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 2(x+1)^4, 4 \rangle$	8
387	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4, 4 \rangle$	8
388	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4, 4(x+1) \rangle$	7
389	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^4 + 4, 4(x+1) \rangle$	7
390	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^2 \rangle$	7
391	$\langle (x+1)^5 + 2(x+1), 2(x+1)^2, 4 \rangle$	8
392	$\langle (x+1)^5 + 2(x+1), 2(x+1)^2 + 4 \rangle$	7
393	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^2 + 4 \rangle$	7
394	$\langle (x+1)^5 + 2(x+1), 2(x+1)^3, 4 \rangle$	8
395	$\langle (x+1)^5 + 2(x+1), 2(x+1)^3 + 4 \rangle$	7
396	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^3 \rangle$	7
397	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^3 + 4 \rangle$	7
398	$\langle (x+1)^5 + 2(x+1), 2(x+1)^4, 4 \rangle$	8
399	$\langle (x+1)^5 + 2(x+1), 2(x+1)^4 + 4 \rangle$	6
400	$\langle (x+1)^5 + 2(x+1), 2(x+1)^4 + 4, 4(x+1) \rangle$	7
401	$\langle (x+1)^5 + 2(x+1), 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
402	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^4 \rangle$	6
403	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^4 + 4 \rangle$	6
404	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
405	$\langle (x+1)^5 + 2(x+1) + 4 + 4(x+1), 2(x+1)^4 + 4 \rangle$	6
406	$\langle (x+1)^5 + 2(x+1) + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
407	$\langle (x+1)^5 + 2(x+1) + 4, 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
408	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^4, 4 \rangle$	8
409	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2, 2(x+1)^4 + 4 \rangle$	7
410	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^4 + 4 \rangle$	7

411	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^2 + 4, 2(x+1)^4 \rangle$	7
412	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4, 4 \rangle$	8
413	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 2(x+1)^4 \rangle$	7
414	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 2(x+1)^4 + 4 \rangle$	5
415	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4 + 4 \rangle$	5
416	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
417	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
418	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
419	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1), 2(x+1)^4 + 4 \rangle$	5
420	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
421	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
422	$\langle (x+1)^5 + 2(x+1)^2, 4 \rangle$	8
423	$\langle (x+1)^5 + 2(x+1)^2, 4(x+1) \rangle$	7
424	$\langle (x+1)^5 + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
425	$\langle (x+1)^5 + 2(x+1)^2, 2(x+1)^3, 4 \rangle$	8
426	$\langle (x+1)^5 + 2(x+1)^2, 2(x+1)^3, 4(x+1) \rangle$	7
427	$\langle (x+1)^5 + 2(x+1)^2 + 4, 2(x+1)^3 \rangle$	5
428	$\langle (x+1)^5 + 2(x+1)^2 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
429	$\langle (x+1)^5 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 \rangle$	6
430	$\langle (x+1)^5 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 \rangle$	6
431	$\langle (x+1)^5 + 2(x+1)^2, 2(x+1)^3 + 4 \rangle$	7
432	$\langle (x+1)^5 + 2(x+1)^2 + 4, 2(x+1)^3 + 4 \rangle$	7
433	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
434	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3, 4(x+1) \rangle$	7
435	$\langle (x+1)^5 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
436	$\langle (x+1)^5 + 2(x+1)^3, 4 \rangle$	8
437	$\langle (x+1)^5 + 2(x+1)^3, 4(x+1) \rangle$	7
438	$\langle (x+1)^5 + 2(x+1)^3, 4(x+1)^2 \rangle$	6
439	$\langle (x+1)^5 + 2(x+1)^3, 4(x+1)^3 \rangle$	5
440	$\langle (x+1)^5 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
441	$\langle (x+1)^5 + 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
442	$\langle (x+1)^5 + 2(x+1)^3 + 4, 4(x+1)^3 \rangle$	5
443	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
444	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1), 4(x+1)^3 \rangle$	5
445	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
446	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
447	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
448	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^4 \rangle$	4
449	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
450	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4, 4(x+1)^3 \rangle$	5
451	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4, 4(x+1)^4 \rangle$	4
452	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
453	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4, 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
454	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3, 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
455	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1), 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
456	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 \rangle$	5
457	$\langle (x+1)^5 + 2(x+1) + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	5
458	$\langle (x+1)^5 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
459	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
460	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
461	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
462	$\langle (x+1)^5 + 2(x+1)^3 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
463	$\langle (x+1)^6, 2 \rangle$	8
464	$\langle (x+1)^6, 2(x+1) \rangle$	7
465	$\langle (x+1)^6, 2(x+1)^2 \rangle$	6
466	$\langle (x+1)^6, 2(x+1)^3 \rangle$	6
467	$\langle (x+1)^6, 2(x+1), 4 \rangle$	8
468	$\langle (x+1)^6, 2(x+1) + 4 \rangle$	7
469	$\langle (x+1)^6, 2(x+1)^2, 4(x+1) \rangle$	7
470	$\langle (x+1)^6, 2(x+1)^2, 4 \rangle$	8
471	$\langle (x+1)^6, 2(x+1)^2 + 4(x+1) \rangle$	6
472	$\langle (x+1)^6, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
473	$\langle (x+1)^6, 2(x+1)^2 + 4, 4(x+1) \rangle$	7

474	$\langle (x+1)^6, 2(x+1)^2 + 4 \rangle$	6
475	$\langle (x+1)^6, 2(x+1)^3, 4 \rangle$	8
476	$\langle (x+1)^6, 2(x+1)^3, 4(x+1) \rangle$	7
477	$\langle (x+1)^6, 2(x+1)^3 + 4 \rangle$	7
478	$\langle (x+1)^6, 2(x+1)^3 + 4(x+1) \rangle$	5
479	$\langle (x+1)^6, 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
480	$\langle (x+1)^6, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
481	$\langle (x+1)^6, 4 \rangle$	8
482	$\langle (x+1)^6, 4(x+1) \rangle$	7
483	$\langle (x+1)^6, 4(x+1)^2 \rangle$	6
484	$\langle (x+1)^6, 4(x+1)^3 \rangle$	7
485	$\langle (x+1)^6 + 4, 2(x+1) \rangle$	7
486	$\langle (x+1)^6 + 4, 2(x+1)^2 \rangle$	6
487	$\langle (x+1)^6 + 4, 2(x+1)^3 \rangle$	6
488	$\langle (x+1)^6 + 4, 2(x+1) + 4 \rangle$	7
489	$\langle (x+1)^6 + 4, 2(x+1)^2, 4(x+1) \rangle$	7
490	$\langle (x+1)^6 + 4, 2(x+1)^2 + 4 \rangle$	6
491	$\langle (x+1)^6 + 4, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
492	$\langle (x+1)^6 + 4, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
493	$\langle (x+1)^6 + 4, 2(x+1)^2 + 4(x+1) \rangle$	6
494	$\langle (x+1)^6 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
495	$\langle (x+1)^6 + 4, 2(x+1)^3 + 4 \rangle$	7
496	$\langle (x+1)^6 + 4, 4(x+1) \rangle$	7
497	$\langle (x+1)^6 + 4, 4(x+1)^2 \rangle$	6
498	$\langle (x+1)^6 + 4, 4(x+1)^3 \rangle$	5
499	$\langle (x+1)^6 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
500	$\langle (x+1)^6 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
501	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
502	$\langle (x+1)^6 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
503	$\langle (x+1)^6 + 4(x+1), 4(x+1)^2 \rangle$	6
504	$\langle (x+1)^6 + 4(x+1), 4(x+1)^3 \rangle$	5
505	$\langle (x+1)^6 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
506	$\langle (x+1)^6 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
507	$\langle (x+1)^6 + 4 + 4(x+1), 2(x+1)^2 \rangle$	6
508	$\langle (x+1)^6 + 4 + 4(x+1), 2(x+1)^3 \rangle$	5
509	$\langle (x+1)^6 + 4 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
510	$\langle (x+1)^6 + 4 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
511	$\langle (x+1)^6 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
512	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 \rangle$	6
513	$\langle (x+1)^6 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1)^2 \rangle$	5
514	$\langle (x+1)^6 + 4(x+1), 2(x+1)^2 \rangle$	6
515	$\langle (x+1)^6 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
516	$\langle (x+1)^6 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
517	$\langle (x+1)^6 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
518	$\langle (x+1)^6 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
519	$\langle (x+1)^6 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
520	$\langle (x+1)^6 + 4 + 4(x+1)^2, 2(x+1)^3 \rangle$	5
521	$\langle (x+1)^6 + 4 + 4(x+1)^2, 2(x+1)^3 + 4(x+1)^2 \rangle$	5
522	$\langle (x+1)^6 + 2, 2(x+1) \rangle$	8
523	$\langle (x+1)^6 + 2(x+1), 2(x+1)^2 \rangle$	7
524	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 \rangle$	6
525	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4 \rangle$	6
526	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5 \rangle$	6
527	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4 \rangle$	6
528	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 \rangle$	6
529	$\langle (x+1)^6 + 2(x+1), 4 \rangle$	8
530	$\langle (x+1)^6 + 2(x+1) + 2(x+1)^2, 4 \rangle$	8
531	$\langle (x+1)^6 + 2(x+1), 2(x+1)^2, 4 \rangle$	8
532	$\langle (x+1)^6 + 2(x+1) + 4, 2(x+1)^2 \rangle$	7
533	$\langle (x+1)^6 + 2(x+1) + 4, 2(x+1)^2 + 4 \rangle$	7
534	$\langle (x+1)^6 + 2(x+1), 2(x+1)^2 + 4 \rangle$	7
535	$\langle (x+1)^6 + 2(x+1)^2, 4 \rangle$	8
536	$\langle (x+1)^6 + 2(x+1)^2, 4(x+1) \rangle$	7

537	$\langle (x+1)^6 + 2(x+1)^2 + 4, 4(x+1) \rangle$	7
538	$\langle (x+1)^6 + 2(x+1)^2 + 4, 4(x+1)^2 \rangle$	6
539	$\langle (x+1)^6 + 2(x+1)^2 + 4, 4(x+1)^3 \rangle$	5
540	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
541	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
542	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 \rangle$	6
543	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3, 4 \rangle$	8
544	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3, 4(x+1) \rangle$	7
545	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3 \rangle$	6
546	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3, 4(x+1) \rangle$	7
547	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 \rangle$	6
548	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 \rangle$	6
549	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 + 4 \rangle$	6
550	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 + 4, 4(x+1) \rangle$	7
551	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
552	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3 + 4 \rangle$	6
553	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3 + 4, 4(x+1) \rangle$	7
554	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
555	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 + 4 \rangle$	6
556	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
557	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 + 4 \rangle$	6
558	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
559	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	6
560	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^3 + 4(x+1) \rangle$	6
561	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
562	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
563	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4, 4 \rangle$	8
564	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4, 4(x+1) \rangle$	7
565	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 \rangle$	6
566	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 \rangle$	6
567	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^4 \rangle$	6
568	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4, 4(x+1) \rangle$	7
569	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4 + 4 \rangle$	6
570	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
571	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
572	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^4 + 4(x+1) \rangle$	6
573	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 + 4 \rangle$	4
574	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
575	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
576	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 + 4, 4(x+1)^3 \rangle$	5
577	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4 \rangle$	5
578	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) \rangle$	5
579	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
580	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
581	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 4(x+1)^4 \rangle$	4
582	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
583	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 4(x+1)^5 \rangle$	3
584	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
585	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^4, 4(x+1)^5 \rangle$	3
586	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	5
587	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 \rangle$	5
588	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) \rangle$	5
589	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
590	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	5
591	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 \rangle$	4
592	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4, 4(x+1)^3 \rangle$	5
593	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) \rangle$	5
594	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
595	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	4
596	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
597	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
598	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^3 \rangle$	4
599	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3, 2(x+1)^4 + 4 \rangle$	4

600	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	4
601	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
602	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^2 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^3 \rangle$	4
603	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^3, 2(x+1)^4 + 4 \rangle$	4
604	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	4
605	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
606	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1)^3, 2(x+1)^4 + 4 + 4(x+1)^3 \rangle$	4
607	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^4 + 4(x+1) \rangle$	6
608	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^4 + 4(x+1) \rangle$	6
609	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^4 + 4(x+1) \rangle$	6
610	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5 \rangle$	6
611	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5, 4 \rangle$	8
612	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5, 4(x+1) \rangle$	7
613	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 \rangle$	6
614	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^5 \rangle$	6
615	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^5 \rangle$	6
616	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5 + 4 \rangle$	7
617	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4 \rangle$	7
618	$\langle (x+1)^6 + 2(x+1)^2, 2(x+1)^5 + 4(x+1) \rangle$	6
619	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4(x+1) \rangle$	4
620	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
621	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4(x+1), 4(x+1)^3 \rangle$	5
622	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
623	$\langle (x+1)^6 + 2(x+1)^2 + 4, 2(x+1)^5 + 4(x+1) + 4(x+1)^3 \rangle$	4
624	$\langle (x+1)^6 + 2(x+1)^2 + 4 + 4(x+1), 2(x+1)^5 + 4(x+1) \rangle$	5
625	$\langle (x+1)^6 + 2(x+1)^2 + 4(x+1), 2(x+1)^5 + 4(x+1) \rangle$	6
626	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 4 \rangle$	8
627	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 4(x+1) \rangle$	7
628	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
629	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4, 4 \rangle$	8
630	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4, 4(x+1) \rangle$	7
631	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4 \rangle$	6
632	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4, 4(x+1) \rangle$	7
633	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1), 2(x+1)^4 \rangle$	6
634	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 \rangle$	6
635	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4 + 4 \rangle$	7
636	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4 + 4 \rangle$	7
637	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3, 2(x+1)^4 + 4(x+1) \rangle$	6
638	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4, 2(x+1)^4 + 4(x+1) \rangle$	6
639	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4 + 4(x+1), 2(x+1)^4 + 4(x+1) \rangle$	6
640	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4(x+1) \rangle$	6
641	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4, 4 \rangle$	8
642	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4, 4(x+1) \rangle$	7
643	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^3 + 2(x+1)^4 + 4, 4(x+1) \rangle$	7
644	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4 \rangle$	8
645	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4(x+1) \rangle$	7
646	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4(x+1)^2 \rangle$	6
647	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4(x+1)^3 \rangle$	5
648	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4(x+1)^4 \rangle$	4
649	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 4(x+1)^5 \rangle$	3
650	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4, 4(x+1) \rangle$	7
651	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1), 4(x+1)^2 \rangle$	6
652	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
653	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
654	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1)^4, 4(x+1)^5 \rangle$	3
655	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5, 4 \rangle$	8
656	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5, 4(x+1) \rangle$	7
657	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5, 4(x+1)^2 \rangle$	6
658	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4, 4(x+1) \rangle$	7
659	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
660	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4(x+1), 4(x+1)^3 \rangle$	5
661	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4(x+1), 4(x+1)^4 \rangle$	4
662	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4(x+1), 4(x+1)^5 \rangle$	3

663	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 2(x+1)^5 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
664	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5, 4 \rangle$	8
665	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5, 4(x+1) \rangle$	7
666	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4, 2(x+1)^5 \rangle$	6
667	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4, 2(x+1)^5, 4(x+1) \rangle$	7
668	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1), 2(x+1)^5 \rangle$	6
669	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4 + 4(x+1), 2(x+1)^5 \rangle$	6
670	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4 \rangle$	7
671	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4, 2(x+1)^5 + 4 \rangle$	7
672	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4(x+1) \rangle$	4
673	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
674	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4(x+1), 4(x+1)^3 \rangle$	5
675	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
676	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4, 2(x+1)^5 + 4(x+1) + 4(x+1)^3 \rangle$	4
678	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4, 2(x+1)^5 + 4(x+1) \rangle$	6
679	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4 + 4(x+1), 2(x+1)^5 + 4(x+1) \rangle$	6
680	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1), 2(x+1)^5 + 4(x+1) \rangle$	5
681	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1), 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
682	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 + 4(x+1) \rangle$	5
683	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1), 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
684	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^4 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
685	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5, 4 \rangle$	8
686	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5, 4(x+1) \rangle$	7
687	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4, 4(x+1) \rangle$	7
688	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4, 4(x+1)^2 \rangle$	6
689	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
690	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
691	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
692	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^4 \rangle$	4
693	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^5 \rangle$	3
694	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
695	$\langle (x+1)^6 + 2(x+1)^2 + 2(x+1)^5 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^4, 4(x+1)^5 \rangle$	3
696	$\langle (x+1)^6 + 2(x+1)^3, 4 \rangle$	8
697	$\langle (x+1)^6 + 2(x+1)^3, 4(x+1) \rangle$	7
698	$\langle (x+1)^6 + 2(x+1)^3, 4(x+1)^2 \rangle$	6
699	$\langle (x+1)^6 + 2(x+1)^3 + 4, 4(x+1) \rangle$	7
700	$\langle (x+1)^6 + 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
701	$\langle (x+1)^6 + 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
702	$\langle (x+1)^6 + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
703	$\langle (x+1)^7, 2 \rangle$	8
704	$\langle (x+1)^7, 2(x+1) \rangle$	7
705	$\langle (x+1)^7, 2(x+1)^2 \rangle$	6
706	$\langle (x+1)^7, 2(x+1)^3 \rangle$	6
707	$\langle (x+1)^7, 2(x+1), 4 \rangle$	8
708	$\langle (x+1)^7, 2(x+1) + 4 \rangle$	7
709	$\langle (x+1)^7, 2(x+1)^2, 4(x+1) \rangle$	7
710	$\langle (x+1)^7, 2(x+1)^2, 4 \rangle$	8
711	$\langle (x+1)^7, 2(x+1)^2 + 4(x+1) \rangle$	6
712	$\langle (x+1)^7, 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
713	$\langle (x+1)^7, 2(x+1)^2 + 4, 4(x+1) \rangle$	7
714	$\langle (x+1)^7, 2(x+1)^2 + 4 \rangle$	6
715	$\langle (x+1)^7, 2(x+1)^3, 4 \rangle$	8
716	$\langle (x+1)^7, 2(x+1)^3, 4(x+1) \rangle$	7
717	$\langle (x+1)^7, 2(x+1)^3 + 4 \rangle$	7
718	$\langle (x+1)^7, 2(x+1)^3 + 4(x+1) \rangle$	5
719	$\langle (x+1)^7, 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
720	$\langle (x+1)^7, 2(x+1)^3 + 4(x+1) + 4(x+1)^2 \rangle$	5
721	$\langle (x+1)^7, 4 \rangle$	8
722	$\langle (x+1)^7, 4(x+1) \rangle$	7
723	$\langle (x+1)^7, 4(x+1)^2 \rangle$	6
724	$\langle (x+1)^7, 4(x+1)^3 \rangle$	5
725	$\langle (x+1)^7 + 4, 4(x+1) \rangle$	7
726	$\langle (x+1)^7 + 4, 4(x+1)^2 \rangle$	6

727	$\langle (x+1)^7 + 4, 4(x+1)^3 \rangle$	5
728	$\langle (x+1)^7 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
729	$\langle (x+1)^7 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
730	$\langle (x+1)^7 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
731	$\langle (x+1)^7 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
732	$\langle (x+1)^7 + 4(x+1), 4(x+1)^2 \rangle$	6
733	$\langle (x+1)^7 + 4(x+1), 4(x+1)^3 \rangle$	5
734	$\langle (x+1)^7 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
735	$\langle (x+1)^7 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
736	$\langle (x+1)^7 + 4, 2(x+1) \rangle$	7
737	$\langle (x+1)^7 + 4, 2(x+1)^2 \rangle$	7
738	$\langle (x+1)^7 + 4, 2(x+1)^3 \rangle$	7
739	$\langle (x+1)^7 + 4, 2(x+1) + 4 \rangle$	7
740	$\langle (x+1)^7 + 4, 2(x+1)^2 + 4 \rangle$	7
741	$\langle (x+1)^7 + 4, 2(x+1)^3 + 4 \rangle$	6
742	$\langle (x+1)^7 + 4, 2(x+1)^3 + 4, 4(x+1) \rangle$	7
743	$\langle (x+1)^7 + 4, 2(x+1)^3 + 4 + 4(x+1) \rangle$	5
744	$\langle (x+1)^7 + 4 + 4(x+1), 2(x+1)^3 + 4 \rangle$	5
745	$\langle (x+1)^7 + 4 + 4(x+1), 2(x+1)^3 + 4 + 4(x+1) \rangle$	6
746	$\langle (x+1)^7 + 4 + 4(x+1), 2(x+1)^3 + 4, 4(x+1)^2 \rangle$	6
747	$\langle (x+1)^7 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4 \rangle$	5
748	$\langle (x+1)^7 + 4 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4 + 4(x+1)^2 \rangle$	5
749	$\langle (x+1)^7 + 4 + 4(x+1)^2, 2(x+1)^3 + 4 + 4(x+1) \rangle$	5
750	$\langle (x+1)^7 + 4(x+1), 2(x+1)^2 \rangle$	6
751	$\langle (x+1)^7 + 4(x+1), 2(x+1)^2 + 4 \rangle$	6
752	$\langle (x+1)^7 + 4(x+1), 2(x+1)^2 + 4 + 4(x+1) \rangle$	6
753	$\langle (x+1)^7 + 4(x+1), 2(x+1)^2 + 4(x+1) \rangle$	6
754	$\langle (x+1)^7 + 4(x+1), 2(x+1)^3 \rangle$	5
755	$\langle (x+1)^7 + 4(x+1), 2(x+1)^3, 4(x+1)^2 \rangle$	6
756	$\langle (x+1)^7 + 4(x+1), 2(x+1)^3 + 4(x+1) \rangle$	6
757	$\langle (x+1)^7 + 4(x+1), 2(x+1)^3 + 4(x+1)^2 \rangle$	5
758	$\langle (x+1)^7 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 \rangle$	5
758	$\langle (x+1)^7 + 4(x+1) + 4(x+1)^2, 2(x+1)^3 + 4(x+1)^2 \rangle$	5
759	$\langle (x+1)^7 + 4(x+1)^2, 2(x+1)^3 + 4(x+1) \rangle$	5
760	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4 \rangle$	6
761	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5 \rangle$	6
762	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^6 \rangle$	6
763	$\langle (x+1)^7 + 2(x+1), 4 \rangle$	8
764	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^4, 4(x+1)^2 \rangle$	6
765	$\langle (x+1)^7 + 2(x+1)^2, 4 \rangle$	8
766	$\langle (x+1)^7 + 2(x+1)^2, 4(x+1) \rangle$	7
767	$\langle (x+1)^7 + 2(x+1)^3, 4 \rangle$	8
768	$\langle (x+1)^7 + 2(x+1)^3, 4(x+1) \rangle$	7
769	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 4(x+1)^2 \rangle$	6
770	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 4(x+1)^3 \rangle$	5
771	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^4, 4 \rangle$	8
772	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^4, 4(x+1) \rangle$	7
773	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^4 + 4(x+1), 4(x+1)^2 \rangle$	6
774	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4, 4 \rangle$	8
775	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4, 4(x+1) \rangle$	7
776	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4 + 4 \rangle$	6
777	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4 + 4, 4(x+1) \rangle$	7
778	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4 + 4(x+1) \rangle$	6
779	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^4 \rangle$	7
780	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^4 + 4 \rangle$	7
781	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 \rangle$	5
782	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4, 4(x+1)^2 \rangle$	6
783	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 \rangle$	5
784	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 \rangle$	4
785	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4, 4(x+1)^2 \rangle$	6
786	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4, 4(x+1)^3 \rangle$	5
787	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) \rangle$	4
788	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6

789	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1), 4(x+1)^3 \rangle$	5
790	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	4
791	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
792	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^3 \rangle$	4
793	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	4
794	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
795	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1)^3 \rangle$	4
796	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
797	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
798	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4(x+1) \rangle$	5
799	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4(x+1), 4(x+1)^2 \rangle$	6
800	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4(x+1) + 4(x+1)^2 \rangle$	5
801	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^4 + 4(x+1)^2 \rangle$	5
802	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 \rangle$	5
803	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 \rangle$	5
804	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) \rangle$	5
805	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
806	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4 + 4(x+1)^2 \rangle$	5
807	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4(x+1) \rangle$	5
808	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4(x+1) + 4(x+1)^2 \rangle$	5
809	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^4 + 4(x+1)^2 \rangle$	5
810	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5, 4 \rangle$	8
811	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5, 4(x+1) \rangle$	7
812	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5 + 4 \rangle$	6
813	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5 + 4, 4(x+1) \rangle$	7
814	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5 + 4 + 4(x+1) \rangle$	6
815	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^5 + 4(x+1) \rangle$	6
816	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^5 \rangle$	7
817	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^5 + 4 \rangle$	7
818	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 \rangle$	5
819	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4 \rangle$	6
820	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4 + 4(x+1) \rangle$	6
821	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1) \rangle$	4
822	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
823	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1), 4(x+1)^3 \rangle$	5
824	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	4
825	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1) + 4(x+1)^2 + 4(x+1)^3 \rangle$	4
826	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^5 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5
827	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 \rangle$	5
828	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 + 4(x+1) \rangle$	5
829	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5
830	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1) + 4(x+1)^2, 2(x+1)^5 + 4(x+1)^2 \rangle$	5
831	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^6, 4 \rangle$	8
832	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^6, 4(x+1) \rangle$	7
833	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^6 + 4 \rangle$	7
834	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^6 + 4(x+1) \rangle$	6
835	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^6 \rangle$	7
836	$\langle (x+1)^7 + 2(x+1)^3 + 4, 2(x+1)^6 + 4 \rangle$	7
837	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^6 \rangle$	5
838	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^6, 4(x+1)^2 \rangle$	6
839	$\langle (x+1)^7 + 2(x+1)^3 + 4(x+1), 2(x+1)^6 + 4(x+1) \rangle$	6
840	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^6, 4 \rangle$	8
841	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^6, 4(x+1) \rangle$	7
842	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^6 + 4(x+1), 4(x+1)^2 \rangle$	6
843	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4 \rangle$	8
844	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4(x+1) \rangle$	7
845	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4(x+1)^2 \rangle$	6
846	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4(x+1)^3 \rangle$	5
847	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4(x+1)^4 \rangle$	4
848	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5, 4(x+1)^5 \rangle$	3
849	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4, 4(x+1) \rangle$	7
850	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4(x+1), 4(x+1)^2 \rangle$	6
851	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4(x+1) + 4(x+1)^2 \rangle$	5

852	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4(x+1)^2, 4(x+1)^3 \rangle$	5
853	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4(x+1)^3, 4(x+1)^4 \rangle$	4
854	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^5 + 4(x+1)^4, 4(x+1)^5 \rangle$	3
855	$\langle (x+1)^7 + 2(x+1)^3, 2(x+1)^4 + 4 + 4(x+1) \rangle$	6
856	$\langle (x+1)^7 + 4, 2(x+1)^3 + 4 + 4(x+1), 4(x+1)^2 \rangle$	6
857	$\langle (x+1)^7 + 4, 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
858	$\langle (x+1)^7 + 4 + 4(x+1), 2(x+1)^3 + 4 + 4(x+1)^2 \rangle$	5
859	$\langle (x+1)^7 + 4 + 4(x+1)^2, 2(x+1)^3 + 4 + 4(x+1) + 4(x+1)^2 \rangle$	5
860	$\langle (x+1)^7 + 2(x+1)^3 + 2(x+1)^6 + 4(x+1) + 4(x+1)^2, 4(x+1)^3 \rangle$	5

IV. CONCLUSION

On the basis of the structure, generators and degree of minimal degree polynomials of a cyclic code of length 2^k over $\mathbb{Z}_8$ given in [6, 7, 8], all non principal cyclic codes have been determined. The rank of such codes have also been obtained. It has been found that there are 860 non principal cyclic codes of length 8 over $\mathbb{Z}_8$, out of which 59 are non principal cyclic codes with leading coefficient 2 or 6 and 801 are non principal monic cyclic codes.

REFERENCES

- [1] Abualrub, T., Oehmke, R. 2003. On Generators of $\mathbb{Z}_4$ Cyclic codes of length 2^e . IEEE Transactions On Information Theory. 49 (9), 2126–2133.
- [2] Abualrub, T., Ghayeb, A., Oehmke, R. 2004. A Mass Formula and Rank of $\mathbb{Z}_4$ Cyclic codes of Length 2^e . IEEE Transactions on Information Theory. 50(12), 3306–3312.
- [3] Dinh, H. Q. 2008. On the linear ordering of some classes of negacyclic and cyclic codes and their distance distributions. Finite Fields Appl., 14(1), 22–40.
- [4] Dinh, H. Q., Lopez Permouth, S. R. 2004. Cyclic and negacyclic codes over finite chain rings. IEEE Trans. Inform. Theory, 50(8), 1728–1744.
- [5] Dougherty S. T., Park Y. H. 2007. On modular cyclic codes. Finite Fields and Their Applications 13, 31 – 57.
- [6] Garg, A., Dutt, S. 2012 Cyclic codes of length 2^k over $\mathbb{Z}_8$. Scientific Research Open Journal of Applied Science Oct - 2012 world Congress on Engineering and Technology. 2, 104 – 107.
- [7] Garg, A., Dutt, S. 2017. On Rank and MDR Cyclic codes of length 2^k over $\mathbb{Z}_8$. LNCS 10156, Algorithms and Discrete Applied Mathematics (CALDAM 2017). 177 – 186.
- [8] Garg A., Dutt, S. 2018. Determining Minimal Degree Polynomials of a Cyclic code of length 2^k over $\mathbb{Z}_8$. LNCS 10743, Algorithms and Discrete Applied Mathematics (CALDAM 2018). 118 – 130.
- [9] Kiah, H.M., Leung, K.H., Ling, S. 2008 Cyclic codes over $GR(p^2, m)$ of length p^k . Finite Fields and their application. 14, 834–846.
- [10] Minjia, S., Shixin, Z. 2008. Cyclic Codes Over The Ring $\mathbb{Z}_{p^2}$ of Length p^e . Journal Of Electronics (China). 25(5), 636–640 .
- [11] Salagean, A. 2006. Repeated-root cyclic and negacyclic codes over a finite chain ring, Discrete Appl. Math., 154(2), 413–419.