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PULSATILE FLOW OF BLOOD IN CAPILLARIES OF SMALL EXPONENTIAL DIVERGENCE WITH MICRO-ORGANISMS AND GRAVITY EFFECT

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Abstract

This paper is concerned with a mathematical analysis of the flow of blood with microorganisms in capillaries of small exponential divergence. A micro-continum approach is used to determine the velocity and flow rate distribution in the tube which taken to diverge exponentially. Analytical expression for velocity and flow rates for both blood and microorganisms increases with a small increase in the non-dimensionalized axial co-ordinate for small values of nt(n-circular frequency, t- time coordinate). It is observed that there is backflow for blood. The phase lag between pressure gradient and flow rate curve of microorganisms is more as compared to blood and it decreases with increase in rate of divergence parameter.

INTRODUCTION:

Problems involving pulsatile fluid motion in closed cylindrical channel are receiving widespread consideration by investigation seeking to describe the dynamics of blood flow in the circulatory system. Blasius [3] considered the flow in tubes with small angle of divergence. Harrison [8] investigated the pressure distribution in a diverging tube. Bond [4] experimented with in compressible Newtonian fluids in cones with large angle of convergence. Abramawitz [1] extended Blasius[3] work and Williams [15] generalized the study for slender tubes by using a similar technique, which produced solutions for converging/diverging, compressible/incompressible flow with various wall profile shapes. Abu-Sitta and Drake [2] have developed analytical solutions, which can be applied to rigid cylindrical ducts of any arbitrary cross-section, so long as the specified shape does not vary along the axial direction. A series solution for both the converging and diverging axisymmetric flow of an incompressible Newtonian fluid was developed by Forester and Young [6]. Sechneck and Walburn [12] have investigated blood flow through divergent channel, obtained linearised solutions for low mean Reynolds number and observed that viscous effects lead to radially dependent phase shifts between different layers of fluid oscillating in the axial direction with phase lags between flow and pressure curves. The vascular resistance of micro-vessels in living tissues with non-uniform cross-section can be more elevated by the presence of blood vessels was reported by Masako Seki [9]. Further they have observed that when a red cell encounters a region of capillary narrowing of lubrication layer. Kareh and Secomb [5] have studied radial motion in diverging capillary bifurcations, as the red cells pass near the dividing surface influences the distribution of orientations in the downstream branches and it affects cell distribution in subsequent bifurcations. Misra and Ghosh [10] used a micro-continum approach to determine the velocity and pressure distribution in exponentially diverging channel.

An attempt is made to study analytically the pulsatile flow of blood in capillaries of small exponential divergence with microorganisms.

FORMULATION OF THE PROBLEM:

The governing equations for unsteady, viscous, incompressible couple-stress fluid and microorganisms through a circular pipe are given by (combining Stokes' [13] and Saffman [11] equation),

$$(1-\phi)\rho\left\{\frac{\partial U}{\partial t}+(V.\nabla_1)U\right\}=(1-\phi)\left\{-gradP+\mu\nabla_1^2U-n\nabla_1^4U\right\}+KN(V-U)+g\sin\psi \quad \textbf{(1)}$$

$$\left\{\frac{\partial V}{\partial t}+(V.\nabla_1)V\right\}=\phi\left\{-gradP+\mu\nabla_1^2U-n\nabla_1^4U\right\}+\frac{K}{m}(U-V)+g\sin\psi \quad \textbf{(2)}$$

$$divU=0 \quad (3)$$

$$\frac{\partial N}{\partial t}+div(NV)=0 \quad (4)$$

Where U and V denote the local velocity vectors of fluid and microorganisms respectively, ρ - density, P- static fluid pressure, ν - kinematic viscosity, N - number density of micro-organisms, K- Stoke's resistance coefficient ($6\pi\mu\varepsilon_1, \varepsilon_1$- the radius of spherical particles), μ - the fluid viscosity, m – mass of microorganisms, g – gravity, ϕ - volume fraction and $\nabla_1^2=\left(\left(\frac{1}{r}\right)\frac{\partial}{\partial r}\right)\left(r\frac{\partial}{\partial r}\right)$

In the present analysis, the fluid is assumed to be homogeneous, isotropic, incompressible couple-stress fluid with microorganisms having a constant viscosity and density under the influence of pressure gradient. It is considered to be undergoing periodic pulsatile motion through a rigid circular channel with impermeable walls, which diverge exponentially as $R(z)=\frac{R_0}{2}\left(1+e^{\frac{\varepsilon z}{z_0}}\right)$ (5)

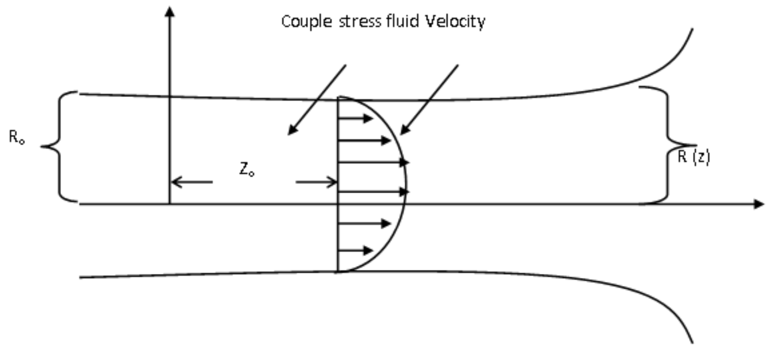

Figure 1: Geometry of the blood flow

Where R_0 is the tube radius at the axial co-ordinate Z = 0, ε the divergence parameter over a characteristic axial distance Z_0 and is considered to be very less than one. The velocity distribution is taking along Z-axial and is considered to be function of r, θ and t. with the above following assumptions, equations for the blood and microorganisms reduces to

$$(1-\phi)\rho\frac{\partial w_1}{\partial t}=(1-\phi)\left[-\frac{\partial p}{\partial z}+\mu\nabla_1^2 w_1-\eta\nabla_1^4 w_1\right]+KN(w_2-w_1)+g\sin\psi \tag{6}$$

$$\frac{\partial w_2}{\partial t}=\phi\left[-\frac{\partial p}{\partial z}+\mu\nabla_1^2 w_1-\eta\nabla_1^4 w_1\right]+\frac{K}{m}(w_1-w_2)+g\sin\psi \tag{7}$$

Where w_1 and w_2 are the velocity components of blood and microorganisms respectively.

The boundary conditions are

w_1, w_2 and $\nabla_1^2(w_1, w_2)$ are all finite at r = 0

$w_1 = 0 = w_2$ and $\nabla_1^2(w_1, w_2) = 0$ at r = R (8)

Introduce the non-dimensional quantities,

$$u=\frac{w_1 n\rho}{M},\ v=\frac{w_2 n\rho}{M},\ y=\frac{r}{R},\ g^*=\frac{g\rho R^2 n}{\mu M},\phi^*=\frac{\phi}{b}$$

$$\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}=\frac{1}{R^2}\left[\frac{\partial^2}{\partial y^2}+\frac{1}{y}\frac{\partial}{\partial y}\right]=\frac{1}{R^2}\nabla^2 \tag{9}$$

And assuming Womersley [16] type of pressure gradient of the form

$$-\frac{\partial p}{\partial z}=M\cos(nt+\theta) \tag{10}$$

Where M is the modulus, n is the circular frequency and θ is the phase in the complex representation of the Fourier coefficient of the velocity analogous to the pressure gradient gives,

$$u=u_0(y)e^{\text{int}}$$

$$v=v_0(y)e^{\text{int}} \tag{11}$$

Using equations (4), (6) and (7), the equations (1) and (2) reduce to

$$(1-\phi)\left\{\nabla^4 u_0 - \bar{\alpha}^2 \nabla^2 u_0 - \bar{\alpha}^2 \alpha^2 e^{i\theta} + i\bar{\alpha}^2 \alpha^2 u_0\right\} - \frac{\bar{\alpha}^2 b}{\beta}(v_0 - u_0) - \bar{\alpha}^2 g \sin\psi = 0 \qquad (13)$$

$$v_0 = \frac{-\phi\beta}{\bar{\alpha}^2 b(1+i\alpha^2\beta)}\left\{\nabla^4 u_0 - \bar{\alpha}^2 \nabla^2 u_0 - \bar{\alpha}^2 \alpha^2 e^{i\theta}\right\} + \frac{u_0}{1+i\alpha^2\beta} + \frac{\beta g \sin\psi}{b(1+i\alpha^2\beta)} \qquad (14)$$

Where $y_1 = R(z)/R_0$ the non-dimentionalized axial co-ordinate, $\bar{\alpha}^2 = \frac{R^2\mu}{\eta}$ is the couple – stress parameter, $\alpha^2 = \frac{R^2 n}{\nu}$ is the Pulsatile Reynolds' number, $b = \frac{mN}{\rho}$ is the mass concentration, $\beta = \frac{m\nu}{KR^2}$ is the relaxation time parameter, $i = (-1)^{1/2}$, $\nu = \frac{\mu}{\rho}$ is kinematics viscosity.

The modified boundary conditions are

u_0, v_0 and $\nabla^2(u_0, v_0)$ are all finite at $y = 0$

$u_0 = 0 = v_0$ and $\nabla^2(u_0, v_0) = 0$ at $y = y_1$ (15)

Solution of Problem:

To solve the fourth order equation in cylindrical polar co-ordinates, the finite Hankel transform technique is applied and a per Tranter[14], it is defined as

$$\overline{u_0}(k) = \int_0^{y_1} u_0(y) y J_0(ky)\,dy \qquad (16)$$

Where k's are the positive roots of the equation

$J_0(ky_1) = 0$

$J_0(ky_1)$ Denotes the Bessel function of the first kind of order zero

The Inverse transform is given by

$$\overline{u_0}(y) = \frac{2}{y_1^2}\sum_k \frac{\overline{u_0}(k) J_0(ky)}{J_1^2(ky_1)} \qquad (17)$$

Applying equation (16), (17) and using boundary conditions (15), the equation (13) yields velocity expression for blood

$$u(y) = \frac{2}{y_1}\sum_k \left\{\frac{\bar{\alpha}^2 \alpha^2 e^{i(nt+\theta)} + L}{\left[k(k^4 + \bar{\alpha}^2 k^2 + X)\right]}\right\}\frac{J_0(ky)}{J_1(ky_1)} \qquad (18)$$

Where $L = \frac{g\bar{\alpha}^2 \sin\psi e^{\text{int}}(2+i\alpha^2\beta)}{1+i\alpha^2\beta(1-\phi b)}$ & $X = \frac{i\bar{\alpha}^2\alpha^2\left[(1-\phi)(1+i\alpha^2\beta)+b\right]}{1+i\alpha^2\beta(1-\phi b)}$ (19)

Using the equation (18) and (10) in (12), we get the velocity for microorganisms in non-dimensional form as

$$v(y)=\frac{2}{y_1}\sum_k\left[\left(\frac{\beta g\sin\psi}{b(1+i\alpha^2\beta)}-M^*L\right)+\frac{\bar{\alpha}^2\alpha^2e^{i(nt+\theta)}+L}{\left[k(k^4+\bar{\alpha}^2k^2+X)\right]}\left(M^*X+\frac{1}{1+i\alpha^2\beta}\right)\right]\frac{J_0(ky)}{J_1(ky_1)} \quad (20)$$

Where $M^*=\dfrac{\phi\beta}{\bar{\alpha}^2b(1+i\alpha^2\beta)}$

Volumetric flow rate Q is defined by

$$Q=2\pi\int_0^{y_1}y\left(u(y),v(y)\right)dy \quad (21)$$

Using equations (18) and (20) in (21), we get

$$Q_u=4\pi\sum_k\left\{\frac{(\bar{\alpha}^2\alpha^2e^{i(nt+\theta)}+L)}{\left\{k^2\left(k^4+\bar{\alpha}^2k^2+X\right)\right\}}\right\} \quad (22)$$

$$Q_v=4\pi\left[\sum_k\frac{1}{k}\left\{\left(\frac{\beta g\sin\psi e^{\text{int}}}{b(1+i\alpha^2\beta)}-M^*L\right)+\frac{\bar{\alpha}^2\alpha^2e^{i(nt+\theta)}+L}{k(k^4+\bar{\alpha}^2k^2+X)}\left(M^*X+\frac{1}{1+i\alpha^2\beta}\right)\right\}\right] \quad (23)$$

RESULTS AND DISCUSSIONS:

In the present study the expression for velocity have been derived for both blood and microorganisms. Fung[8] has observed that the velocity profile is blunted in the vicinity of the axis when the blood flows through a narrow vessel. This is due to the non-Newtonian character of the blood, considered as couple-stress fluid, with suspended micro-structured particals(RBC). This phenomenon has been shown through Fig. 4.1, the blunted character depends on the divergence parameter ε. From angle 90^0 and onwards-back flow is observed for blood only. For various values of pulsatile Reynold's number and divergence parameter and for a fixed value of couple-stress parameter. Fig 4.2 show that there is not much difference in both the velocities with increase in the value of couple-stress and divergence parameter, also there is no back flow for microorganisms. In fig 4.3 there is increase in both velocities with increasing of Reynolds number and divergence parameter. The back flow is observed only for blood, at and above 90^0, for $\alpha^2 = 6$ back flows is observed for microorganisms also. In fig 4.4 there is slight increase in the velocity of blood and microorganisms with increase in the value of volume fraction and divergence parameter upto 60^0, at and above 60^0 the velocity of blood start decreasing and at 75^0, the velocity of microorganisms is more compared with that of velocity of blood and there is back flow for both blood and microorganisms at and above 90^0.

From fig 4.5 it has been noticed that as gravity increased the velocity of microorganisms is much more compared to the velocity of blood. The flow rates have been computed for different values of $\alpha, \overline{\alpha}$ and nt for 1st , 2nd and 3rd approximations and a good convergence has been observed from fig 4.6 to fig 4.9.

Hence, the present mathematical model conclude that a careful choice of the values of the parameters of the blood flow around the tube in capillaries of small exponential divergence in the presence of micro-organisms will affect the flow characteristics. We hope that the present analysis help not only people working in the field of physiological fluid dynamics but also help to the medical practitioners to detect, diagnose and treat the patients suffering from blood born diseases like Cardio-vascular at an early stage.

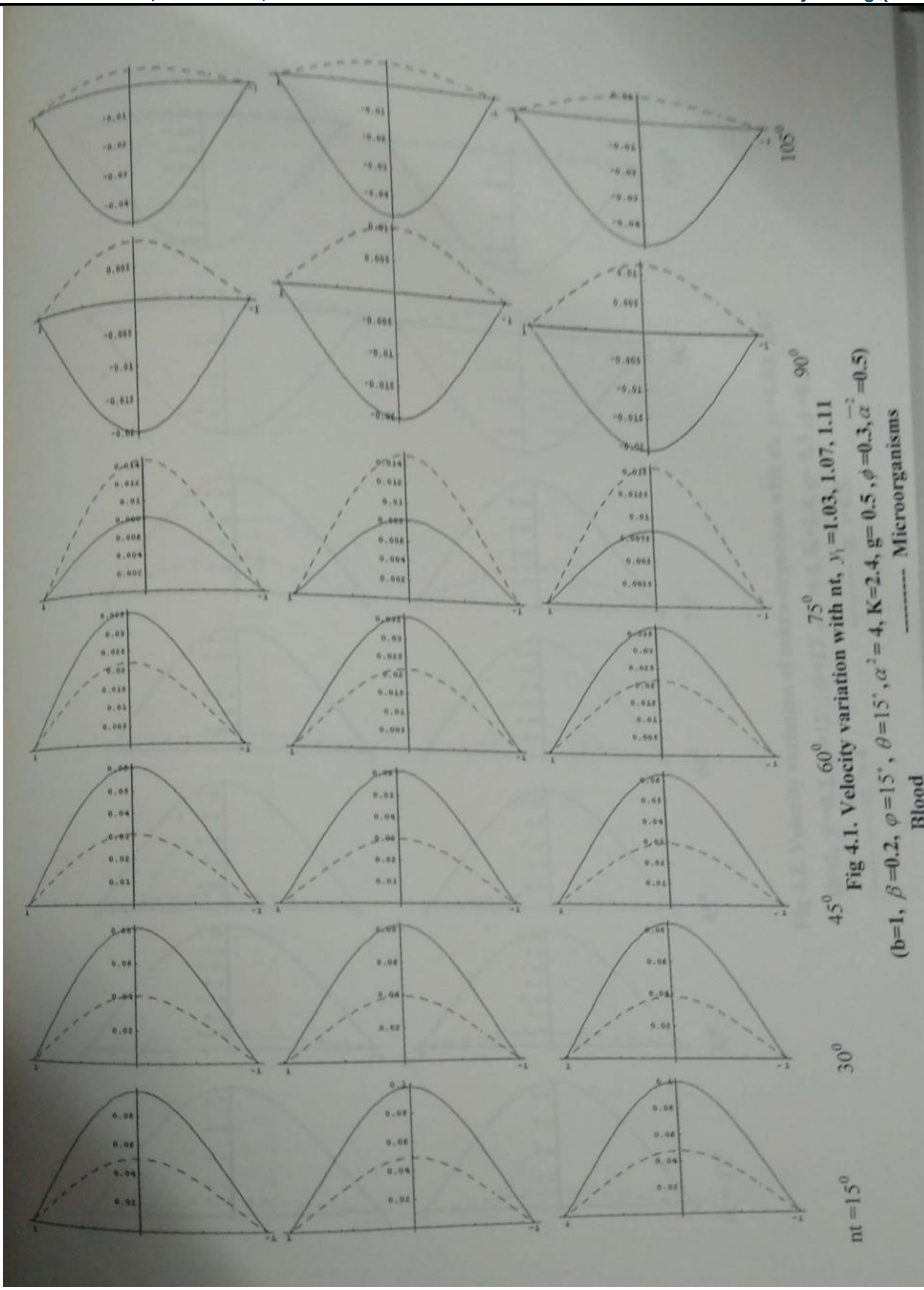

Fig 4.1. Velocity variation with nt, y_1=1.03, 1.07, 1.11

(b=1, β=0.2, φ=15°, θ=15°, α^2= 4, K=2.4, g= 0.5, ϕ=0.3, $\bar{\alpha}^{-2}$=0.5)

——— Blood ------- Microorganisms

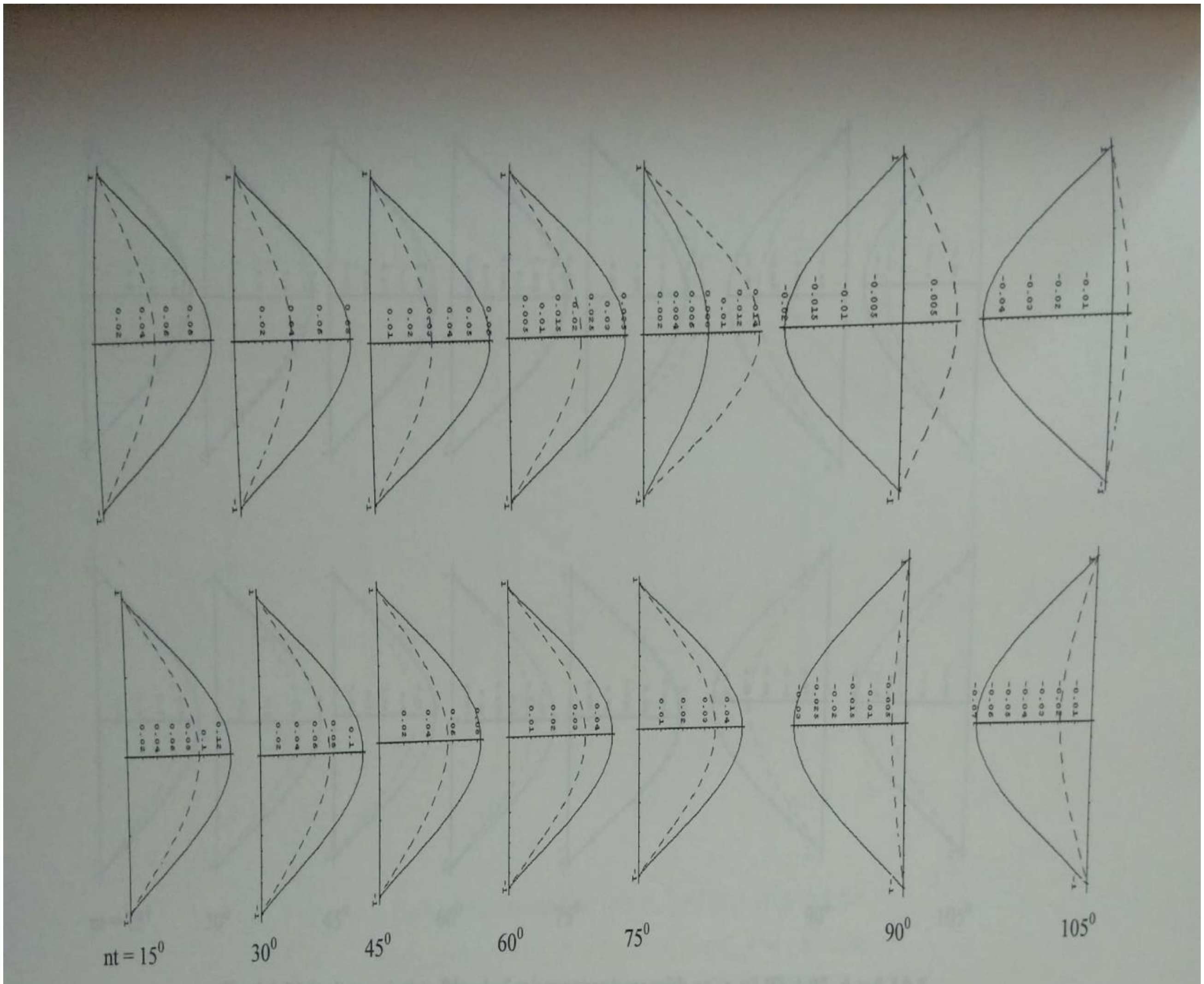

Fig 4.3. Velocity variation with nt, y_1=1.03, 1.07, α^2= 4, 6

(b=1, β=0.2, φ=15°, θ=15°, K=2.4, g= 0.5, ϕ=0.3, $\bar{\alpha}^{2}$=0.5)

_______ Blood --------- Microorganisms

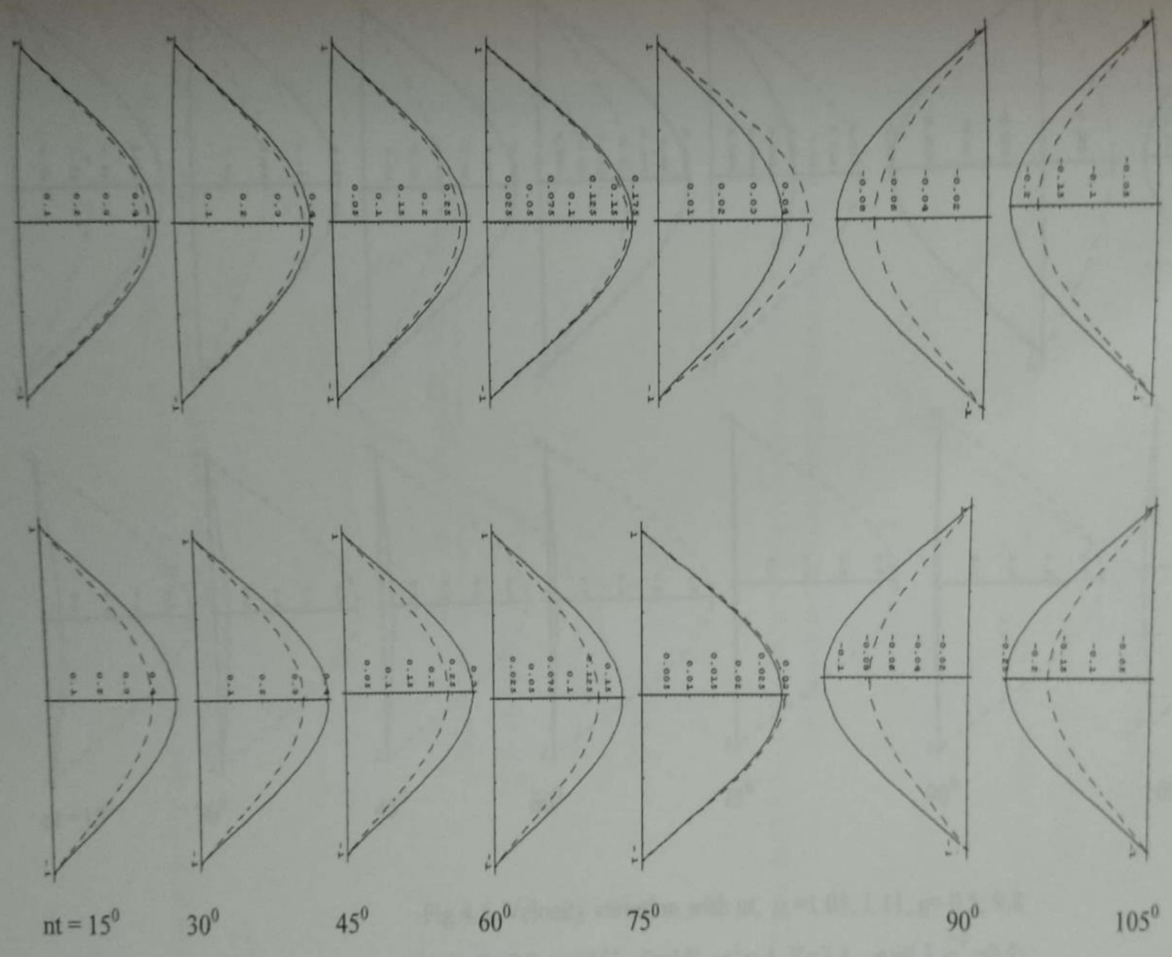

Fig 4.4. Velocity variation Blood of microorganisms with nt, y_1=1.03, 1.07, ϕ = 0.3,0.5

(b=1, β=0.2, φ=15°, θ=15°, K=2.4, g= 0.5 , α^2=4, $\bar{\alpha}^2$ = 0.5)

_______ Blood --------- Microorganisms

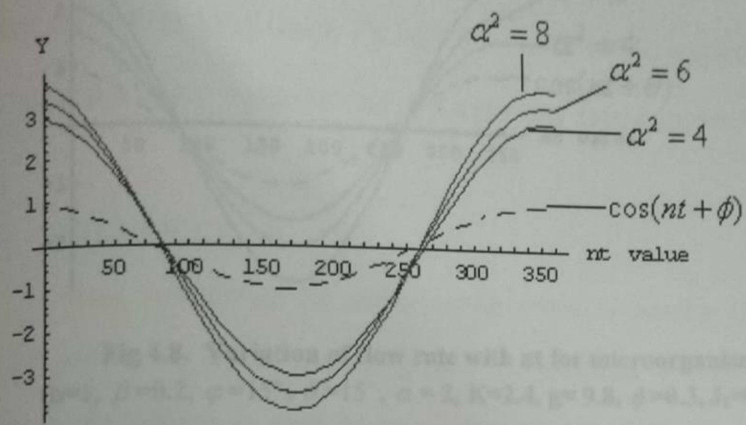

Fig 4.6. Variation of flow rate with nt for blood
(b=1, β=0.2, φ=15°, θ=15°, $\bar{\alpha}$ = 2, K=2.4, g= 9.8, ϕ=0.3, J_1=0.5202)

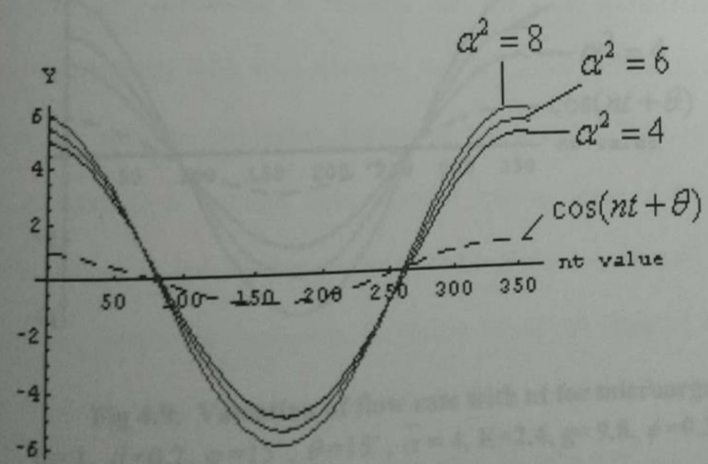

Fig 4.7. Variation of flow rate with nt for blood
(b=1, β=0.2, φ=15°, θ=15°, $\bar{\alpha}$ = 4, K=2.4, g= 9.8, ϕ=0.3, J_1=0.5202)

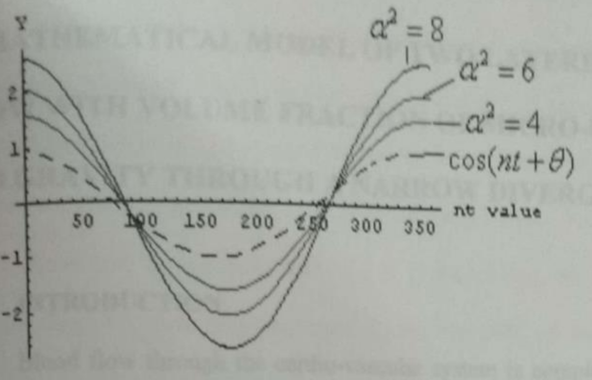

Fig 4.8. Variation of flow rate with nt for microorganisms
(b=1, β=0.2, φ=15°, θ=15°, $\bar{\alpha}$ = 2, K=2.4, g= 9.8, ϕ=0.3, J_1=0.5202)

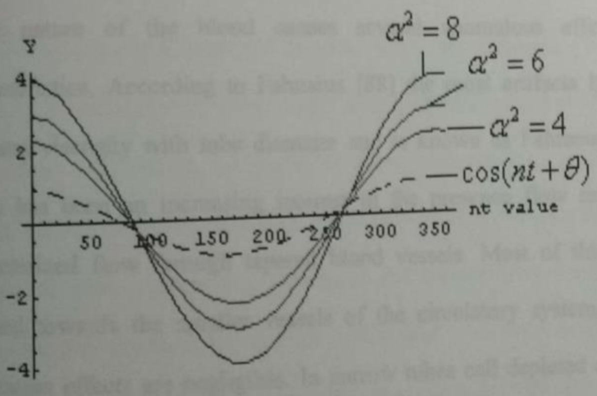

Fig 4.9. Variation of flow rate with nt for microorganisms
(b=1, β=0.2, φ=15°, θ=15°, $\bar{\alpha}$ = 4, K=2.4, g= 9.8, ϕ=0.3, J_1=0.5202)

References:

1. Abramawitz M., on back flow of a viscous fluid in a diverging channel, J. Math. Phys. 28, 1, (1949).
2. Abu-Sitta A.M and Drake D.G., Oscillating flow in ducts of arbitrary cross-section, Aeronautical Journal. 73, 10, 894-896, (1969).
3. Blasius H., Lamare Stromung in Kanalen Wechselnder Briete. Z. Math. U. physik 58, 225, (1910).
4. Bond B.A., Viscous flow through wide angled cones, Phil. Mag. 50, 1058, (1925).
5. El-Kareh, A.W and Secomb, T.W., Red cell motion in diverging capillary bifurcations, effect of shape on hematocrit distribution, Biorheology free communications: Micro ciralration-I.
6. Forrester J.H and young D.F., Flow through a converging-diverging tube and its implications in occlusive vascular disease, I.J. Biomechanics. 3, 297, (1970).
7. Fung, Y.C.Biomechanics: Mechanical properties of living tissues. New York. Springer-Verleg, pp.433 (1993).
8. Harrison W.J., The pressure in a viscous liquid moving through a channel with diverging boundaries. Proc. Cambridge Phil. Soc., 19, 307, (1919).
9. Masako Suchiha- Seki., Study of micro-vessels with lumen of irregular geometries, Japanese Soc. Biorheology. 33, 4, 5, 413, (1995).
10. Misra J.C and Ghosh S.K., A Mathematical model for the study of blood flow through a channel with permeable walls, Acta Mechanica. 122, 137-153, (1997).
11. P.G.Saffman, on the stability of laminar flow of a dusty gas, Journal of Fluid Mechanics, **13,** 120-128 (1962).
12. Sechneck D.J and Walburn F.J., Pulsatile blood flow in a channel of small exponential divergence –part-II : Steady streaming due to the interaction of viscous effects wit convected inertia, Journal of fluid Engineering. 98, 707-714, (Dec. 1976).
13. Stokes V.K., Couple-stresses in fluids, the physics of fluids. 9, 1709-1715, (1966).
14. Tranter C.J., Editor: B.L. Wornshop, Intergral transforms in mathematical physics, Infinite transforms, 8, 2nd edition London (1956).
15. Williams J.C. III, Viscous compressible and incompressible flow in slender channels. AIAA, J. 1, 186, (1963).