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SOME SOLUTIONS FOR VISCOUS FLUID BIANCHI TYPE-IX COSMOLOGICAL MODELS

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ABSTRACT :

The present paper provides solution for Bianchi Type – IX Cosmological Model by assuming suitable relations between metric potential A,B and by taking $\eta \propto \theta$ and $\zeta =$ constant where η is the co-efficient of shear viscosity and θ is the scalar of expansion and ζ is coefficient of bulk viscosity. We have obtained the solution in different cases depending on relations between metric functions A and B. Various physical and geometrical properties of the model have been obtained and discussed in presence and absence of viscosity.

Key words :

Cosmological model, viscosity, expansion, shear, fluid.

1. INTRODUCTION

In recent years various relativists have shown their keen interest in the study of evolution of the universe and have investigated cosmological models with a fluid containing viscosities. Belinskii and Khalatni-kov [10] investigating a Bianchi type

Icosmological model under the influence of viscosity, found the important property that near the initial singularity the gravitational field creates matter. Szydlowski and Heller [26] have constructed world models filled with interacting matter and radiation including bulk viscosity dissipation. They have shown the existence of stationary solutions in which the bulk viscosity term can be interpreted as phenomenologically describing the creation of matter and radiation. Santos et al [25] obtained exact solutions of an isotropic homogeneous cosmology with general viscosity for open closed, and flate universes. Banerjee and Santos [7, 8] obtained some exact solutions for a homogeneous anisotropic model using certain restrictions. Banerjee et al. [9] obtained some Bianchi type I solutions for the case of stiff matter by using the assumptions that shear viscosity co-efficients are power functions of the energy deneity. However the bulkviscosity co-efficients in the model are zero or constant. Recently Huang [15(a)] presented exact solution of a Bianchi type I cosmological model with bulk viscosity without introducing Shear viscosity – However, he adobted the restriction that the viscous co-efficients are constant or proportional to the energy density. Finally, Huang [15(a)] studied various Physical aspects of the problem.

Bianchi Type – IX Cosmological models are interesting because these models allow not only expansion but also rotation and shear and in general are anisotropic. Many relativists have been taken interest in studying Bianchi type – IX universe because familiar solutions lives Robertson walker universe with positive curvature the de-sitter universe, the Taub – NUT solutions e.t.c. are Bianchi type-IX space-time. In these models, neutrino viscosity does not guarantee isotropy at the present epoch. Viscosity is important in cosmology for a number of reason. Misner [20, 21] has studied the effect of viscosity on the evolution of cosmological models, Collins and stewart [11] have studied the effect of viscosity on the formation of galaxies. Murphy [19] has studied the influence of viscosity on the formation of initial singularity. Weinberg [29] derived general formula for bulk and shear viscosity and used these to evaluate the rate of cosmological entropy production. Heller and Klimek [15] have investiaged viscous universe without initial singularity. They have shown that introduction of bulk viscosity removes the initial singularity. Roy and Prakash [23, 24] investigated viscous fluid cosmological models of petrov type-ID and non-degenerate petrov type I in which co-efficient of viscosities are constant.

Krori et al. [16] and Wang [27, 28] studied the exact solutions of string cosmology for Bianchi type – II, VI_0, VIII and IX space – times. Pradhan et a. [22] have investigated the generation of Bianchi type – v cosmological models with varying $\wedge$ term.

Bali and Jain [45] have obtained some expending and shearing Bianchi Type – I viscous fluid cosmological models in which co-efficient of shear viscosity is proportional to the rate of expansion in the model and free gravitational field is Petrov Type – ID and non=degenerate. Bali et. al. [2, 3] have studied Bianchi type-IX viscous their cosmological models in general relativity. Robertson walker cosmological models with bulk viscosity and equation of state $P = (\gamma - 1)\rho$, $0 < \gamma \leq 2$ is investigated by Mohanty and Pradhan [18]. Cademi and Febric [12] have carried out the research on homogeneous viscous universe and investigated models Bianchi type V – Type, VIII and Type – IX some other workers in this field are Zimdahl [30], Banerjee et. al. [6], Baliond [1]. Dubey et. al. [13].

Here in this paper we have studied Bianchi Type – IX viscous fluid cosmological model. To obtain a deterministic models we have assumed suitable relations between metric potential A,B and by taking $\eta \propto \theta$ and $\zeta =$ constant where η is the co-efficient of shear viscosity and θ is the scalar of expansion and ζ is coefficient of bulk viscosity. We have obtained the solution in different cases depending on relations between metric functions A and B Various physical and geometrical properties of the model have been obtained and discussed in presence and absence of viscosity.

2. The field equations

(2.1) $T_i^j = (\rho + p)u_i u^j + p g_i^j - \eta\left(u_i^j + u_i^j + u^j u_\alpha + u_j u^\alpha u_{;\alpha}^j\right)$

$$-\left(\zeta - \frac{2}{3}\eta\right)u^\alpha\left(g_i^j + u_i u^j\right)$$

Where p is the isotropic pressure, ρ the density, η and ζ are the co-efficient of viscosity, u^i the flow vector satistying

(2.2) $g_{ij}u^i u^j = -1$

In co-moving co-ordinates, we have

(2.3) $u^1 = 0 = u^2 = u^3$ and $u^4 = 1$

Where A = A(t) and B = B(t)

Here we take Bianchi type-IX line element written as

(2.4) $ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + (B^2\sin^2 y + A^2\cos^2 y)$

$$dZ^2 - 2A^2\cos y\, dx\, dz$$

The Einstein's field equations are

(2.5) $R_i^j - \frac{1}{2}R\, g_i^j + \wedge g_i^j = -8\pi T_i^j$

where R_{ij} is Ricci tensor, R is scalar of curvature tensor, g_{ij} is metric and $\wedge$ is cosmological constant.

where we have used gravitational units (i.e. C = ζ = 1)

The field equation (5.2.4) for the metric (5.2.4) are

$$(2.6)\quad \left[\frac{2B_{44}}{B}+\frac{B_{44}^2}{B^2}+\frac{1}{B^2}-\frac{3A^2}{4B^4}+\wedge\right]=-8\pi\left[p-2\eta\frac{\alpha_4}{\alpha}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right]$$

$$(2.7)\quad \left[\frac{A_{44}}{A}+\frac{A_4}{AB}+\frac{B_{44}}{B}+\frac{A^2}{4B^2}+\wedge\right]=-8\pi\left[p-2\eta\frac{B_4}{B}-\left(\zeta-\frac{2}{3}\eta\right)\theta\right]$$

$$(2.8)\quad \left[\frac{2A_4B_4}{AB}+\frac{B_4^2}{B^2}-\frac{A^2}{4B^4}+\frac{1}{B^2}+\wedge\right]=8\pi\rho$$

here 4 after A and B denotes ordinary differentiation with respect to t and expansion θ is given by

$$(2.9)\quad \theta=u^i_{;i}$$

3. Solution of the field equations

We have six unknows A, B, ρ, p and η in three equations (2.6) (2.8) thus the system is indeterminate. To make the system determinate we need three more relations or equations. For this we firstly assume co-efficients of shearviscouty η directly proportional to expansion θ and co-efficient of bulk viscosity ζ to be constant i.e.

$$(3.1)\quad \eta\alpha\theta \text{ and}$$

$$(3.2)\quad \zeta=\text{constant}$$

Further we are free to choose one more relation. For this assume suitable relations between metric co-efficients A and B. We solve the field equations in the following different cases (Models)

Case I (a) :

Here we choose

(3.3) $A = \alpha B^{1/3}$, α = constant

The from equation (2.7) and (2.8) we get

(3.4) $$\left[\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{A_{44}}{A} - \frac{A_4 B_4}{AB} - \frac{A^2}{B^4} + \frac{1}{B^2}\right] = 16\pi\eta\left(\frac{A_4}{A}\frac{B_4}{B}\right)$$

Now from (3.1) (3.2) we get

(3.5) $$\eta = C\left(\frac{A_4}{A} + \frac{2B_4}{B}\right)$$

Equations (3.1), (3.4) and (3.5) provide

(3.6) $$B_{44} + \lambda B_4^2 = \frac{3}{2}B^{-4/3} - \frac{3}{2}$$

To avoid mathematical complerity we choose $\alpha = 1$

Also we have

(3.7) $$\lambda = \frac{4}{3}[1 + 2\delta\pi C]$$

Then from equation (3.6) we find

(3.8) $$\frac{d}{dB}(\phi^2) + \frac{2\lambda}{B}(\phi^2) = 3B^{-7/3} - \frac{3}{B}$$

where

(3.9) $B_4 = \phi\ (B)$

(3.10) $$[B_4]^2 = \frac{3B^{-4/3}}{2\left(\frac{1}{B} + \lambda\frac{-2}{3}\right)} + \frac{\mu}{B^{2\lambda}} - \frac{3}{2\lambda}$$

(3.11) $$ds^2 = -\left(\frac{dt}{dB}\right)^2 dB^2 + B^{2/3}dx^2 + B^2dy^2 + (B^2\sin^2 y$$

$$+B^{2/3}\cos^2 y)dz^2 - 2b^{1/3}\cos y\, dx\, dz.$$

(3.12) $$ds^2 = \frac{dT^2}{\left[\frac{3T^{-4/3}}{2\left(\lambda - \frac{2}{3}\right)} + \frac{\mu}{T^{2\lambda}} - \frac{3}{2\lambda}\right]} + T^{2/3} d\times 2$$

$$+T^2 dY^2 + (T^2 \sin^2 y + T^{2/3}\cos^2 y)dz^2 - 2T^{2/3}\cos y\, dx\, dz$$

Where B = T, x = X, y = Y, z = Z

Case 1 (b)

Here we have

(3.13) $$A = \alpha_1 \sqrt{B}$$

where α_1 is constant

Now equation (2.7) and (2.8) provide

(3.14) $$\left[\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{A_{44}}{A} - \frac{A_4 B_4}{AB} - \frac{A^2}{B^4} + \frac{1}{B^2}\right] = 16\pi\eta\left(\frac{A_4}{A} - \frac{B_4}{B}\right)$$

Conditions (3.1) and (3.2) leads to

(3.15) $$\eta = C_1\left(\frac{A_4}{A} + \frac{2B_4}{B}\right)$$

Where C_1 is constant

Equation (3.1), (3.4) and (3.5) yield

(3.16) $$BB_{44} + \lambda_1 B_4^2 = 2\left(\frac{1}{B} - 1\right)$$

Here we have taken $\alpha_1 = 1$ to avoid mathematical Complexity

and

$$(3.17)\quad \lambda_1 = \frac{3}{2} + 4\pi C_1$$

Equation (3.16) gives

$$(3.18)\quad \frac{d}{dB}\left(\phi_1^2\right) + \frac{2\lambda_1}{B}\left(\phi_1^2\right) = \frac{4}{B^2} - \frac{4}{B} = \frac{4}{B^2}(1-B)$$

$$(3.19)\quad B_4 = \phi_1(B)$$

Equation (3.18) provides

$$(3.20)\quad \left[BB_4\right]^2 = \frac{4B}{2\lambda_1 - 1} + \frac{D}{(\lambda_1 - 1)} - \frac{2B^2}{\lambda_1}$$

Where D is constant

Now metric (2.1) goes to the form

$$(3.21)\quad ds^2 = -\left(\frac{dt}{dB}\right)^2 + Bdx^2 + B^2dy^2 + (B^2\sin^2 y + B\cos^2 y)dz^2$$

$$-2\sqrt{B}\cos y\, dx\, dz$$

Use of (3.20) reduces (3.21) to the form

$$(3.22)\quad ds^2 = \frac{dT^2}{\frac{4T^{-1}}{2\lambda_1 - 1} + \frac{D}{T^2\lambda_1}} + Tdx^2 + T^2dy$$

$$+(T^2\sin^2 Y + T^2\cos^2 Y)dz^2 - 2T\cos Y dX\, dZ$$

Where we have used

$$B = T, x = X, y = Y, z = Z$$

(ii) Physical and Geometrical Aspects (case 1(b))

Pressure and density for the model (3.22) are given by

$$(3.23)\ 8\pi p = \frac{320\pi C_1 - 6\lambda_1 + 27}{12(2\lambda_1 - 1)T^3} + \frac{D(80\pi C_1 + 18\lambda_1 - 3)}{12T^{2(\lambda+1)}} + \frac{3 - 80\pi C_1}{6\lambda_1 T^2}$$

$$+20\pi\zeta\left(\frac{4}{(2\lambda_1 - 1T^3} + \frac{D}{T^{2(\lambda_1+1)}} - \frac{2}{\lambda_1 T^2}\right)^{1/2} - \wedge$$

and

$$(3.24)\ 8\pi\rho = \frac{33 - 2\lambda_1}{(2\lambda_1 - 1).4T^3} + \frac{2D}{T^{2(\lambda_1+1)}} + \frac{\lambda_1 - 4}{\lambda_1 T^2}$$

The energy condition given by Ellis [11] are

(i) $(\rho + p) > 0$ and (ii) $(\rho + 3p) > 0$

The condition (i) leads to

$$(3.25) \left[\frac{63 - 6\lambda_1 + 160\pi C_1}{6(2\lambda_1 - 1)T^3} + \frac{6\lambda_1 - 80\pi C_1 - 21}{6\lambda_1 T^2} + \frac{D(80\pi C_1 + 18\lambda_1 + 21)}{12T^{2(\lambda_1+1)}}\right.$$

$$\left. +12\pi\zeta\left(\frac{4}{(2\lambda_1 - 1)T^3} + \frac{L}{T^{2(\lambda_1+1)}} - \frac{2}{\lambda_1 T^2}\right)^{1/2}\right] > 0$$

$$(3.26) \left[\frac{80\pi C_1 + 15 - 2\lambda_1}{(2\lambda_1 - 1)T^3} + \frac{L}{4}\frac{(80\pi C_1 + 18\lambda_1 + 5)}{T^{2(\lambda_1-1)}} + \frac{2\lambda_1 - 80\pi C_1 - 5}{2\lambda_1 T^2}\right.$$

$$\left. +60\pi\zeta\left(\frac{4}{(2\lambda_1 - 1)T^3} + \frac{D}{T^{2(\lambda_1-1)}}\frac{2}{\lambda_1 T^2}\right)^{1/2}\right] > 2\wedge$$

Which puts restriction on $\wedge$. Thus energy condition (i) $\rho + p > 0$ (ii) $\rho + 3p > 0$ are satisfied

The expansion (θ) and shear (σ) in the model (3.22) are given by

$$(1.3.27)\ \theta = \frac{5}{2}\left[\frac{4}{2(\lambda_1 - 1)T^3} + \frac{L}{T^{2(\lambda_1+1)}}\left(1 - \frac{2T^{2\lambda_1}}{D\lambda_1}\right)\right]$$

$$(3.28)\ 0 = \frac{1}{\sqrt{6}}\left[\frac{4}{(2\lambda_1 - 1) + 3} + \frac{1}{T^{2(\lambda_1+1)}}\left(1 - \frac{2T^{2D}}{D\lambda_1}\right)\right]^{1/2}$$

In the absence of viscosity i.e. when $C_1 \rightarrow 0$the metric redues to

$$(3.29)\ ds^2 = \frac{-3dT^2}{6T^{-1} + 3Dt^{-3-4}} + Tdx^2 + T^2dy^2$$

$$+\left(T^2 \sin^2 y + T\cos^2 y\right)dz^2 - 2T\cos y dx\, dz$$

From (2.6) we get

$$(3.30)\ \lambda_1 = 40\pi C_1 + \frac{3}{2}$$

From the above equation we find that in the absence of viscosity $\lambda_1 = \frac{3}{2}$ as $C_1 = 0$.

The expression for pressure, density, expansion (θ) and (σ) are given by

$$(3.31)\ 8\pi p = \frac{3}{4T^3} + \frac{2D}{T^5} - \frac{1}{3T^2} - \wedge$$

$$(3.32)\ 8\pi\rho = \frac{15}{4T^3} + \frac{2D}{T^5} - \frac{5}{3T^2} - + \wedge$$

$$(3.33)\ \theta = \frac{5}{2}\left[\frac{2}{T^3} + \frac{D}{T^5} + \frac{4}{3T^2}\right]^{1/2}$$

$$(3.34)\ \sigma = \frac{1}{\sqrt{6}}\left[\frac{2}{T^3} + \frac{D}{T^5} + \frac{4}{3T^2}\right]^{1/2}$$

The energy condition (i) $\rho + p > 0$ and (ii) $\rho + 3p > 0$ leads to

$$(3.35)\ \frac{1}{T^3} + \frac{2L}{T^5} + \frac{1}{T^2} > 0$$

(3.36) $\frac{16D}{T^5} - \frac{5}{T^3} - \frac{16}{3T^2} > 4\wedge$

Which gives condition on $\wedge$. In the absence of viscosity. The expansion in the model starts with a big bang at T = 0 and the expansion in the model decreases with time when $T \to o$ then $\rho \to \infty, p \to \infty$ and when $T \to \infty$ then $\rho \to \wedge$ and $p \to -\wedge$ since $\lim T \to \infty \left(\frac{\sigma}{\theta}\right) \neq 0$. Hence the model does not approach isotropy for large values of T in the absence of viscosity in general.

4. Solution of field equations Case – II

Case II(a)

Here η and ζ are same as in case I and

(4.1) $A = \alpha_2 B^2$

Now equation (1.2.7) and (1.2.8) gives

(4.2) $\left[\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{A_{44}}{AB} - \frac{A^2}{B^4} + \frac{1}{B^2}\right] = 16\pi\eta\left(\frac{A_4}{A} - \frac{B_1}{B}\right)$

From condition (3.1) and (3.2) we have

(4.3)

$$\mu = \left(\frac{A_4}{A} + \frac{2B_4}{B}\right)$$

Here μ is constant

Equation (4.2), (4.1) and (4.3) yield

(4.4) $BB_{44} + kB_4^2 = 1 - B^2$

Here we have taken $\alpha_2 = 1$, to avoid mathematical complexity and

(4.5) $K = 64\pi\mu + 3$

Equation (.4.4) leads to

(4.6) $\frac{d}{dB}(\phi_2^2) + \frac{2K}{B}(\phi_2)\frac{2}{B}2B$

(4.7) where $B_4 = \phi_2(B)$

(4.8) $(BB_4)^2 = -\frac{B^4}{K+1} + \frac{4B}{2} + \frac{B^2}{M}$

Where M is constant

Now metric (2.4) takes the form

(4.9) $ds^2 = \left(\frac{dt}{dB}\right)^2 + dB^2 + B^4dx^2 + B^2dy^2$

$+(B^2\sin^2 y + B^4\cos^2 y)dz^2 + 2B^2\cos y dxdz$

Use of (1.4.8), redues (4.9) to the form

(4.10) $ds^2 = \frac{dT^2}{-\frac{T^2}{(1+K)} + \frac{M}{T^{2K}} + \frac{1}{K}} + T^4dx^2 + T^2dy^2$

$+\left(T^2\sin^2 Y + T^4\cos^2 Y\right)dz^2 - 2T^4\cos Y\ dx\ dz$

Where we use $B = T, x = X, Y = Y, z = Z$

(ii) **Physical and geometrical aspects (for cases II):** pressure and density for the model (4.10) are fourd to be

(4.11) $8\pi p = \frac{256\pi\mu + 81 - 3K}{12(K+1)} + \frac{M(-64\pi\mu + 9K - 12)}{3T^{2(K+1)}}$

$+\left(\frac{64\pi\mu + 12}{-3K}\right).\frac{1}{T^2} + 32\pi\zeta\left(\frac{1}{-(K+1)} + \frac{M}{T^{2(K+1)}} + \frac{1}{KT^2}\right)^{1/2} - \wedge$

$$(4.12)\ 8\pi\rho\frac{K+21}{M(K+1)}+\frac{5M}{T^{2(K+1)}}+\frac{K+5}{KT^2}+\wedge$$

The energy conditions lead to

$$(4.13)\ \left[\frac{12\pi\mu-3K+9}{6(K+1)}+\frac{3K-64\pi\mu+3}{3KT^2}+\frac{M(9K-64\pi\mu+3)}{T^{2(K+1)}}\right.$$

$$\left.+32\pi\zeta\left(\frac{M}{T^{2(K+1)}}-\frac{1}{K+1}-\frac{1}{KT^2}\right)^{1/2}\right]>0$$

and the condition ρ + 3p > 0 leads to

$$(4.14)\ \left[\frac{64\pi\nu-K+15}{1+K}+\frac{(9K-64\pi\mu-7)}{T^{2(K+1)}}\right.$$

$$\left.+\frac{3K-64\pi\mu-9}{KT^2}+96\pi\zeta\left\{\frac{1}{KT^2}-\frac{1}{K+1}+\frac{M}{T^{2(K+1)}}\right\}^{1/2}\right]>2\wedge$$

Also θ and σ are given by

$$(4.15)\ \theta=4\left\{\frac{4K+T^{2K}}{KT^{2K+1}}-\frac{1}{K+1}\right\}^{1/2}$$

$$(4.16)\ \sigma=-\sqrt{\frac{2}{3}}\left\{\frac{MK+T^{2K}}{T^{2(K+1)}}-\frac{1}{K+1}\right\}$$

In the absence of viscosity i.e. $\mu\to 0$ the metric (4.10) reduces to the form

$$(4.17)\ ds^2=\frac{-dT^2}{\frac{T^2}{-4}+\frac{M}{T^6}+\frac{1}{3}}+T^4dx^2+T^2dy^2$$

$$+\left(T^2\sin^2 y+T^4\cos^2 y\right)dz^2-2T^4\cos y\,dx\,dz$$

From equation (4.5)

(4.18) $K = 3 + 64\pi\mu$

In the absence viscosity K = 3 and μ = 0. The expression for pressure, density, Expansion (θ) and shear (σ) are given by

(4.19) $8\pi P = \frac{4}{3T^2} + \frac{5M}{T^8} + \frac{3}{2} - \wedge$

(4.20) $8\pi\rho = -\frac{3}{2} + \frac{5M}{T^8} + \frac{8}{3T^2} + \wedge$

(4.21) $\theta = 4\left(\frac{M}{T^8} - \frac{1}{3T^2} - \frac{1}{4}\right)^{1/2}$

(4.22) $\sigma = -\sqrt{\frac{2}{3}}\left(\frac{M}{T^8} - \frac{1}{4} - \frac{1}{3T^2}\right)^{1/2}$

The energy condition (i) $\rho + p > 0$ and (ii) $\rho + 3p > 0$ leads to

(4.23) $\frac{10M}{T^8} - \frac{4}{T^2} - \frac{5}{2} > 0$

and

(4. 24) $\frac{-11}{2} + \frac{20M}{T^8} - \frac{20}{3T^2} > 2\wedge$

Which gives condition on $\wedge$. In the absence of viscosity, the expansion in the model starts with a big hand at T = 0 and expansion in the model decreases with time, when $T \to 0$ and expansion in the model decreases with time, when $T \to 0$. Then $\rho \to \infty, p \to \infty$ and when $T \to \infty$ Then $\rho \to \rho - \wedge -\frac{3}{2}$ and $p \to \frac{3}{2} - \wedge$. Since $\lim T \to \infty (\sigma/\theta) \neq 0$. Hence this model also does not approach isotropy for large values of T in the absence of viscosity in general.

Case II (b)

Here η and ζ are same as in case II(a) and

(4.25) $A = \alpha_3 B^2$

Also equation (4.2) is same

Also from (3.1) and (32), we have

(4.26) $\eta = \mu_1 \left(\frac{A_4}{A} + \frac{\zeta B_4}{B} \right)$

Where μ, is constant then from equations (4.2), (4.25) and (4.26) we have

(4.27) $BB_{44} + K_1 B_4^2 = \frac{1}{2} - 2B^4$

Where we taken $\alpha_3 = 1$ to avoid mathematical complexity and

(4.28) $K_1 = 2[40\pi\mu_1 - 1]$

Equation (4.27) leads to

(4.29) $\frac{d}{dB}\left(\phi_1^2\right) + \frac{2K_1}{B}\left(\phi_1^2\right) = \frac{1}{B} - B^3$

Where

(4.30) $B_4 = \phi_1(B)$

Then equation (4.4) gives

(4.31) $B_4^2 = \frac{1}{2K_1} + \frac{M_1}{B^{2k_1}} - \frac{B^4}{2(K_1 - 2)}$

Where M_1 is constant of integration

The metric (2.4) reduces to

$$(4.32)\ ds^2 = \left(\frac{dt}{dB}\right)^2 dB^2 + B^6 dx^2 + B^2 dy^2 +$$

$$(B^2 \sin^2 y + B^6 \zeta B^2 y) dz^2 - 2B^3 \cos y\, dx\, dz$$

Use of (4.31) reduces (4.32) into the form

$$(4.33)\ ds^2 = \frac{dT^2}{\left[\frac{M_1}{T^{2K_1}} + \frac{1}{2K_1} - \frac{T^4}{2(K_1+2)}\right]} + T^6 dx^2 + T^2 dy^2$$

$$+\left(T^2 \sin^2 Y + T^6 \cos^2 Y\right) dZ^2 - 2T^6 \cos Y\, dX\, dZ$$

Where we have used B = T, x = X, y = Y and z = Z

Physical and geometrical cospects of this model can be discussed as in previous cases.

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