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Two-dimensional Non-linear Steady Solution to Thermal Plumes with high Prandtl number

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Abstract : Purely buoyancy-driven flows generated by temperature-induced density variations are commonly referred to as thermal plumes. Typical examples include the curling smoke rising from burning sticks, agarbatti, or a cigarette tip. In the present study, a two-dimensional, steady thermal plume is analysed under the Boussinesq approximation, wherein density variations are retained only in the body force term of the momentum equation as a function of temperature. The resulting governing equations form a coupled, non-linear system describing the interaction between velocity and temperature fields. The focus of the investigation is on the high Prandtl number limit, which is of particular relevance to bubble flows, droplet combustion, and several geophysical and engineering applications. Such an analysis also holds significance for cloud dynamics and fire modelling, where thermal plumes play a dominant role. Non-dimensional forms of the governing equations are employed to examine the influence of the Prandtl number on the flow and thermal characteristics of the plume. The results indicate that near the source region, the non-dimensional velocity increases with increasing Prandtl number, while at locations far from the source the velocity decreases as the Prandtl number increases. Similarly, the non-dimensional temperature at the source is found to be higher for larger Prandtl numbers. These findings provide insight into the structure and behaviour of thermal plumes in high Prandtl number fluids and contribute to a better understanding of buoyancy-driven flows in both natural and industrial processes.

IndexTerms - Thermal Plumes, Prandtl number, Two-dimensional Non-linear, non-dimensional temperature.

I. INTRODUCTION

Purely buoyancy-driven flows are known as plumes – a typical example of a plume is the curling smoke coming from out of burning sticks, agarbatti, cigarette-tip. The density difference is created by the temperature difference between the tip and its surroundings. Using the Boussinesq approximation, where the density is written as a function of temperature for the body force term in the momentum equation, to model the plume. The two- dimensional steady flow is considered for the study. Study of thermal plumes is very important in cloud modelling, fire modelling. High Prandtl number limit solution is important for study of the bubble flows, droplet combustion.

II. FORMULATION

Basic Equations Let x and y be the vertical and horizontal co-ordinates with the origin at the heat source; u and v be the velocity components in the x and y directions, respectively. For the steady two-dimensional convection above a line heat source, the equations of motion are:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} &= g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \\ u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} &= \alpha\frac{\partial^2 T}{\partial y^2} \end{aligned}$$

g = gravitational acceleration

β = volumetric thermal expansion coefficient

T = fluid temperature

ν = kinematic viscosity of the fluid

T_∞ = ambient temperature.

The velocity and temperature distributions must be symmetrical about the x-axis, and the temperature and the vertical velocity component at a distance from the heat source are not to be affected by it. Hence the boundary conditions may be expressed as:

$$v = 0, \quad \frac{\partial u}{\partial y} = 0, \qquad \text{at } y = 0,$$

$$u = 0, \quad T = T_\infty, \qquad \text{at } y = \infty.$$

We further assume the existence of a similarity solution for the velocity and temperature fields such that let u be the stream function such that

$$\frac{\partial \psi}{\partial y} = u, \qquad \frac{\partial \psi}{\partial x} = -v$$

$$\xi = Gr^{1/5}\,\frac{y}{x}$$

$$\psi = \nu\, Gr^{1/5}\, f(\xi)$$

$$T = Gr^{-1/5}\,\Theta\, h(\xi),$$

where

$$Gr = \frac{x^3 g \beta \Theta}{\nu^2}$$

and Θ is an arbitrary constant with the dimension of temperature. Substituting the above expressions for u, v and T into the equations of motion, we obtain the following ordinary differential equations:

$$f''' + \frac{3}{5} f f'' - \frac{1}{5}(f')^2 + h^2 = 0,$$

$$h'' + \frac{3}{5}\,\Pr(fh)' = 0,$$

with boundary conditions:

$$\xi = 0: \quad f = 0, \quad f' = 0, \quad h' = 0$$

$$\xi = \infty: \quad f' = 0, \quad h = 0,$$

$$\int_0^\infty f' h \, d\xi = \frac{1}{2}. ,$$

where *Pr* is the Prandtl number, defined as the ratio between the kinematic viscosity and thermal diffusivity:

$$Pr = \frac{\nu}{a}.$$

$$\xi = 0: \quad f = 0, \quad f' = 0, \quad h' = 0$$
$$\xi = \infty: \quad f' = 0, \quad h = 0,$$
$$\int_0^\infty f' h \, d\xi = \frac{1}{2}.$$

,

where *Pr* is the Prandtl number, defined as the ratio between the kinematic viscosity and thermal diffusivity:

$$Pr = \frac{\nu}{a}.$$

The analytical solution of the ordinary differential equations for *Pr* = 2 is given by (Fujii 1963)

$$f = \left(\frac{10a}{3}\right)^{1/2} \tanh\left[\left(\frac{3a}{10}\right)^{1/2} \xi\right]$$

$$h = \frac{4}{5} a^2 \operatorname{sech}^4\left[\left(\frac{3a}{10}\right)^{1/2} \xi\right],$$

,

,

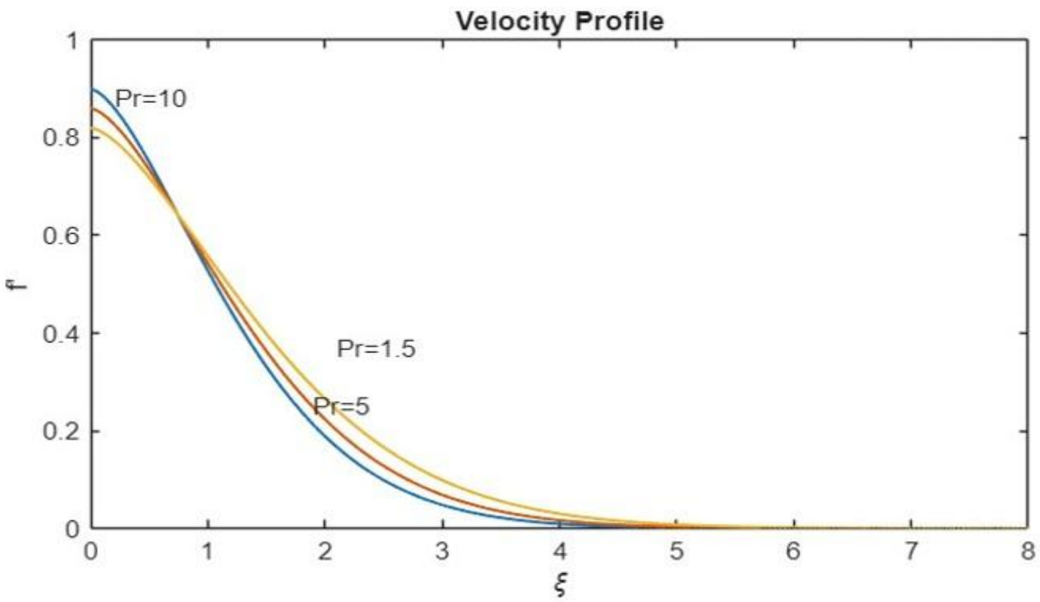
where

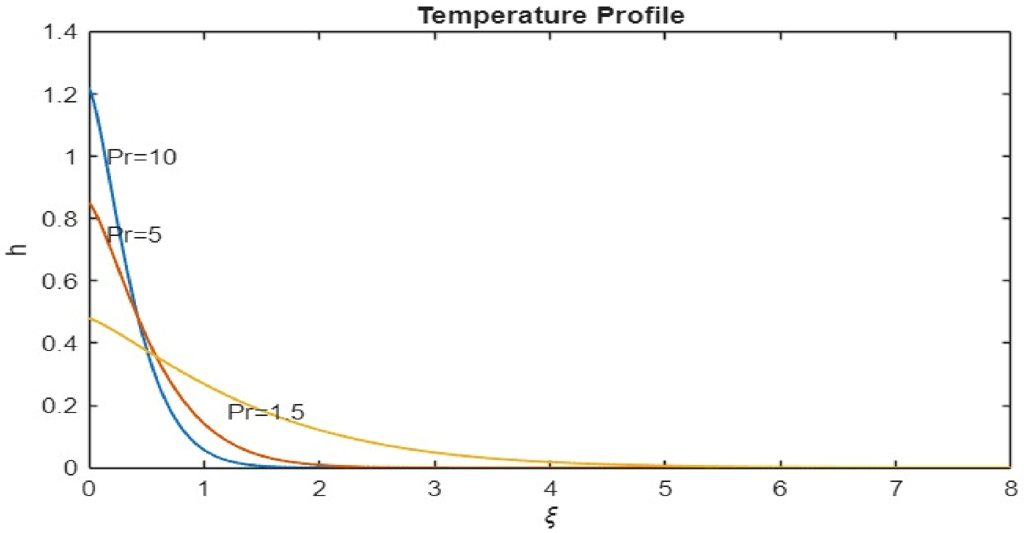
$$f' = a \operatorname{sech}^2\left[\left(\frac{3a}{10}\right)^{1/2} \xi\right]$$

$$a = \frac{1}{4}\left(\frac{15}{2}\right)^{1/3} = 0.837.$$

For other values of Prandtl numbers, we have solved the base flow equations using the standard Runge– Kutta method of the MATLAB software. Some typical plots for velocity and temperature fields are shown in Figures 1 and 2, respectively. If the Prandtl number goes to infinity, bubble plumes are likely to behave as thermal plumes.

III. Results





IV. CONCLUSION

- At source, the non-dimensional velocity is higher as the Prandtl number increases.
- At far away from source, the non-dimensional velocity is lower as the Prandtl number increases.
- At source the non-dimensional temperature is higher as the Prandtl number increases.
- At far field, the non-dimensional temperature is lower as the Prandtl number increases.
- The ratio of momentum diffusion to thermal diffusion plays important role in the steady solution.

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