



# COMPARATIVE STUDY OF THE HIDDEN MARKOV MODEL AND FUZZY HIDDEN MARKOV MODEL – REVIEW

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**Abstract:** This paper presents an overview of the study comparing Hidden Markov Models (HMMs) and Fuzzy Hidden Markov Models (FHMMs). The research aims to analyse and evaluate the two models in terms of modelling capabilities, representation methods, inference and learning algorithms, and future usefulness. This paper sheds light on the strengths, and drawbacks of the similarities and differences between HMMs and FHMMs, highlighting their importance in different areas, through an intensive review of the literature and analysis of experimental results. The application of HMMs and FHMMs has dominated in recent years, as seen by the many articles released. In this investigation, 101 publications from journals were used. The authors review the literature using HMM and FHMM, which have been applied to a variety of application sectors. Based on collected publications, this report provides a brief review of research on the comparative analysis of HMM and FHMM.

**Keywords:** *Hidden Markov Model (HMM), Fuzzy Hidden Markov Model (FHMM), Algorithms*

## I. INTRODUCTION

Hidden Markov Models were first developed in the 1960s by Leonard E. Baum and his colleagues, who added hidden states to the Markov chain theory. A significant research study "A Maximisation Technique Occurring in the Statistical Analysis of Probabilistic Functions of Markov Chains," by Baum, Petrie, Soules, and Weiss in 1970, inspired HMMs. HMMs developed widespread acceptance in the 1970s and 1980s and were used in a variety of industries, including bioinformatics, pattern recognition, and speech recognition. Researchers like Bhiksha Raj and Lawrence R. Rabiner (1970-1980) were early adopters of HMMs in speech recognition.

Hidden Markov Models (HMMs) and Fuzzy Hidden Markov Models (FHMMs) are extensively used probabilistic models with applications in speech recognition, bioinformatics, Time series, Medical Diagnosis, Handwriting Recognition and Fault Diagnosis are discussed in this paper. These models offer a framework for modelling sequential data and be successful in capturing complicated interactions and making predictions based on observed data. HMMs are statistical models that are based on the assumption that the underlying system is a Markov process with hidden states. They are made up of observable states and hidden states that are not immediately observable but can be deduced from the observable states.

FHMMs, on the other hand, extend HMM capabilities by introducing fuzzy logic to deal with ambiguity and inaccuracy. FHMMs use fuzzy sets to represent states and observations, allowing for the representation of selective memberships and fuzzy limits. As a result, FHMMs are appropriate for conditions in which the limits between states are not well-defined or when the observations are ambiguous.

While both HMMs and FHMMs have been extensively studied and utilised in a variety of domains, a comparison of both models can provide useful insights into their comparisons, variations, and respective strengths and drawbacks. A comparative study of this type can assist investigators and users in making educated decisions about which model to use for specific applications. As a result, the goal of this research is to undertake a thorough comparison of HMMs and FHMMs. The research will look into their modelling abilities, representation methodologies, inference and learning algorithms, performance evaluation criteria, and applicability. This work seeks to provide a deeper knowledge of the relative benefits of HMMs and FHMMs in various circumstances by examining existing literature and analysing experimental outcomes. Furthermore, this research will look at recent advances in both models, such as hybrid models or extensions that combine the strengths of HMMs and FHMMs. These recent developments will be reviewed, as will expected future research and application prospects.

Finally, this comparison study will lead to a deeper understanding of the similarities and differences between HMMs and FHMMs, providing insights into their strengths, limitations, and applicability. The study's findings will be useful for researchers and practitioners looking to use these models in a variety of areas, ultimately improving the effectiveness of modelling sequential data and making educated decisions based on observed data.

## II. HIDDEN MARKOV MODEL

A Hidden Markov Model is a stochastic automaton that is finitely learnable. The two aspects of it can be summed up as a type of double stochastic process:

- A finite number of states that are each typically connected to numerous variable probability distributions form the initial stochastic process. A collection of probabilities known as transition probabilities are used to statistically analyse the transitions between the various states.
- In the second stochastic process, an event can be observed in any state. The states are "hidden" from the observer since we only analyse what we observe without examining the states at which it occurred; this is how the name "Hidden Markov Model" came about states, state probabilities, transition probabilities, emission probabilities, and initial probabilities all together to form a hidden Markov model.

### 2.1 Assumptions of HMM

1. Markov's assumption
2. Stationary assumption
3. Output independence assumption

### 2.2 Elements of HMM

The hidden Markov models are characterized by the following

1. The number of states in the model, N. The set of states is denoted as

$$S = \{ S_1, S_2, S_3, \dots, S_N \} \text{ and the state at time } t \text{ in } q_t$$

2. The number of distinct observations symbols per state, M (discrete alphabet size). The individual symbols are denoted by

$$V = \{ v_1, v_2, v_3, \dots, v_m \}$$

3. The state transition probability distribution,  $A = \{ a_{ij} \}$ ;

$$a_{ij} = P\{q_{t+1} = S_j / q_t = S_i\}; 1 \leq i, j \leq N \tag{2.1}$$

4. The observation symbol probability distribution in the state

$$\{b_j(k) = P\{v_k = t / q_t = S_j\}; 1 \leq j \leq N, 1 \leq k \leq M \tag{2.2}$$

5. The initial state distribution

$$\pi = \pi_i = P\{q_1 = S_i\}; 1 \leq i \leq N \tag{2.3}$$

The HMM  $\lambda = (A, B, \pi)$  generate an observation sequence,  $O = O_1, O_2, O_3, \dots, O_T$ . In By values, A, B and  $\pi_i$  is one of the symbols from V and T denotes the number of observations in the sequence

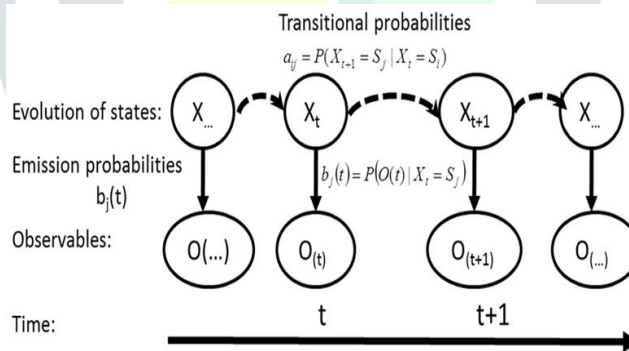


Fig.1: A hidden Markov model system's structure and historical evolution process<sup>[45]</sup>

### 2.3 Types of HMM

<p><b>Ergodic Model</b></p>		<p>Ergodic HMMs enable transitions between any two hidden states, resulting in a completely linked graph. Ergodic HMMs, unlike left-to-right HMMs, do not impose any order or constraint on state transitions.</p>
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<p><b>Left-right Model</b></p>		<p>Left-to-Right HMMs are a type of HMM where the fundamental states in a directed acyclic graph change from left to right. For modelling sequences that have a temporal or sequential order, such as speech recognition or handwriting recognition, this type of HMM is frequently used.</p>
<p><b>Hybrid model</b></p>		<p>A hybrid Hidden Markov Model is a version of the classic HMM that enhances its capabilities by combining the HMM framework with other modelling methodologies. A hybrid HMM often includes additional components or characteristics that increase the HMM's modelling and prediction accuracy.</p>

**2.4 Basic Problems for HMM**

<p>Evaluation</p>	<p>Given <math>O = O_1, O_2, \dots, O_T</math> and <math>\lambda = (A, B, \pi)</math>, compute <math>P(O \lambda)</math>, the probability of the observation sequence, given the model</p>	<p>To find the probability of the observations generated by the models -Forward Algorithm</p>
<p>Decoding</p>	<p>Given <math>O = O_1, O_2, \dots, O_T</math> and <math>\lambda = (A, B, \pi)</math>, find the state sequence <math>Q = q_1, q_2, \dots, q_T</math> that best explains the observation.</p>	<p>To identify the optimal state sequences – Viterbi Algorithm</p>
<p>Learning</p>	<p>How to adjust the model parameters <math>\lambda = (A, B, \pi)</math>, to maximize <math>P(O \lambda)</math>.</p>	<p>To optimise the model parameters to best describe how a given observation sequence comes out Baum- Welch Algorithm</p>

**III. FUZZY HIDDEN MARKOV MODEL**

A fuzzy hidden Markov model (FHMM) is a variant of the classic hidden Markov model (HMM) that includes uncertainty or fuzziness in the model parameters and data. It is a probabilistic model for analysing sequential data in which the underlying system is considered to be in a hidden state that evolves. The hidden states in an FHMM are not directly observed, but the model looks for them based on the observed data. The representation of uncertainties in the model distinguishes an FHMM from a normal HMM. FHMM allows for uncertainty in model parameters and data rather than presuming that they are crisp or precise values.

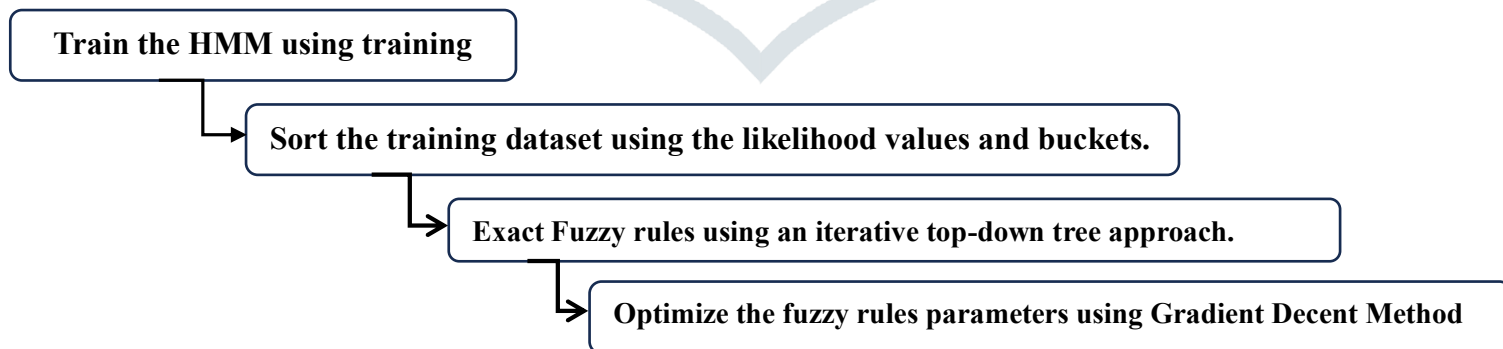


Fig.2: The Block Diagram of the HMM-Fuzzy Model

**IV. ALGORITHMS CREATED IN HMM AND FHMM**

Numerous methods have been developed for both Hidden Markov Models (HMMs) and Fuzzy Hidden Markov Models (FHMMs) to handle a variety of tasks such as model training, state estimation, and decoding. Here are some HMM and FHMM algorithms that are used frequently,

#### 4.1 Hidden Markov Model (HMM) Algorithms

- **Forward Algorithm:** Given an HMM model, this technique calculates the likelihood of observing a specific set of observations. It effectively determines the forward probability for each time step using dynamic programming.
- **Backward Algorithm:** The backward method computes the likelihood of viewing the remaining sequence of observations from a specific condition at a specific time, much like the forward algorithm does. The backward probabilities are also calculated using dynamic programming.
- **Viterbi Algorithm:** The most likely set of hidden states (Viterbi path) that corresponds to a given set of observations is found using the Viterbi algorithm. It uses dynamic programming to quickly search the state space and identify the best order.
- **Baum-Welch Algorithm:** An HMM model is trained using this approach using a set of observed sequences. It iteratively estimates the model's parameters, such as transition probabilities and emission probabilities, using the Expectation-Maximization (EM) technique.

#### 4.2 Fuzzy Hidden Markov Model (FHMM) algorithms:

- **Fuzzy State Estimation Algorithm:** The fuzzy membership values of each state at each time step are estimated using this approach. To address uncertainties and express the level of membership of each state at a specific time, it uses membership functions and fuzzy logic.
- **Fuzzy Forward-Backward Algorithm:** This approach computes the fuzzy forward and backward probabilities, much as the forward-backwards algorithm in HMMs. Taking into account both current and future observations (ahead) or current and past observations (backwards), these probabilities describe the possibility of being in a specific fuzzy state at a specific time.
- **Fuzzy Viterbi Algorithm:** The traditionally used Viterbi method is expanded by the fuzzy Viterbi algorithm to accommodate fuzzy states. By taking into account the fuzzy membership values and their transitions, it determines the most likely fuzzy state sequence (fuzzy Viterbi path) that corresponds to a given set of observations.
- **Fuzzy Baum-Welch Algorithm:** To estimate the model parameters of an FHMM, this approach extends the Baum-Welch algorithm. Based on the observed sequences, the fuzzy transition and emission probabilities are computed using fuzzy logic and membership functions.

#### V. LITERATURE SURVEY

Applications	HMM		FHMM	
	Authors	Description	Authors	Description
Speech Recognition	Steve Young <sup>[81]</sup>	HMM-based continuous speech recognition system and then explain the major areas of refinement incorporated into contemporary day systems	Dat Tran and M. Wagner <sup>[16]</sup>	FHMM for speech and speaker recognition
	Mark Gales and Steve Young <sup>[58]</sup>	HMM-based LVCSR system, and then detail the numerous improvements required to attain state-of-the-art performance.	Jia Zeng and Zhi-QiangLiu <sup>[43]</sup>	Extension of HMM based on the type-2 fuzzy set
	SJ Young and PC Woodland <sup>[71]</sup>	HMM-based phone recognition on the TIMIT database	A.D. Cheok, et al. <sup>[1]</sup>	New generalised fuzzy HMM to speech recognition
	MJF Gales and SJYoung <sup>[54]</sup>	Describes the issue of automatic voice identification in the face of background noise	IntanNurmaYulit a,TheHouwLion g and Adiwijaya <sup>[39]</sup>	Developed for the classification of Indonesian speech
	JAN Flores and SJ Young <sup>[41]</sup>	Robust voice recognition at low signal-to-noise ratios and evaluation of the CSS-PMC approach using the Noisex 92 database	Dat Tran and Michael Wagner <sup>[17]</sup>	Statistical modelling methods for voice recognition that is generalised fuzzy
	V.Valtchev, etal. <sup>[93]</sup>	Big vocabulary recognition system based on continuous density HMMs using the maximum mutual information estimation (MMIE) criterion	Asghar Taheri, Mohammad Reza Tarihi and HadiVafadarAli <sup>[85]</sup>	Fuzzy NN models and fuzzy hidden Markov models for speaker recognition
	M.J.F Gales and S J. Young <sup>[22]</sup>	Automatic speech identification in the face of interfering noise	Tu Van Le, D. Tran and M. Wagner <sup>[94]</sup>	Fuzzy codebooks are represented using the Gibbs distribution.
	Silke Witt and Steve Young <sup>[79]</sup>	Automated speech recognition techniques based on HMM	Jia Zeng and Zhi-QiangLiu <sup>[99]</sup>	Interval type-2 FHMMs by extending HMMs with fuzzy sets (FSs) and fuzzy logic systems

	D.Pye,P.C.Woodl and S.J.Young <sup>[15]</sup>	Overview of the HTK speech recognition system	Jia Zeng and Zhi-QiangLiu <sup>[100]</sup>	Type-2 FHMMs and a unique extension of HMMs
	Kai Yu, Heiga Zen, Francois Mairesse and Steve Young <sup>[47]</sup>	Experiments on a word-level emphasis synthesis problem reveal that all context-adaptive training approaches outperform the conventional full-context-dependent HMM strategy	Rania.M.GhoniemandKhaledShaa lan <sup>[23]</sup>	Presented a revolutionary Arabic speaker verification technique that is not dependent on text

	Steve Young <sup>[82]</sup>	Evolution of the statistical modelling approaches that, underlie modern systems, namely HMMs and N-grams		
	Habib Ibrahim and Asaf Varol <sup>[27]</sup>	Described the investigation of some models of speech recognition systems, their classification of speech, their relevance, and their application		
Bioinformatics	Mario Stanke and Stephan Waack <sup>[56]</sup>	The problem of discovering genes in eukaryotic DNA sequences	N.P. Bidargaddi, M.Chety and J. Kamruzzaman <sup>[8]</sup>	Sequence preference involved in protein structures using Fuzzy profile HMM
	De Fonzo, et al. <sup>[18]</sup>	Detailed information such as important algorithms, useful comparison, advantages and disadvantages inHMMs in bioinformatics.	ChrysaCollyda,et al. <sup>[13]</sup>	FHMM-based technique for matching multiple genomic or proteomic sequences.
	Lukas Zimmermann, et al. <sup>[52]</sup>	Profile HMMs for numerous model organisms and domain datasets	Bidargaddi, et al. <sup>[9]</sup>	Examine the characteristics and performances of three different fuzzy profile HMMs.
	Han Xu, eal. <sup>[29]</sup>	HMM technique to identify differential histone modification sites across the genome using ChIP-seq data		
	Mario Stankeet al. <sup>[57]</sup>	Gene prediction in eukaryotes using a generalised hidden Markov model		
	K.Karplus, C. Barrett and R. Hughey <sup>[46]</sup>	HMM method (SAM-T98) for locating remote homologs of protein sequences		
	Ronesh Sharma et al. <sup>[69]</sup>	HMM profiles to predict MoRFs in protein sequences		
	Jonathan Widomet al. <sup>[51]</sup>	Predict nucleosome placement Model of Hidden Markov Chains		
	Kiyoshi Asai, Satoru Hayamizu and Ken'ichiHanda <sup>[48]</sup>	HMM predicts the secondary structure of proteins	C Collyda, et al. <sup>[12]</sup>	Phylogenetic analysis is improved by fuzzy hidden Markov model alignments
	SL Cawley and L Pachter <sup>[78]</sup>	HMM, sampling to gene discovery and alternative splicing.		
	Pei Chen, RuiLiu et al. <sup>[62]</sup>	HMM detects crucial states before phase transitions in complicated biological systems		
	Jakob Skou Pedersen and Jotun Hein <sup>[42]</sup>	HMM of genome structure and evolution is used to discover genes		
Soren Mork and Ian Holmes <sup>[80]</sup>	Evaluate bacterial gene-finding HMM structures			
<b>Time Series</b>	YingjianZhang <sup>[96]</sup>	HMM deals with two major issues of financial time series modelling	Sheng-Tun Li and Yi-Chung Cheng <sup>[75]</sup>	HMM has been presented and demonstrates the improved accuracy of the suggested model over conventional fuzzy time series models

	Tim Oates, Laura Firoiu and Paul R. Cohen <sup>[89]</sup>	Time Series Clustering Using Hidden Markov Models and Dynamic Time Warping	Ahmed.Salawud en, et al. <sup>[72]</sup>	HMM with Particle Swarm Optimisation and Genetic Algorithm to forecast fuzzy time series
	ShimaGhassemp our, Federico Girosi and AnthonyMaeder <sup>[77]</sup>	Multivariate time series with variables that can take both category and continuous values	JinboLi, et al. <sup>[44]</sup>	HMM-based fuzzy model by comparing it to a set of artificial and freely accessible multivariate time series
	Francesco Lagona, Marco Picone, and Antonello Maruotti <sup>[20]</sup>	Bivariate time series of intensities and angles, a new hidden Markov model	Yi-Chung Cheng and Sheng-Tun Li <sup>[95]</sup>	HMM-based forecasting model that addresses performance degradation by creating a novel fuzzy smoothing technique
	HajoHolzmann, et al. <sup>[28]</sup>	Time series based on Hidden Markov models	Saurabh Bhardwaj,etal. <sup>[74]</sup>	To study the sun radiation with accuracy by using FHMM
	Muhammad Iqbal Ch, et al. <sup>[32]</sup>	A new hidden Markov model is constructed, its parameters are estimated, and the state space model is explained	Ahmet.Salawude en, et al. <sup>[2]</sup>	HMM and genetic Algorithmbased FTS forecasting model
	Mark Thyer and George Kuczera <sup>[59]</sup>	HMM was used to calibrate the model to multi-site rainfall data	M.A.Teixeiraand. Zaverucha <sup>[88]</sup>	FHMM predictor, a new hybrid system that combines fuzzy logic with dynamic Bayesian networks
	Bernhard Knab et al. <sup>[6]</sup>	Hidden Markov Model-Based Clustering and its Application to financial time-series Data	S. Sridevi, et al. <sup>[70]</sup>	New HMM and fuzzy logic-based framework for time series data forecasting
	LngmarVisser,M aareE.J.Raijmake rs and Han L.J.vander Maas <sup>[38]</sup>	Hidden Markov models for studying and characterising (individual) time series	Sheng-Tun Li and Yi-Chung Cheng <sup>[76]</sup>	Handle two-factor forecasting issues and offer a unique FHMM-based forecasting model
	Alina Delia Calin <sup>[5]</sup>	HMM is used for variation in the accuracy of multiple time series classifiers	Md.Rafiul Hassan, et al. <sup>[35]</sup>	Fuzzy logic and the HMM for creating a fuzzy model to forecast non-linear time series data
<b>Medical Diagnosis</b>	Laurent Jeanpierre and François Charpillet <sup>[50]</sup>	Proposed medical diagnosis using Hidden Markov Models.	Harun Uğuz , Ahmet Arslan and İbrahim Türkoğlu <sup>[91]</sup>	A biomedical diagnosis system for pattern recognition with normal and abnormal classes has been developed, and the current study demonstrates that choosing the initial FHMM
	Harun Uğuz, Ahmet Arslan and İbrahim Turkoglu <sup>[33]</sup>	A biological diagnosis system for pattern recognition with normal and pathological classes has been created	Zairan Li, et al. <sup>[97]</sup>	Enhanced FHMM, plantar pressure image fusion for diabetes mellitus comfort is performed
	Alexandre Bureau, Stephen Shiboski and James P. Hughes <sup>[4]</sup>	Continuous time-hidden Markov models are used to investigate misclassified disease outcomes	Harun Uğuz, et al. <sup>[90]</sup>	A biomedical system in HMM has been placed to categorise the TCD signals and proposed fuzzy technique
	Farzan Madadzadeh, et al. <sup>[19]</sup>	The hidden Markov Model is used to predict liver disease	Harun Uğuz, et al. <sup>[92]</sup>	Fuzzy discrete HMM for heart valve disease detection.
	RıdvanSaraçoğlu <sup>[68]</sup>	Heart valve disease classification using a Hidden Markov model and PCA for dimension reduction.		
	Rafid Sukkar, et al. <sup>[67]</sup>	Hidden Markov Models are used to model disease development		
	Francisco J, et al. <sup>[21]</sup>	Alzheimer's Disease Using Hidden Markov Models-based Paths		
	Changyue Song <sup>[11]</sup>	ECG-based discriminative Hidden Markov Model approach for detecting obstructive Sleep Apnea		
	Alessandro Daidone, et al. <sup>[3]</sup>	HMM in a comprehensive framework and a formalism for		

		modelling such over-time diagnosis scenarios and finding effective answers		
	Tarik Al-Ani, et al. <sup>[87]</sup>	Diagnosis of sleep apnea syndrome and automatic diagnosis approach based on HMM is developed.		
	Hang Wu, Sahong Kim, and KeunsungBae <sup>[30]</sup>	Heart disease detection using an HMM using heart sound signals		
	R.V. Andraeo, B. Dorizzi and J. Boudy <sup>[66]</sup>	HMM strategy for real-time beat segmentation and ECG categorization		
<b>Handwriting Recognition</b>	Michel Gilloux <sup>[60]</sup>	HMMs in handwriting recognition for various circumstances and reviewed some alternative techniques	R. Budsayaploron, W. Asdornwiset and S. Jitapunkul <sup>[10]</sup>	The HMM and fuzzy logic classifier used for online Thai handwritten character identification
	Isa Ibrahim <sup>[40]</sup>	Hand Gesture Recognition System Based on the Hidden Markov Model for Computer Vision Applications	N. Rodrigues Gomes and Lee Luan Ling <sup>[24]</sup>	Identifying features in handwritten words based on fuzzy set theory
	SanasamInungan bi <sup>[73]</sup>	HMM in a comprehensive survey of handwritten document recognition and analysis.	Azizah Suliman, et al. <sup>[83]</sup>	A hybrid approach to handwritten character recognition using HMM and fuzzy logic
	H. Bunke, et al. <sup>[25]</sup>	Off-line detection approach for cursive handwriting based on hidden Markov models	Ashutosh Malaviya and Liliane Peters <sup>[55]</sup>	Complex handwriting patterns can be described using fuzzy HMM linguistic principles
	R. El-Hajj, L. Likforman-Sulem and C. Mokbel <sup>[65]</sup>	An analytical method explains a 1D HMM offline handwriting recognition system	T. R. Indhu and V. K. Bhadran <sup>[37]</sup>	Online handwriting recognition method for Malayalam utilising SFAM artificial neural networks
	Biadsy, et al. <sup>[7]</sup>	HMM-based approach to address the majority of the problems associated with recognising Arabic script		
	M. Zimmermann and H. Bunke <sup>[53]</sup>	Optimise the number of states of linear left-to-right HMM and compared different length modelling schemes with a handwriting recognition system using off-line images of cursively handwritten English words		
	Hanhong Xue and V. Govindaraju <sup>[31]</sup>	Discrete symbols and continuous attributes into structural handwriting features and models them using HMM that emit transitions and states		
	Prem Natarajan, et al. <sup>[63]</sup>	A script-independent methodology based on HMM for multilingual offline handwriting recognition		
	Cristian López, et al. <sup>[14]</sup>	Simulated outer race fault and an application phase in which the CHMM is utilised to estimate the characteristic frequency directly from the unprocessed measured signal	YuZhao, YongqianLiu and RuimingWang <sup>[101]</sup>	Application of fuzzy scalar quantization using a hidden Markov model for wind turbine fault diagnosis
	Mitchell Yuwono, et al. <sup>[61]</sup>	An automatic bearing fault diagnosis approach based on Hidden Markov Models		
	Liang Cao, et al. <sup>[49]</sup>	A coupled hidden Markov fault diagnosis approach based on a Gaussian mixture model.		
	Tao Liu, et al. <sup>[86]</sup>	CHMM is presented to detect the bearing failure based on the chosen singular characteristics		

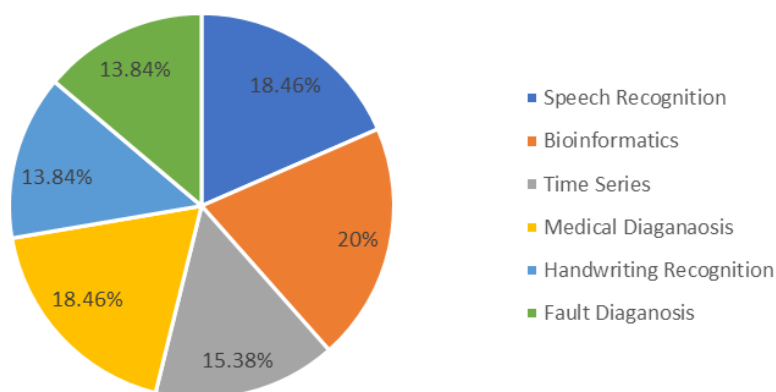
	H. Ocak and K.A. Loparo <sup>[26]</sup>	Bearing fault identification and diagnosis method based on vibration signal hidden Markov modelling		
	Zefang Li, et al. <sup>[98]</sup>	HMM for data-driven bearing fault identification		
	T. Marwala, U. Mahola and F.V. Nelwamondo <sup>[84]</sup>	The fractals-based bearing Fault Detection using HMMs and Gaussian Mixture Models		
	QunZhao <sup>[64]</sup>	Coupled HMM in bearing fault diagnosis and performance degradation assessment		
	Hongwu Xu <sup>[36]</sup>	Bearing defect diagnosis approach based on the Hidden Markov Model		

**VI. COMPARATIVE STUDY**

The two model methods applied in a variety of areas are Fuzzy Hidden Markov Models (FHMM) and Hidden Markov Models (HMM). HMMs are statistical models that depend upon the idea that observable data is produced by a Markov process with hidden states. They are widely utilised in fields like bioinformatics and speech recognition and are computationally effective. HMMs do not, however, explicitly take into consideration fuzziness or uncertainty in the transitions and emissions of hidden states. However, FHMMs extend above HMMs by adding fuzzy logic to model uncertainty.

To describe the degree of membership of data points to various hidden states, they offer fuzzy sets and membership functions. This enables a more adaptable modelling strategy that can account for fuzziness and ambiguity. FHMMs are useful in fields where uncertainty is prevalent, such as control systems and uncertain decision-making. FHMMs offer more complex representations of hidden states, but they could also be more computationally taxing and require more work during parameter estimation. In comparison research, performance on certain tasks would be assessed while taking into account variables like accuracy, resilience, computing efficiency, and interpretability.

**Percentage of paperwork taken to consider the study of various applications**



**Fig. 3: Number of Papers in HMM**

**Percentage of Paperwork in FHMM**



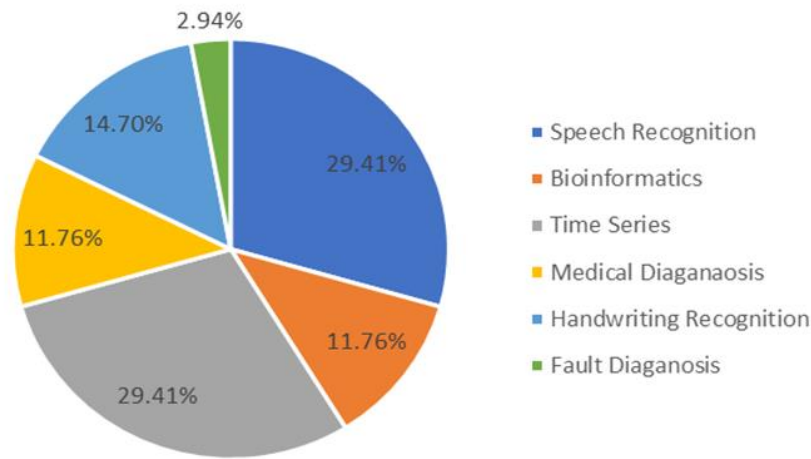


Fig.4: Number of Papers in FHMM

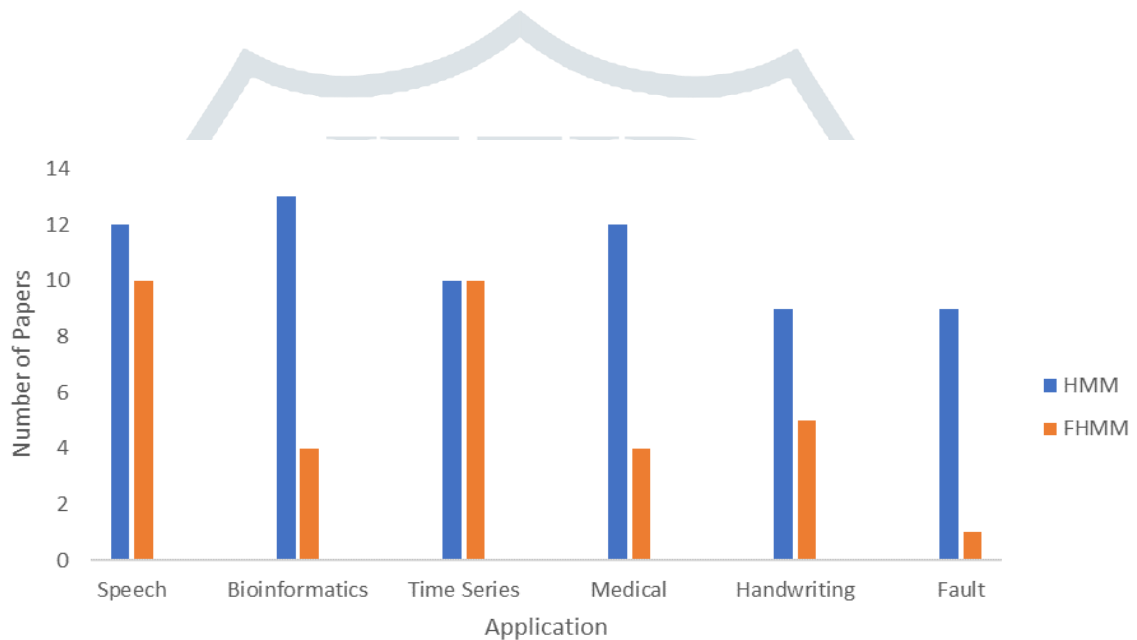


Fig.5: Represent the Bar diagram of HMM and FHMM

**VII. CONCLUSION**

In conclusion, the comparison of HMMs and FHMMs shows that based on the particular situation and requirements, both models have different characteristics and advantages. HMMs are excellent for applications where the data is straightforward and computational efficiency is important since they are simple, scalable, and interpretable. By incorporating a hierarchical structure, FHMMs, on the other hand, offer a higher level of representation capacity and enable the modelling of complicated systems with numerous interacting variables. FHMMs are more complicated and computationally difficult, but they perform well in situations when the data is hierarchical. In the end, the decision between HMMs and FHMMs should be made based on a variety of considerations, including the system complexity, the need for interpretability, the availability of computational resources, and the trade-off between model complexity and performance.

**VIII. FUTURE WORK**

The comparative study's upcoming work will concentrate on several areas, the development of advanced inference algorithms to increase computational effectiveness and scalability, the investigation of enhanced hybrid models that combine HMMs and FHMMs with other machine learning techniques, the consideration of robustness to noisy or incomplete data, and the development of adaptive model selection methods.

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