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Determining The Profit Maximisation Input Levels of Ecolab Inc. Using Multivariable Optimization

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1 Abstract

1.1 Statement of Task and Rationale

The purpose of this study is to optimize the product function in order to maximize the output level or minimize the cost function (with respect to the other constant), which will both result in maximum profit. Utilizing this information, the most efficient combination of Input Levels through substitution in the Cost and Product functions will be determined.

The objective of this study is to utilize multivariable calculus to optimize the *x (capital)* and *y (labour)* variables in the context of a production function. This will be achieved by identifying the point of intersection between the Production Quantity Function & Cost Function, at a predetermined production level. Subsequently, the research will expand on this analysis by incorporating a cost constraint function in order to optimize production levels within the constraints of a specified budget. This approach accounts for the reality that many businesses have financial limitations on their investment capabilities. For the purpose of illustrating the generalised equations, I have used the financial data of an American public limited company - Ecolab Inc. I have calculated the optimal combination of input levels for profit maximization based on the interpolated data to obtain a product function.

2 Cobb Douglas Function Base Model

2.1 Background

Several methods can be employed to model a business' production level. Product function determines what level of resources are required to achieve a given output level. For this study, I will utilize the most widely recognized function- the Cobb-Douglas Function.

This study employs the economic model proposed by Knut Wicksell, which was subsequently supported by statistical evidence provided by Charles Cobb and Paul Douglas, commonly known as the Cobb-Douglas Function. However, it is important to acknowledge that this model has received criticism and has limitations.

In order to determine the degree to which the model reflects real-life working levels, these factors must be taken into account (see the analysis section).

The Cobb-Douglas Function, which is widely used in economics and econometrics, can take on a variety of forms, with the most general being a continuous function of **m** with different types of inputs. This can be mathematically represented as follows:

$$W = T\,(y_1, y_2, y_3, y_4 \ldots y_m) = B\,\left(y_1^{z_1} y_2^{z_2} y_3^{z_3} \ldots y_m^{z_m}\right) = B \prod_{l=1}^{m} z_l^{zl}$$

$$where \sum_{l=1}^{m} z_l = 1$$

The Cobb-Douglas Function can be represented mathematically as a continuous function **T**, with inputs: $y_1, y_2, y_3, y_4 \ldots y_m$ (the diverse inputs required for goods production: Capital, Labour, etc. where $y_m, m \in \mathbb{Z}^+$) used to produce the quantity of production (**W**). The other variables are: **B** = Total Factor Productivity (considers changes in technology and business efficiency over time) $z_1, z_2 \ldots z_m$ = Model Coefficients that represent the product's elasticity parameters. Elasticity parameters describe the responsiveness of quantity demanded or supplied of a good to changes in factors such as price, income, and substitute goods (for example: PED- Price Elasticity Demand). This investigation operates under the assumption that the goods under examination exhibit constant returns to scale (CRTS). CRTS refer to the property where an increase in all inputs by a certain proportion leads to an equivalent proportionate increase in output. Given this assumption, a necessary condition for the Cobb-Douglas production function to hold is that the sum of the model coefficients must be equal to 1[1]:

$$\sum_{l=1}^{m} z_l = 1$$

2.2 Providing CRTS in a Cobb-Douglas Function

As a result of its simplicity and ability to be manipulated mathematically for further differential equation computations, I have assumed CRTS making it the ideal profit-maximisation situation.

CRTS is the most favourable operational stage for a business from a profit perspective. The economies of scale behaviour exhibited by businesses is depicted in Fig. 1.1. CRTS yield the lowest average costs for firms, making it the most profitable period for them.

[1] Arrow, K. J., et al. "Capital-Labor Substitution and Economic Efficiency." *The Review of Economics and Statistics*, vol. 43, no. 3, 1961, pp. 225–50. *JSTOR*, https://doi.org/10.2307/1927286. Accessed 31 Jan. 2023.

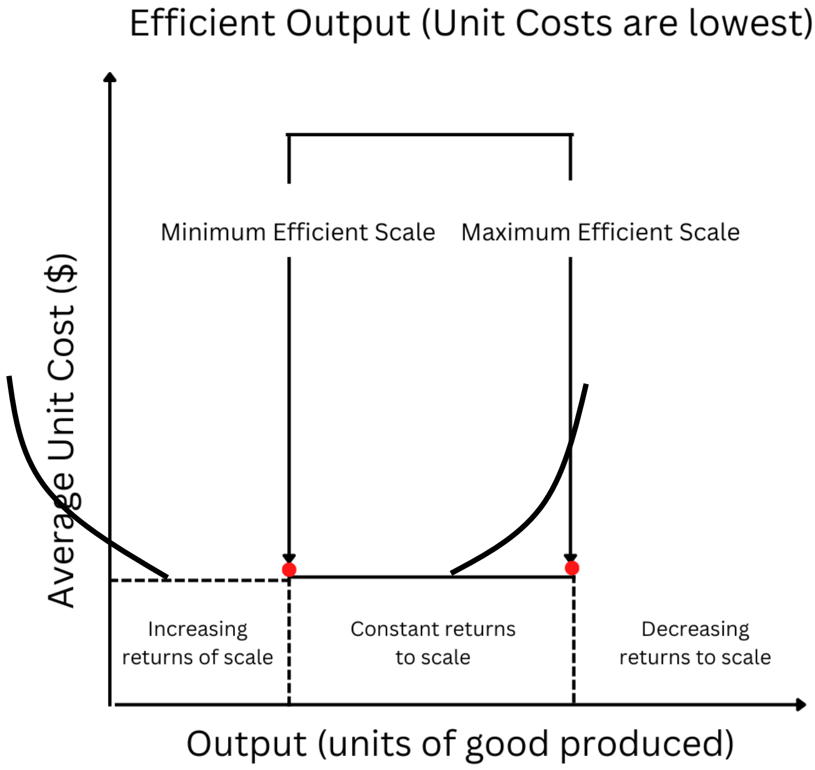

Fig. 1: A Firm's Economies of Scale in the Long Run (Average Cost vs Total Output)[2]

The graph above illustrates the economies of scale behaviour of firms in the long run in terms of the output (x-axis) and the average unit cost (y-axis). Economies of scale are cost advantages reaped by companies when production becomes efficient.[3] The firm's implementation of economies of scale, such as lower costs, increased productivity and greater returns, has led to a sustained increase in production. As a result, fixed costs are spread across a larger quantity of items, leading to decreased expenditure and higher profits. While utilizing economies of scale, the firms eventually enter the CRTS period (depicted as the horizontal line in the graph) which represents the profit-maximizing stage, as the firm operates at the most efficient scale, leading to a minimum unit cost of production. However, beyond this point, the firm experiences diseconomies of scale - when a firm grows so large that the costs per unit increase – due to increased disturbances in the supply chain: communication breakdown, lack of coordination, and loss of focus by the management and employees. This period is characterized by decreasing returns as it surpasses the efficiency scale.

Analysing using the Cobb Douglas function:

$$\boldsymbol{T\,(y_1, y_2, y_3, y_4 \dots y_m) = B\,(y_1^{z_1} y_2^{z_2} y_3^{z_3} \dots y_m^{z_m})}$$

In this case, increasing each variable by the scale factor of *n* implies:

$$\boldsymbol{T\,(ny_1, ny_2, ny_3 \dots ny_{m)} = B\,((ny_1)^{z_1}(ny_2)^{z_2} \dots (ny_m)^{z_3})}$$

$$\boldsymbol{T\,(ny_1, ny_2, ny_3 \dots ny_{m)} = B\,(n^{z_1+z_2+z_3+\cdots+z_m})(y_1^{z_1} y_2^{z_2} \dots y_m^{z_m})}$$

Only when $z_1 + z_2 + z_3 + \cdots + z_m = 1$ or better represented as

[2] "Long Run Cost Analysis and Economies of Scale- Supply and Production Theory". *Coursera, University of Michigan*

[3] Dorton, Ian, and Jocelyn Blink. *IB Economics*. Oxford: Oxford UP, 2015. Print.

$$\sum_{l=1}^{m} z_l = 1$$

Since it implies that $T\ (ny_1, ny_2, ny_3 \dots ny_m) = nT(y_1, y_2, y_3 \dots y_m)$

The above expression implies that the quantity of production will enhance by n factor when each variable is increased by a factor of n.

2.3 Mathematical Variables

Numerous variables have been utilized, to make it easier to understand, I have defined them at the beginning.

VARIABLE SPECIFICATION	**SYMBOL**
Function T with inputs $y_1Y_2Y_3Y_4 \dots Y_m$ (Generalised Cobb-Douglas Function)	$\boldsymbol{T}\ (y_1, y_2, y_3, y_4 \dots y_m)$
Total Factor Productivity (Generalized Cobb-Douglas Product Function)	B
The Coefficients of the Model which denote the Elasticity Parameters in the Generalized Cobb-Douglas Production Function	$z_1, z_2, z_3, z_4 \dots z_m$
Total Quantity Produced (Cobb-Douglas Production Function)	W
First Model Coefficient (Two Factor Cobb-Douglas Production Function)	α
Second Model Coefficient (Two Factor Cobb-Douglas Production Function)	β
Total Factor Productivity (Two Factor Cobb-Douglas Production Function)	γ
Amount of Additional Quantity Produced with an Additional Capital Unit (Marginal Product of Capital)	MP_I
Amount of Additional Quantity Produced with an Additional Labour Unit (Marginal Product of Labour)	MP_O
Input: Capital	K
Input: Labour	O
Market Exchange Rate: Price Ratio of the Input Levels of Labour and Capital.	MGF
Marginal Rate of Technical Substitution (Substitution Rate of Inputs): one input unit is substituted for one unit of another input at an Equivalent Level of Production.	$MGIS$
Rental Rate of Capital	D
Wage Rate	J
Cost Function	$C(K, O)$
Maximum Budget Limit	U

3 Two Factor Cobb-Douglas Function

Firms typically utilize two **FoP (Factors of Production)**: labour and capital. Hence, for this study, I'll take only these into account. In order to mathematically model the production levels **($\boldsymbol{W}$)** of a firm based on Capital ($\boldsymbol{K}$) and Labour ($\boldsymbol{O}$), the Two-Factor Cobb-Douglas production function will be employed:

$$W = T\ (K, O) = \gamma K^{\alpha} O^{\beta}$$

Where α and β are constants (previously referred to as model coefficients) that may fluctuate based on the business' nature. γ: total factor productivity constant (formerly referred to as **B** in the product function). It is important to acknowledge that the condition of CRTS is upheld in the context under consideration i.e. $\alpha + \beta = 1$

4 Data Interpolation to Find Product Function of Ecolab Inc.

I've obtained data from Ecolab Inc.'s[4] annual financial reports from the financial year 2006 to the financial year 2021, to find the function ***T*** of the firm's production, through identifying the constants α, β, & γ.

Year	Relative Stock Capital (Year 2006=100) (K)	Relative Number of Workers (Year 2006=100) (L)	Relative Production (P) Index of Manufacturers (Year 2006 = 100)
2006	100	100	100
2007	114.4	112.6	111.7
2008	79.1	114.8	125.4
2009	102.1	112.1	120.5
2010	115.3	114.5	124.4
2011	136.8	173.8	138.9
2012	165.5	176.7	241.8
2013	249.6	196.3	270.7
2014	257	205.1	291.7
2015	281.2	203.2	276.7
2016	294.3	205.6	268.7
2017	330.4	209.3	282.7
2018	382.7	211.8	295.5
2019	477.9	217	304.5
2020	555.6	190.2	269.2
2021	577.9	203.2	283.2

Table 1.1: Ecolab Inc's Data for Capital, Labour and Production over 16 years

In order to establish the functional form T (by determining the constants α, β and γ), interpolation of the available data for x-Capital, y-Labour and z-Quantity was performed. This was done by utilizing Grapher (graphing software) in order to derive the Two Factor Cobb Douglas Production Function.

[4] *Annual reports* (2006-2021) *Ecolab Inc. - Financials - Annual Reports.* Available at: https://investor.ecolab.com/financials/annual-reports/default.aspx (Accessed: December 3, 2022).

Fig. 2: Data's 3D Graph, optimized curve of *E(K,O)*

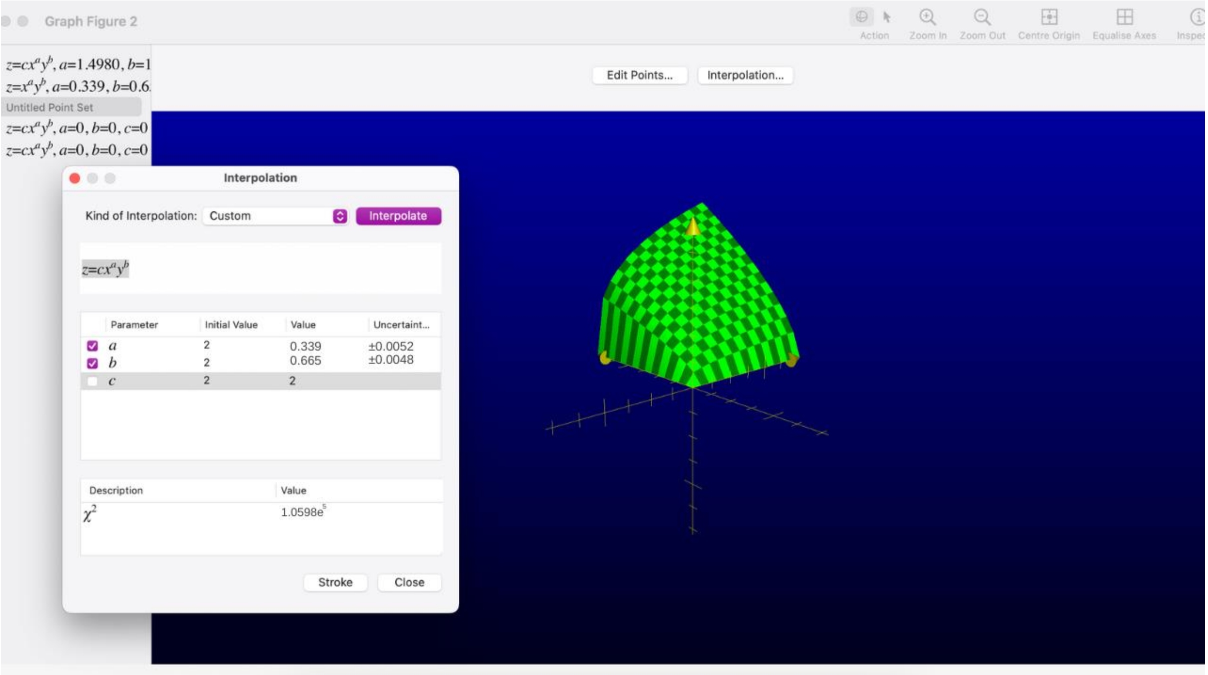

Fig. 2.1: Interpolation calculation for the data

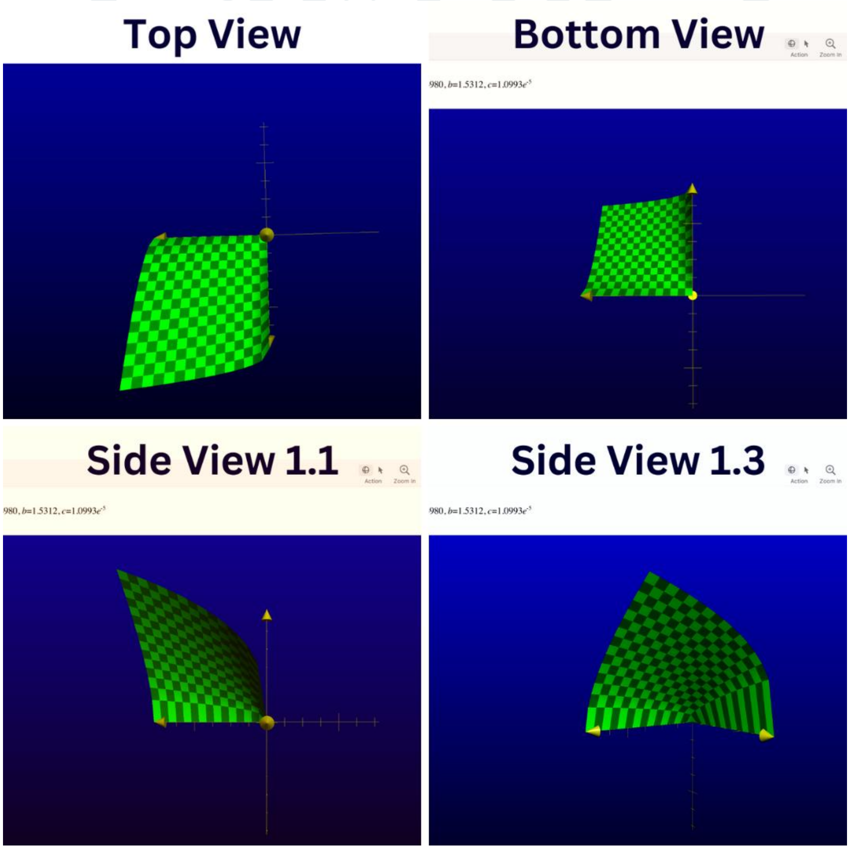

Fig. 2: Top, Bottom, Side View 1.1 and Side View 1.3 of the Curve

The three-dimensional optimized graph of the interpolation of the Cobb-Douglas production function provides a visual representation of the relationship between three variables - capital, labour, and production quantity - for Ecolab Inc. The graph highlights the optimized curve and helps in the derivation of the Two-Factor Cobb-Douglas Production Function. The multiple views of the graph offer a deeper understanding of the curve, data trend, and function by allowing for a comprehensive examination of the data.

I obtained the following results for these constants and the following function after doing interpolation:

α	0.339 ± 0.0052
β	0.655 ± 0.0048
γ	1

Hence, the product function for Ecolab Inc. is:

$$W = 1 \times K^{0.339}O^{0.655}$$

Furthermore, Ecolab Inc. is also demonstrating CRTS: $0.339 + 0.655 = 0.994 \approx 1$ taking the margin for error into consideration.

5 Input Choice Calculation to Minimize the Costs and Maximize the Quantity

This Ecolab Inc. production function can be utilized to identify the optimal levels of capital and labour that minimize costs for a specific production quantity. For this purpose, I will first define a number of other variables:

$$MP_O = Marginal\ Product\ of\ Labour = value\ of\ an\ additional\ unit\ of\ labour$$
$$Or, the\ amount\ of\ additional\ quantity\ produced\ with\ an\ additional\ unit\ of\ labour.$$

$$= \frac{\partial W}{\partial K} = \frac{\partial}{\partial K}T(K,O)$$
$$= \alpha K^{\alpha-1}O^{\beta}$$
$$= 0.339K^{-0.661}O^{0.655}$$

$$MP_I = Marginal\ Product\ of\ Capital = value\ of\ an\ additional\ unit\ of\ capital$$
$$Or, the\ amount\ of\ additional\ quantity\ produced\ with\ an\ additional\ unit\ of\ capital.$$

$$= \frac{\partial W}{\partial O} = \frac{\partial}{\partial O}T(K,O)$$
$$= \beta K^{\alpha}O^{\alpha-1}$$
$$= 0.655K^{0.339}O^{-0.335}$$

A key economic condition is that profits are maximized when the price level equals marginal revenue. A business must produce at its optimal conditions (as illustrated in Fig. 3 below) in order to maximize profits.

$$MGIS = MGF$$

$MGIS$ = Marginal Rate of Technical Substitution (substitution rate of inputs): No. of other inputs used to substitute an input, in order to sustain an identical production level:

$$MGIS = \frac{MP_O}{MP_I} = \frac{\frac{\partial W}{\partial K}}{\frac{\partial W}{\partial O}}$$

MGF = Market Exchange Rate: price ratio of both the input levels (Capital and Labour). By considering labour & capital as distinct products, the trade-offs involved when one input is increased or decreased at the expense of the other can be identified. This will allow us to evaluate how changes in the level of one input impacts the overall production, and how much benefit is derived from such changes. As a basis for further calculations, I have assumed a wage rate of $250 ($J$), and a rental rate of $100 ($D$), based on national averages. In order to determine the optimality condition, one must identify the point at which the substitutability curve meets the Cost Function plane, i.e., $C(K, O) = JK + DO = 250K + 1000$

$$MGF = \frac{J}{D} = \frac{250}{100}$$

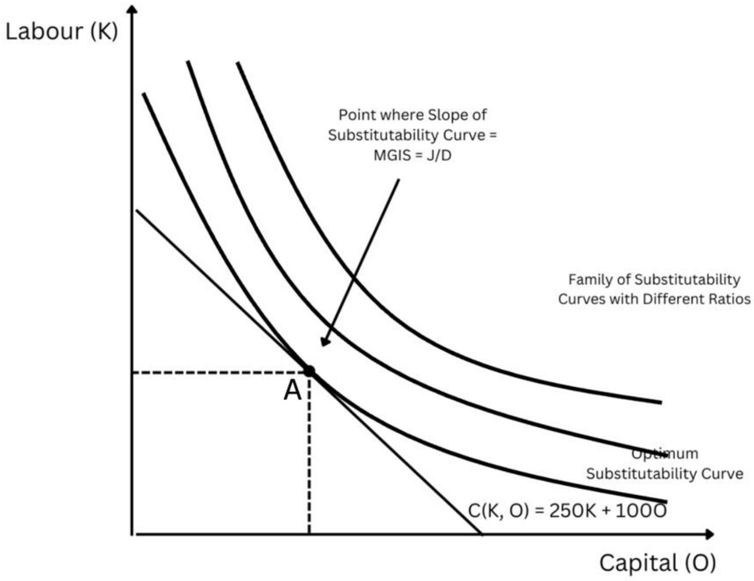

Fig. 3: Isocost Curve for 2 Different Inputs' Relationship & their Tangential Substitutability Relationship with Cost Curve C.

The Isocost Curve is a graphical representation of the relationship between two inputs - labour and capital - and their substitutability with respect to the cost curve C. The optimality condition is at the point where the slope of the substitutability curve, defined as the ratio of the wage rate to the rental rate, intersects with the cost function plane $C(K, O)$ at point A. A represents the combination of inputs that yields the minimum costs and, therefore, maximum profits.

Achieving optimality situation: (as depicted in the diagram that the condition of minimum costs occurs when $MGIS = MGF$)

$$MGIS = MGF$$
$$\Rightarrow \frac{MP_O}{MP_I} = \frac{J}{D}$$
$$\therefore \frac{0.339K^{-0.661}O^{0.655}}{0.655K^{0.339}O^{-0.335}} = \frac{250}{100}$$
$$\therefore \frac{O^{0.655+0.335}}{K^{0.339+0.661}} = \frac{2.5 \times 0.655}{0.339}$$
$$\therefore \frac{O}{K} = 4.830$$
$$\Rightarrow O = 4.830\ K$$

Therefore, Ecolab Inc. must satisfy the following condition in order to achieve cost-minimizing input levels: $O = 4.830\ K$. Upon substituting this condition into our initial product function:

$$W = K^{0.339}O^{0.655}$$
$$W = K^{0.339}(4.830\ K)^{0.655}$$
$$W = K^{0.339+0.655} \times (4.830)^{0.655}$$
$$W = 2.805K^{0.994}$$

Taking into consideration the demand for the product, the input cost minimization level can be determined by assuming a specific quantity of units to be produced. For example, assuming a hypothetical quantity of 300 units is required to be produced, the input cost minimization level will be:

$$300 = 2.805K^{0.994}$$
$$\Rightarrow K = \sqrt[0.994]{106.952}$$
$$\therefore K = 110.011 \approx 110 \text{ units}$$
$$\Rightarrow O = 531.353 \approx 531 \text{ units}$$

Therefore, the most cost-effective combination of units to produce 300 units: 110 units of labour and 531 units of capital, with cost $C = (110 \times 250) + (531 \times 100) = 27{,}500 + 53{,}100 = \$80{,}600$

5.1 Graphically Verifying the Result

To validate the accuracy of the method, described above, the graphical approach was utilized. As the objective is to optimize the production function for a specific quantity of 300 units, two initial conditions can be seen:

$$W = 300 \textit{ production quantity}$$
$$W = 1 \times K^{0.339}O^{0.655}$$
$$\textit{Note that the } Z - \textit{dimension is the quantity produced } (W) \textit{ or production level.}$$

The 3D graphs shown below were generated by inputting the above data into *Grapher* (graphing software).

Fig. 4: Optimization Condition

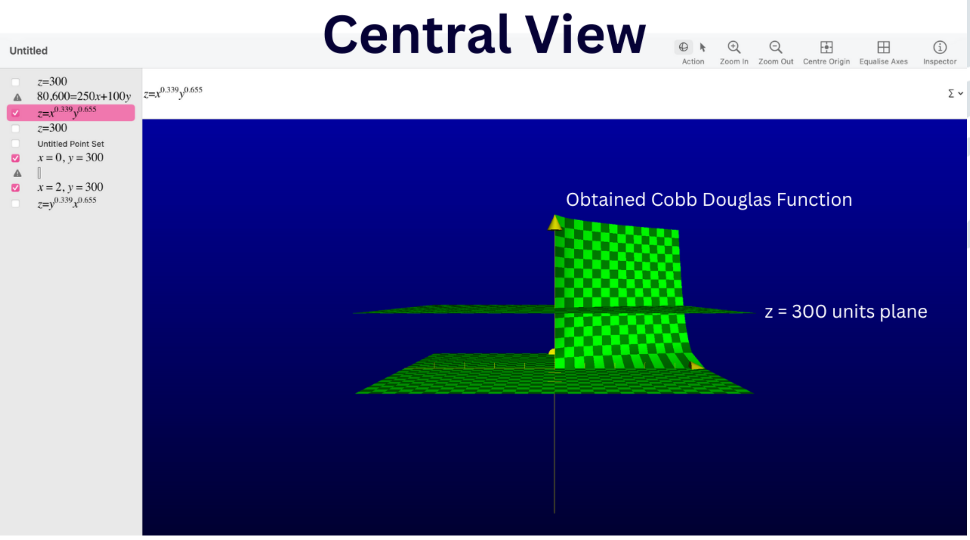

Fig. 4.1: Central View of the Optimization Conditions

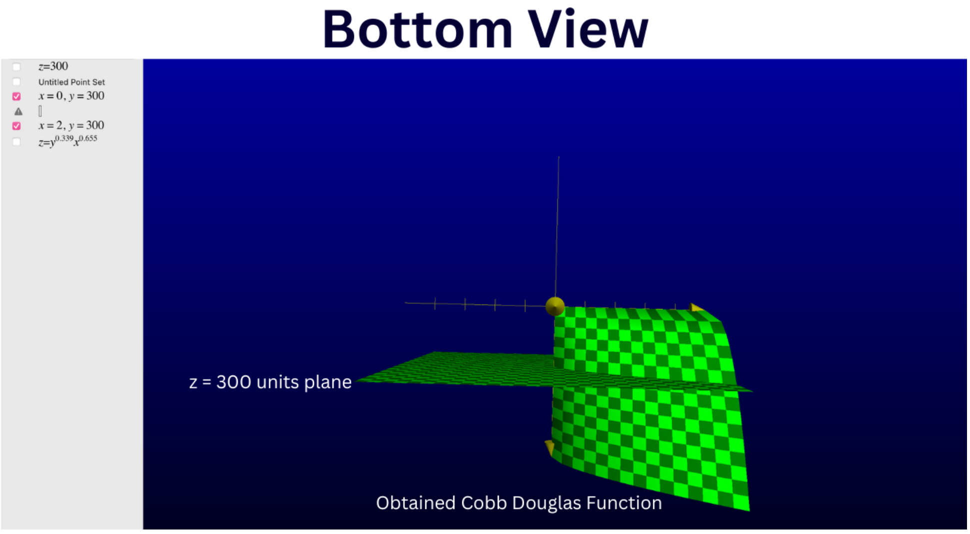

Fig. 4.2: Bottom View of the Optimization Conditions

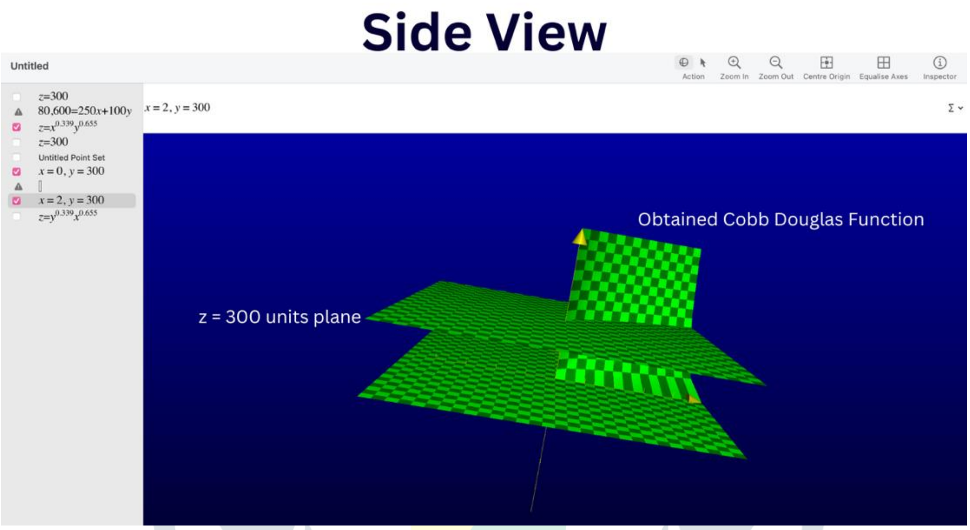

Fig. 4.3: Side View of the Optimization Conditions

For the answer to be true the following must be accurate: the plane of the equation $\$80{,}600 = 250K + 100O$ intersect the Cobb Douglas Product Function at the exact point that $H = 300$ plane intersects the function, that is $(K, O, W) = (110, 531, 300)$. This is depicted in Fig. 5. In this particular point of intersection, both curves intersect, thereby verifying the answer obtained by the method described above.

Fig. 5: Verifying the Calculation

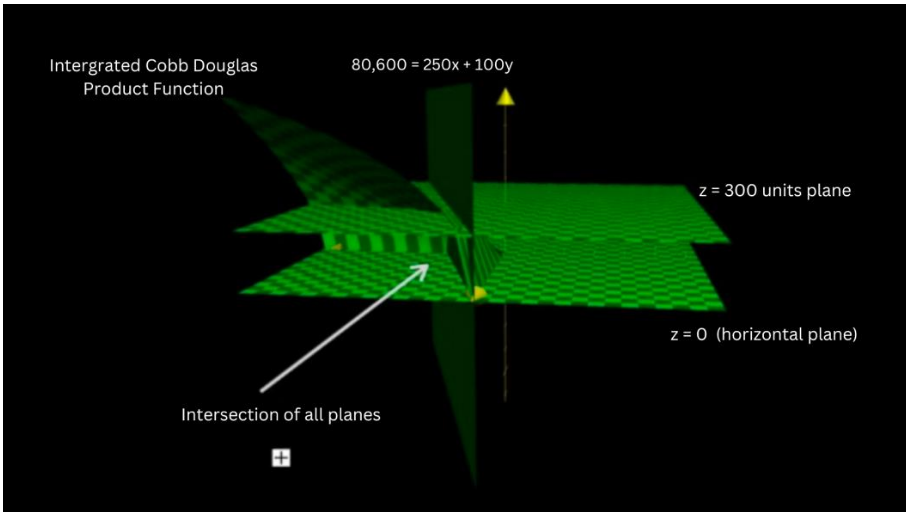

Fig. 5.1: Central View of the Graph

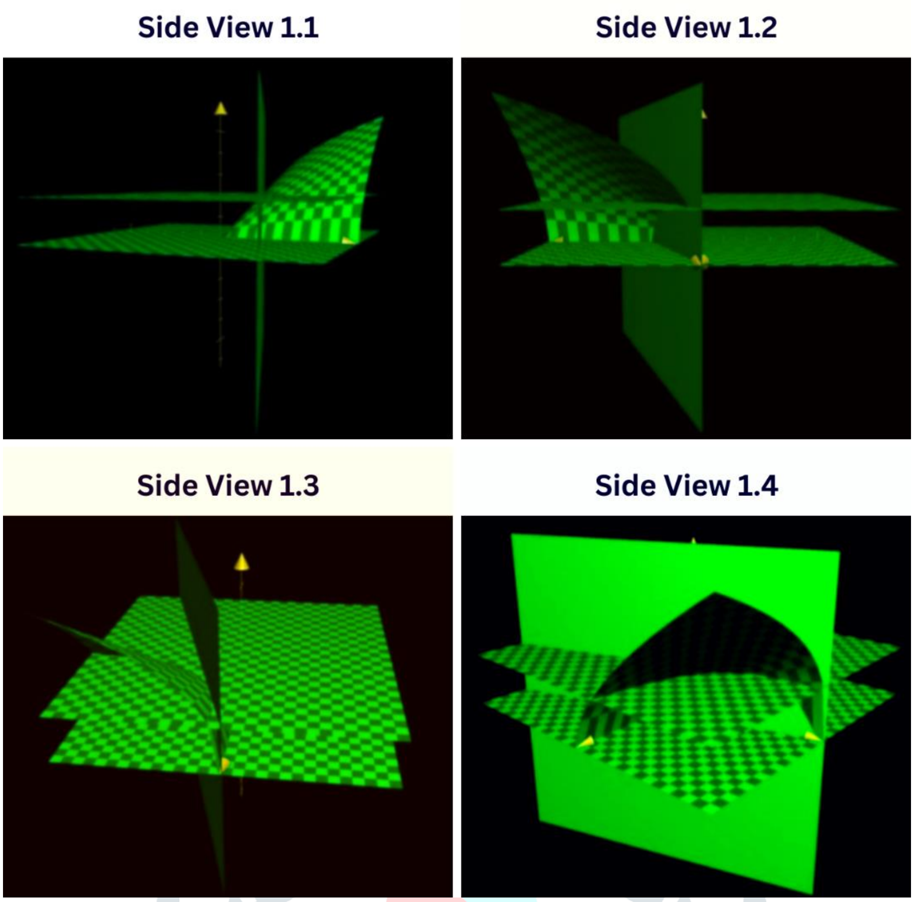

Fig. 5.2: Side View 1.1, 1.2, 1.3, 1.4 of the Graph

The graph presents the calculation, intersection and the integrated Cobb-Douglas Production Function in the third dimension. The multiple views of the graph offer a more in-depth understanding of the optimal input combinations for businesses and the integrated function by allowing for a comprehensive examination of the data.

This method is suitable for identifying optimal input combinations for businesses that operate without any limitations on costs. However, that only works in theory and businesses in the real world have to take budget and cost constraints into account. For instance, if the mentioned method gave Ecolab Inc. the input level production of capital (531 units) and labour (110 units), that constitutes an expense of:

$$(110 \times 250) + (531 \times 100) = 80{,}600$$

However, a dilemma will arise if the business is unable to sustain an input cost of $80,600. What if the firm's maximum budget constraint is $75,000, and they're unable to exceed this limit? Profit maximization can be achieved by identifying the highest producible quantity within a set cost constraint, taking into consideration the limitations of the budget function.

It is possible to describe the cost function here: C(K, O) where J and D are constants.

$$C(K,O) = WK + DO$$

Based on the assumption that X (a constant) represents a business firm's budget, which is the maximum amount of money that the firm can allocate to the production operation, we can describe the production function as follows: $W = W(K,O) = \gamma K^{\alpha} O^{\beta}$ While holding the constraint that $C(K,O) = X = WK + DO$. This conclusion directs us to the relationship between the cost and production function, which can be demonstrated through graphical representation:

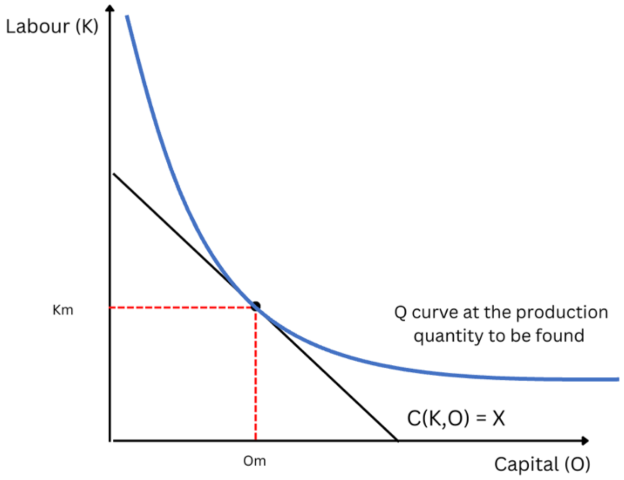

Fig. 6: Maximum Production Level's Horizontal Cross-Sectional Curve

The horizontal cross-sectional curve shows the relationship between capital and labour for a firm operating at its optimal level of efficiency. This information is used in decision-making regarding production and costs. Based on this, it is possible to establish a relationship between variables K and O and their gradients in order to solve for them. It can be deduced that where the functions are in tangency, their slopes will be parallel or proportional. It is imperative to acknowledge that the gradients cannot be equivalent. To establish this correlation, the Lagrange Multiplier technique will be employed for minimizing multi-variable functions within limitations. Over here we have to maximise the function,

$$W = \gamma K^{\alpha} O^{\beta} \; subject\ to\ constraint\ WK + DO,$$

$$\nabla C = \nabla(WK + DO) = \begin{pmatrix} W \\ D \end{pmatrix}$$

while considering that,

$$\nabla W = \nabla\left(\gamma K^{\alpha} O^{\beta}\right) = \begin{pmatrix} \frac{\partial W}{\partial K} \\ \frac{\partial W}{\partial O} \end{pmatrix} = \begin{pmatrix} \gamma \alpha K^{\alpha-1} O^{\beta} \\ \gamma \beta K^{\alpha} O^{\beta-1} \end{pmatrix}$$

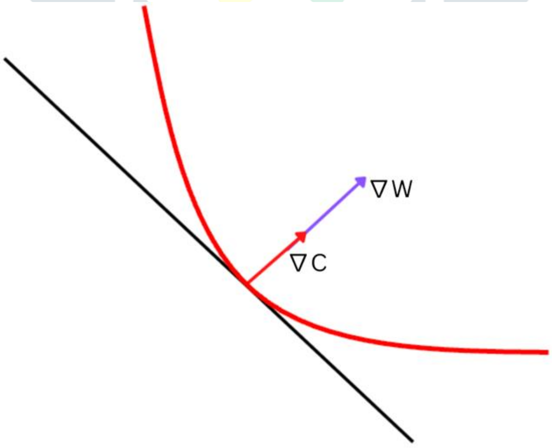

Fig. 7: Representing W and C functions' gradients

The figure illustrates the directly proportional relationship between the gradients of the total quantity produced (∇W) and the cost function (∇C). Given that $\nabla W \propto \nabla C$
$\Rightarrow \nabla W = \lambda \nabla C$ (λ is the proportionality constant)

By satisfying the optimality condition, we derive the Lagrangian Function (the proportionality constant λ has not yet been determined, and hence it's also an input to the function):

$$e\,(K, O, \lambda) = W(K, O) - \lambda C(K, O)\ so\ \nabla e = 0 \text{ at } (K_n, O_n, \lambda_n)$$
$$\nabla W(K_n, C_n) = \lambda\, \nabla W((K_n, C_n)$$

(Where $K_n, C_n$is the point to be determined for minimizing the costs; λ is the Lagrangian Multiplier = proportionality constant). Take a look at Fig. 7.

Therewith,
I've equated the gradients in both directions:

$$\begin{pmatrix} \gamma\alpha K^{\alpha-1}O^{\beta} \\ \gamma\beta K^{\alpha}O^{\beta-1} \end{pmatrix} = \lambda \begin{pmatrix} W \\ D \end{pmatrix}$$

In this way, we are able to obtain the equations:

$$1.\, \gamma\alpha K^{\alpha-1}O^{\beta} = \lambda W$$
$$2.\, \gamma\beta K^{\alpha}O^{\beta-1} = \lambda D$$

Amongst this set of unknown variables, K, O, λ. There are 3 unknown variables in this equation, and one more equation will be needed in addition to these two to solve the system. In this case, I will use the initial equation:

$$X = WK + DO$$

(Note that $X, \alpha, \beta, \gamma, W\ and\ D$ are known constants from the start) Solving the equation given above,

$$\gamma\alpha K^{\alpha-1}O^{\beta} = \lambda W$$
$$\Rightarrow \lambda = \frac{\gamma\alpha}{W}K^{\alpha-1}O^{\beta}$$

and

$$\gamma\beta K^{\alpha}O^{\beta-1} = \lambda D$$
$$\Rightarrow \lambda = \frac{\gamma\beta}{D}K^{\alpha}O^{\beta-1}$$

Thereby,

$$\frac{\gamma\beta}{D}K^{\alpha}O^{\beta-1} = \frac{\gamma\alpha}{W}K^{\alpha-1}O^{\beta}$$
$$\Rightarrow \frac{W_{\beta}}{D_{\alpha}} = K^{-1}O^{1}$$
$$\Rightarrow \frac{KW_{\beta}}{D_{\alpha}} = O$$

Substituting this into the constraint equation,

$$X = WK + DO$$
$$X = WK + \frac{KW_{\beta}}{\alpha}$$
$$X = K\left(W + \frac{W_{\beta}}{\alpha}\right)$$
$$\Rightarrow K = \frac{X}{\left(W + \frac{W_{\beta}}{\alpha}\right)}$$

As we've obtained **K**'s value in terms of constants, we can replace it into the initial cost constraint function, thus allowing us to determine C. Therefore, we can determine both constant values.

$$75{,}000 = 250K + 100O$$

Substituting for K and hence solving for O:

$$75{,}000 = 250\left(\frac{X}{\left(W + \frac{W_\beta}{\alpha}\right)}\right) + 100O$$

$$\Rightarrow O = \frac{75{,}000 - 250\left(\frac{X}{\left(W + \frac{W_\beta}{\alpha}\right)}\right)}{100}$$

Hence, to find the profit-maximizing quantity of production,

$$W = \gamma K^{\alpha} O^{\beta}$$

$$W = \gamma\left(\frac{X}{\left(W + \frac{W_\beta}{\alpha}\right)}\right)^{\alpha} \frac{75{,}000 - 250\left(\frac{X}{\left(W + \frac{W_\beta}{\alpha}\right)}\right)^{\beta}}{100}$$

$X, \alpha, \beta, \gamma, W \text{ and } D$ are known constants.

In our initial case study, by applying this same formula, we can identify the most cost-efficient input levels for generating W units while adhering to a cost constraint of \$75,000. By re-defining the constants, $X = 75{,}000, W = \$250, \alpha = 0.339, \beta = 0.655, \gamma = 1$,

$$K = \frac{X}{\left(W + \frac{W_\beta}{\alpha}\right)} = \frac{75{,}000}{250 + \left(\frac{250 \times 0.655}{0.339}\right)} = 102.313 \approx 102\, units$$

Substituting for K = 102 units,

$$O = \frac{75{,}000 - 250(102)}{100}$$

$$O = 495\ capital\ units$$

Finally, to establish the most profit-generating quantity, the obtained values of K and O units will be incorporated into the product function.

$$W = 102^{0.339} 495^{0.655}$$

$$W = 279\ units$$

Verifying this graphically:
By utilizing the cost constraint equation- $75{,}000 = 250K + 100O, W = 279$, and the Cobb-Douglas Production Function for the business- $K^{0.339}O^{0.655}$ the intersection of these planes can be determined as $K = 102, O = 495, W = 279$.

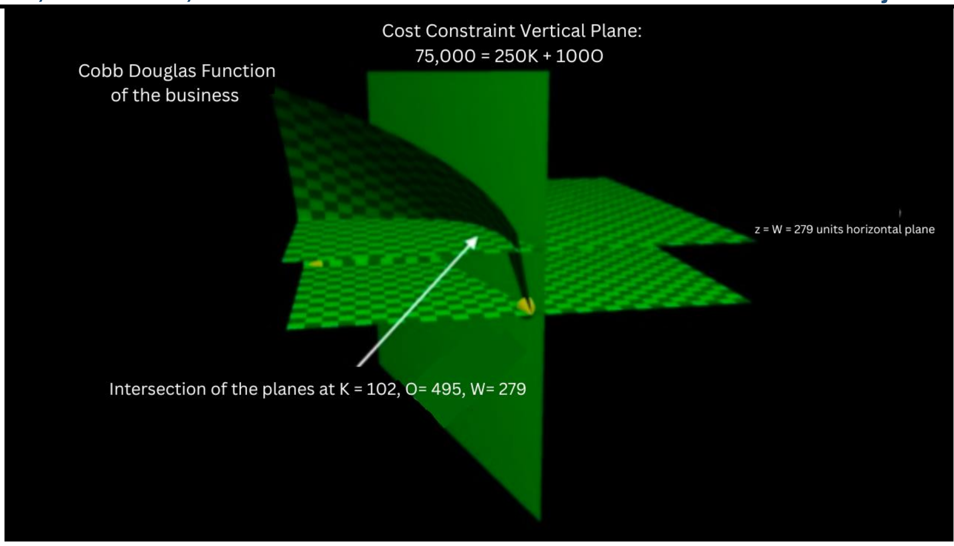

Fig. 8: Verifying the Calculation

Therefore, given a budget constraint of $75,000, the quantity of production that minimizes costs for this business is 279 units.

6 Conclusion

6.1 Analysis

According to the circumstances of the business, two methods of profit maximization were adopted in this study, and reasonable solutions were obtained for both:
1. Finding profit-maximizing input levels based on the known quantity of production (total cost to be found). Example: For Ecolab Inc., when targeting a production quantity of 300 units, the optimal input levels of labour and capital are 110 units and 531 units respectively, resulting in a cost of $80,600.

2. Determining the optimal input levels for profit maximization by taking into consideration a specific cost constraint (total production quantity to be determined). Example: Ecolab Inc., when operating under a constrained budget of $75,000, the minimum cost input levels are 102 units of labour and 495 units of capital, resulting in a production of 279 units of the product.

During the calculation process for Ecolab Inc. Observing Fig. 3 and Fig. 6, I found: the further the company's input-level combinations deviate from the tangent point, the more costly and profit-reducing the decisions became for the company. Therefore, it is advisable for companies to strive to continually manufacture at the input-level combination that corresponds to the tangential point.

6.2 Investigation's Limitations

Cobb-Douglas Production Function's Accuracy

- The function operates on the assumption of CRTS (initial condition of $\alpha + \beta = 1$). It is, however, very difficult to find such firms in real life, because the majority of businesses either have increasing returns or decreasing returns. Because of their scared and invisible nature, it is extremely difficult to combine the different FoP.
- This production function is applicable in limited industries. For example, it cannot be used in the agriculture industry.
- The function is based on the perfect competition market structure, which is a theoretical construct and doesn't exist in the real world. Most markets are monopolies or oligopolies.

- Although the function is in accordance with the substitutability of factors, it neglects to take into account the complementary nature of certain FoP.

Data's Reliance & Reliability:

- Conducting such computations is feasible only if the company possesses sufficient data to evaluate over time, and has experimented with various input-level combinations. For very small companies that have recently entered the market, are inexperienced, and have a limited budget, collecting this data may not be practical due to limited time, money, and resources. Therefore, this model may not be applicable to them, or they may have to rely on data collected by other companies in the same sector. This data may not be entirely representative of their firm's operations.

6.3 Investigation's Strengths

Strengths of the Cobb-Douglas Production Function

- This approach is appropriate for making comparisons between companies situated in different countries and across specific industries.
- Is applicable to '*m*' FoP to generalize the results.
- Facilitates the calculation of unknown variables such as α,β and γ in addition to the production functions.
- This method is widely recognized and accepted within academic and research circles.

Reliance and reliability of the data:

- The data was obtained from Ecolab Inc.'s annual financial reports from the financial year 2006 to the financial year 2021. The origin of the data ensures the validity and dependability of the data acquisition process.
- It is important to note that the primary objective of this study was to develop a general model/procedure aimed at identifying the most optimal input combination and the majority of the research was generalized in terms of various variables, the data does not impact the credibility and pertinence of mathematical computations. An example business model was used in order to illustrate the model and illustrate how it works in terms of the data set.

Key advantages to society:

- A higher level of profitability for smaller businesses operating in extremely competitive markets.
- Due to efforts in research and development like this initiative, economic growth will increase, with its ripple effects.
- The production process is more sustainable because fewer FoP are wasted.

6.4 Conclusion

The results of this investigation have provided me with a better understanding of multivariable calculus as a method for optimizing quantities in Business Studies and Economics, as well as a deeper understanding of the fact that firms can save substantial amounts of input levels by modelling and investing in calculating and analysing their data. In terms of environmental and sustainability issues, placing emphasis on business profits over environmental concerns can have detrimental effects. Studies such as this can contribute to a more positive macro-impact. During recessions, certain large companies that constitute a significant percentage of a country's economy can have a positive impact. It is suggested that for further research and expansion of this study, utilize alternative production functions, such as ones other than the Cobb-Douglas function, in the computation process. In addition, these forecasts' precision needs to be evaluated & examined in the context of realistic business conditions. It is possible that certain unforeseen factors may have been overlooked in this study.

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