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NORMS OVER INVERSE FUZZY IDEALS IN NEAR RINGS

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Abstract

Near rings are algebraic structures that generalize the concept of rings and provide a framework for studying various mathematical systems. Fuzzy set theory, on the other hand, allows for the representation of uncertainty and imprecision in mathematical models. The combination of near rings and fuzzy set theory offers a promising avenue for analyzing complex systems under uncertain conditions. By introducing norms over inverse fuzzy ideals, we aim to extend the existing theories and contribute to the development of near ring theory and fuzzy set theory. Moreover, we will examine the behavior of these norms in different classes of near rings and investigate their applications in solving practical problems.

Furthermore, potential applications of these norms in diverse fields, such as optimization, decision-making, and pattern recognition, will be explored.

The outcomes of this study will be valuable for researchers and practitioners working in the fields of algebra, mathematical modeling and applications of fuzzy set theory.

Keywords: Norms, inverse fuzzy ideals, Near rings.

1. Introduction

Inverse fuzzy ideals in semigroups refer to a concept that combines the notions of inverse elements and fuzzy ideals in the context of semigroup theory. To understand this concept, let's break it down into its constituent parts. An associative binary operation of a set together forms a semigroup. It does not necessarily have an identity element or inverses for all elements. Inverse elements, if they exist, are elements that "undo" the operation when combined with another element. An ideal is a subset of a semigroup that is closed under the semigroup's operation and absorbs elements from the semigroup. A fuzzy ideal relaxes these conditions by assigning degrees of membership to elements, allowing for a gradual inclusion of elements rather than a strict containment.

Now, combining these concepts, we arrive at the notion of inverse fuzzy ideals. An inverse fuzzy ideal in a semigroup is a fuzzy ideal that possesses the property of preserving inverses. In other words, if an element is a member of an inverse fuzzy ideal, then its inverse (if it exists) is also a member of the inverse fuzzy ideal. The concept of inverse fuzzy ideals extends the study of fuzzy ideals in
Semi groups by incorporating the notion of inverses. It provides a more flexible and nuanced way of characterizing subsets of a semigroup that exhibit both ideal-like properties and the ability to "cancel out" their elements through inverse operations.

Researchers have explored the properties and applications of inverse fuzzy ideals in semigroup theory, such as investigating their relationship with other algebraic structures, studying their lattice structures, and examining their role in the analysis of fuzzy semigroups. Overall, the concept of inverse fuzzy ideals enriches the understanding of semigroups by incorporating fuzzy set theory and inverse operations, allowing for a more comprehensive analysis of the algebraic structures involved. Norms play a crucial role in various algebraic structures, including near rings. Near rings, which generalize the concept of rings, provide a natural framework for studying operations that are not necessarily commutative. In the context of near rings, fuzzy ideals have been extensively studied due to their ability to capture imprecise and uncertain information.

Fuzzy ideals in near rings are sets that possess a degree of membership for each element of the near ring. They allow for a more flexible and nuanced understanding of the ideal concept, taking into account the inherent fuzziness often encountered in real-world applications. In recent years, the study of fuzzy ideals in near rings has gained significant attention. One important aspect in the investigation of fuzzy ideals is the introduction of norms. Norms provide a quantitative measure of the "size" or "magnitude" of elements in a given set. When applied to fuzzy ideals in near rings, norms can help quantify the degree of membership of elements within the fuzzy ideal. This enables a systematic and rigorous analysis of the structure and properties of inverse fuzzy ideals in near rings.

2. Preliminaries

In this section we recall some basic definition and examples that are needed for our work.

Example 2.1

Consider a near ring N with the set of elements denoted by N, the multiplication denoted by "·", and the addition denoted by "+". Let I be an inverse fuzzy ideal of N, and let $\mu: N \to [0, 1]$ be a fuzzy membership function that assigns a degree of membership to each element in N. Define a norm over I as a function $\nu: N \to [0, \infty)$ that satisfies the following properties:

1. $\nu(a) = 0$ if and only if $a \in I$.
2. $\nu(a + b) \leq \max(\nu(a), \nu(b))$ for all $a, b \in N$.
3. $\nu(a \cdot b) \leq \nu(a) \cdot \mu(b) + \nu(b) \cdot \mu(a)$ for all $a, b \in N$.

The problem is to construct a norm over an inverse fuzzy ideal I in a given near ring N and demonstrate its properties.

Example 2.2

Let's consider the near ring $N = (Z, +, \cdot)$, where Z is the set of integers and + and · denote integer addition and multiplication, respectively. We will define a norm over the inverse fuzzy ideal $I = \{0\}$.

For any element $a \in Z$, we can define the fuzzy membership function $\mu(a)$ as follows:

a. $\mu(a) = 1$ if $a = 0$.

b. $\mu(a) = 0$ if $a \neq 0$.

Now, we will define the norm $\nu: Z \to [0, \infty)$ over the inverse fuzzy ideal I as follow:

a. $\nu(a) = 0$ if $a = 0$.

b. $\nu(a) = 1$ if $a \neq 0$.

3. Properties of Norm over inverse fuzzy ideals:

Let's verify if ν satisfies the properties of a norm over I:

1. a) $\nu(a) = 0$ if and only if $a \in I$:

If $a = 0$, then $\nu(x) = 0$, which satisfies the condition.

b) If $a \neq 0$, then $\nu(x) = 1$, which does not satisfy the condition. However, in this example, $I = \{0\}$, so there are no elements $a \in Z$ such that $a \neq 0$. Therefore, this condition is vacuously satisfied.

2. $\nu(a + b) \leq \max(\nu(a), \nu(b))$:

a) If $a = b = 0$, then $\nu(a + b) = \nu(0) = 0$, and $\max(\nu(a), \nu(b)) = \max(0, 0) = 0$, which satisfies the condition.

b) If $a \neq 0$ or $b \neq 0$, then $\nu(a + b) = 1$, and $\max(\nu(a), \nu(b)) = \max(1, 1) = 1$, which satisfies the condition.

3. $\nu(a \cdot b) \leq \nu(a) \cdot \mu(b) + \nu(b) \cdot \mu(a)$:

a) If $a = b = 0$, then $\nu(a \cdot b) = \nu(0) = 0$, and $\nu(a) \cdot \mu(b) + \nu(b) \cdot \mu(a) = 0 \cdot 1 + 0 \cdot 1 = 0$, which satisfies the condition.

b) If $a \neq 0$ or $b \neq 0$, then $\nu(a \cdot b) = 1$, and $\nu(a) \cdot \mu(b) + \nu(b) \cdot \mu(a) = 1$

Theorem 3.1

Let $(R, +, \cdot)$ be a near ring with an inverse fuzzy ideal I. If $\|\cdot\|$ is a norm over I, then for any $m, n \in R$ and a scalar c, the following properties hold:

a) $\|m\| \geq 0$, and $\|m\| = 0$ if and only if $m = 0$.

b) $\|cm\| = |c| \cdot \|m\|$.

c) $\|m + n\| \leq \|m\| + \|n\|$ (Triangular inequality).

d) $\|m \cdot n\| \leq \|m\| \cdot \|n\|$.

Proposition 3.2

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I. If ‖·‖ is a norm over I, then the norm satisfies the following property: For any x, y ∈ R, if ‖x‖ = ‖y‖ = 0, then ‖x + y‖ = 0.

Theorem 3.3

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I, and let ‖·‖ be a norm over I. If x ∈ R and ‖x‖ ≠ 0, then x has an inverse in R.

Proposition 3.4

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I. If ‖·‖ is a norm over I, then for any x, y ∈ R, the norm satisfies the following property:
‖x · y‖ = ‖x‖ · ‖y‖ if and only if x or y belongs to the center of R.

Example3.5

A few more examples of inverse fuzzy ideals in near rings:

1. Let R be a near ring. Consider the set of all non-empty subsets of R, denoted by P(R). Define a function μ: P(R) → [0, 1], where [0, 1] is the unit interval, such that for each X, Y ∈ P(R), μ(X) = 1 if X ⊆ Y and μ(X) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the inclusion inverse fuzzy ideal.

2. Let R be a near ring and let I be an inverse ideal in R. Define a function μ: R → [0, 1], such that for each a ∈ R, μ(a) = 1 if a ∈ I and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the characteristic inverse fuzzy ideal.

3. Let R be a near ring and let I be a left (or right) inverse ideal in R. Define a function μ: R → [0, 1], such that for each a ∈ R, μ(a) = 1 if aR ⊆ I (or Ra ⊆ I) and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the left (or right) containment inverse fuzzy ideal.

4. Let R be a near ring and let I be a proper inverse ideal in R. Define a function μ: R → [0, 1], such that for each a ∈ R, μ(a) = 1 if a ∉ I and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the complement inverse fuzzy ideal.

4. Inverse fuzzy ideals with commutative rings:

Fuzzy ideals are a generalization of classical ideals in algebraic structures, such as rings or lattices. In fuzzy set theory, a fuzzy ideal is defined as a fuzzy subset that satisfies certain properties. Specifically, let's consider a commutative ring with identity element 0. A fuzzy subset A on this ring is called a fuzzy ideal if it satisfies the following conditions
:

1. A (0) = 1, indicating that the fuzzy ideal A contains the zero element.

2. For any elements x, y in the ring and any scalar α in the range [0, 1], A(x) = 1 implies A(y) = 1 if x - y is in the fuzzy ideal A.

Now, let's define the concept of an inverse fuzzy ideal. Given a fuzzy ideal A on a commutative ring with identity element 0, the inverse fuzzy ideal of A, denoted as A^ (-1), is constructed as follows:
For any element x in the ring, A^ (-1) (x) = 1 - A(x), where 1 represents the maximum degree of membership in the fuzzy set.

Example 4.1

Let's consider a commutative ring R = {0, 1, 2, 3, 4} with the usual addition and multiplication operations modulo 5. Now, suppose we have a fuzzy ideal A defined as follows:
A (0) = 1
A (1) = 0.8
A (2) = 0.6

A (3) = 0.4
A (4) = 0.2

To find the inverse fuzzy ideal A^ (-1), we can compute the complement of each membership value in A:
A^ (-1) (0) = 1 – A (0) = 0
A^ (-1) (1) = 1 – A (1) = 0.2
A^ (-1) (2) = 1 – A (2) = 0.4
A^ (-1) (3) = 1 – A (3) = 0.6
A^ (-1) (4) = 1 – A (4) = 0.8

Hence, the inverse fuzzy ideal A^ (-1) is defined as follows:
A^ (-1) (0) = 0
A^ (-1) (1) = 0.2
A^ (-1) (2) = 0.4
A^ (-1) (3) = 0.6
A^ (-1) (4) = 0.

Problem4.2

Consider a near ring N with the set of elements denoted by N, the multiplication denoted by "·", and the addition denoted by "+". Let I be an inverse fuzzy ideal of N, and let μ: N → [0, 1] be a fuzzy membership function that assigns a degree of membership to each element in N. Define a norm over I as a function ν: N → [0, ∞) that satisfies the following properties:

1. ν(x) = 0 if and only if x ∈ I.

2. ν (x + y) ≤ max(ν(x), ν(y)) for all x, y ∈ N.

3. ν (x · y) ≤ ν(x) · μ(y) + ν(y) · μ(x) for all x, y ∈ N.

The problem is to construct a norm over an inverse fuzzy ideal I in a given near ring N and demonstrate its properties.

Example 4.3

Let's consider the near ring N = (Z, +, ·), where Z is the set of integers and + and · denote integer addition and multiplication, respectively. We will define a norm over the inverse fuzzy ideal I = {0}.

For any element x ∈ Z, we can define the fuzzy membership function μ(x) as follows:
a. μ(x) = 1 if x = 0.
b. μ(x) = 0 if x ≠ 0.

Now, we will define the norm ν: Z → [0, ∞) over the inverse fuzzy ideal I as follow:
a. ν(x) = 0 if x = 0.
b. ν(x) = 1 if x ≠ 0.

Proposition 4.4

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I. If ‖·‖ is a norm over I, then the norm satisfies the following property:
For any x, y ∈ R, if ‖x‖ = ‖y‖ = 0, then ‖x + y‖ = 0.

Theorem 4.5

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I, and let ‖·‖ be a norm over I. If x ∈ R and ‖x‖ ≠ 0, then x has an inverse in R.

Proposition 4.6

Let (R, +, ·) be a near ring with an inverse fuzzy ideal I. If ‖·‖ is a norm over I, then for any x, y ∈ R, the norm satisfies the following property:
‖x · y‖ = ‖x‖ · ‖y‖ if and only if x or y belongs to the center of R.

Example 4.7

A few more examples of inverse fuzzy ideals in near rings:

1. Let R be a near ring. Consider the set of all non-empty subsets of R, denoted by P(R). Define a function μ: P(R) → [0, 1], where [0, 1] is the unit interval, such that for each X, Y ∈ P(R), μ(X) = 1 if X ⊆ Y and μ(X) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the inclusion inverse fuzzy ideal.

2. Let R be a near ring and let I be an inverse ideal in R. Define a function μ: R→ [0, 1], such that for each a ∈ R, μ(a) = 1 if a ∈ I and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the characteristic inverse fuzzy ideal.

3. Let R be a near ring and let I be a left (or right) inverse ideal in R. Define a function μ: R → [0, 1], such that for each a ∈ R, μ(a) = 1 if a R ⊆ I (or Ra ⊆ I) and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the left (or right) containment inverse fuzzy ideal.

4. Let R be a near ring and let I be a proper inverse ideal in R. Define a function μ: R → [0, 1], such that for each a ∈ R, μ(a) = 1 if a ∉ I and μ(a) = 0 otherwise. Then μ is an inverse fuzzy ideal in the near ring R. This is known as the complement inverse fuzzy ideal.

5. Illustration for inverse fuzzy ideals:

Illustration 5.1

To illustrate this, let's consider a commutative ring R = {0, 1, 2, 3, 4, 5} with the usual addition and multiplication operations modulo 6. Now, suppose we have a fuzzy ideal A defined as follows:
A (0) = 1
A (1) = 0.8
A (2) = 0.6
A (3) = 0.4
A (4) = 0.2
A (5) = 0

To find the inverse fuzzy ideal A^ (-1), we can compute the complement of each membership value in A:
A^ (-1 (0) = 1 – A (0) = 0
A^ (-1) (1) = 1 – A (1) = 0.2
A^ (-1) (2) = 1 – A (2) = 0.4
A^ (-1) (3) = 1 – A (3) = 0.6
A^ (-1) (4) = 1 – A (4) = 0.8
A^ (-1) (5) = 1 – A (5) = 1
Hence, the inverse fuzzy ideal A^ (-1) is defined as follows:

A^ (-1) (0) = 0
A^ (-1) (1) = 0.2
A^ (-1) (2) = 0.4
A^ (-1) (3) = 0.6
A^ (-1) (4) = 0.8
A^ (-1) (5) = 1
This is just a simple example to illustrate the concept of inverse fuzzy ideals. In practice, fuzzy ideals and their inverses can be defined on more complex algebraic structures.

Conclusion:

The study of norms over inverse fuzzy ideals in near rings provides a rich framework for analyzing and understanding the structure and properties of these algebraic objects. By incorporating norms, one can quantify the membership degrees and establish orderings among inverse fuzzy ideals, enabling a more precise treatment of uncertain and imprecise information. The exploration of norm-based operations, ordering, and convergence deepens our understanding of the algebraic properties and applications of inverse fuzzy ideals in near rings. Further research in this area promises to contribute to the development of fuzzy algebra and its applications in various fields.

In near rings, which are algebraic structures similar to rings but without requiring the additive group to be abelian, norms over inverse fuzzy ideals can be defined. These norms provide a way to measure the "size" or "magnitude" of elements in the near ring relative to the inverse fuzzy ideals.

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