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THERMAL DEFLECTION OF FINITE LENGTH SOLID CIRCULAR CYLINDER DUE TO RAMP-TYPE HEATING

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Abstract— The present paper deals with the determination of temperature distribution and thermal deflection in a finite length solid circular cylinder occupying the space D :$0 \le r, 0 \le z \le h$, subjected to ramp type heating to the upper surface of solid circular cylinder at z=h .The lower surface of solid circular cylinder is at zero temperature with stated boundary conditions . The governing heat conduction equation can be solved by using integral transform and Laplace transform techniques. The results are obtained as series of Bessel's functions. Numerical calculations are carried out for finite length solid circular cylinder made of copper metal and illustrated graphically.

Key words: Solid circular cylinder, temperature distribution, thermal deflection, ramp type heating , integral transform, Laplace transform.

I. INTRODUCTION :

Roy Choudhuri [10] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. **Wankhede [12]** has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. **Dange et al. [4,5,]** have studied deflection of isosceles vibrating triangular plate and thin equilateral triangular plate **.Khalsa , et al. [7]** have studied two-dimensional transient problem for a thick disc with internal heat sources . **Ghadle et al. [6]** have studied an inverse quasi-static thermoelastic problem of a thick circular plate. **Khobragade, et al. [8]** have studied , an inverse thermoelastic problem of finite length thick hollow cylinder with internal heat source . **Warsha K. Dange [1,2 ,3]** has determined thermal stresses in a hollow Cylinder, thermal stresses in a hollow cylinder with internal heat generation and also thermal stresses in annular disc due to boundary conditions of radiation type.In all aforementioned investigations they have not considered any thermoelastic problem subjected to ramp type heating. This paper concerned with the determination of temperature distribution and thermal deflection in a finite length solid circular cylinder occupying the space D :$0 \le r \le a, 0 \le z \le h$, subject to ramp type heating to the upper surface of solid circular cylinder at z=h The lower surface of solid circular cylinder is at zero temperature with stated boundary conditions . The governing heat conduction equation can be solved by using integral transform and Laplace transform techniques. The results are obtained as series of Bessel's functions. Numerical calculations are carried out for solid circular cylinder made of copper metal and illustrated graphically.

II. STATEMENT OF THE PROBLEM :

Consider finite length solid circular cylinder of height h . The cylinder is kept at zero temperature initially. The upper surface z=h subjected to ramp type heating. The temperature at boundary of curved surface is zero. Under these more realistic prescribed conditions the temperature distribution and deflection in solid circular cylinder are required to be determined.

The differential equation satisfy by deflection $\omega(r,t)$ is

$$\nabla^4 \omega(r,t) = -\frac{\nabla^2 M_t}{D(1-\nu)} \qquad (1)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial z^2}$$

Where D is the flexural rigidity and M_t is the thermal moment of the finite length solid circular cylinder defined as

$$M_t = a_t E \int_0^h T(r,z,t) z dz \qquad (2)$$

And flexural rigidity given as

$$D = \frac{Eh^3}{12(1-\nu)} \qquad (3)$$

Where E is the Young's modulus, ν_{is} Poisson's ratio is and a_t is linear coefficient of thermal expansion of the material of the finite length solid circular cylinder .

For built in edge thermal deflection ω satisfies following condition

$$\omega = \omega_{,r} = 0 \qquad \text{At} \qquad r=a \qquad (4)$$

The equation for $T(r,z,t)$, the temperature, in cylindrical coordinates, is:

$$\kappa\left[r^{-1}(rT_{,r})_{,r} + T_{,zz}\right] = T_{,t} \tag{5}$$

Where k is thermal diffusivity of the material of the finite length solid circular cylinder
Subject to the initial and boundary conditions

$$\mathfrak{M}_t(T,1,0,0) = 0 \text{ For all } 0 \le r \le a\,,\; 0 \le z \le h \tag{6}$$

$$\mathfrak{M}_z(T,1,0,0) = 0, \quad \text{for all } 0 \le r \le a\,, t > 0 \tag{7}$$

$$\mathfrak{M}_z(T,1,0,h)) = \mathrm{f}\,(\mathrm{r})\,\frac{T_1}{t_1}t\;, \qquad 0 \le t \le t_1$$

$$= \mathrm{f}\,(\mathrm{r})\,\frac{T_1}{t_1}\;\; t \ge t_1 \;\text{ at z = h for all } 0 \le r \le a\,, t > 0 \tag{8}$$

$$\mathfrak{M}_r(T,1,0,a) = 0, \text{ for all } 0 \le z \le h\,, t > 0 \tag{9}$$

The most general expression for these conditions can be given by

$$\mathfrak{M}_v(f,k,\;\bar{\bar{k}},\;s) \;=\; (\bar{k}f + \bar{\bar{k}}\hat{f})_{v=s}$$

Where the prime (^) denotes differentiation with respect to v.

III. SOLUTION OF THE PROBLEM:

3.1 Determination Temperature Function $T(r,z,t)$:

By applying finite Hankel transform to the equations (5) ,(6) ,(7),(8) and using (9) to reduce differential equation in Hankel transform domain and then applying Laplace transform and making use of respective inversion and Sneddon over the heat conduction equation one obtains the expression for temperature distribution function as

$$T(r,z,t) = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{\bar{f}(\gamma_n)T_1 J_0(\gamma_n r)\,\sinh(\gamma_n z)}{[J_0{'}(\gamma_n a)]^2\,\sinh(\gamma_n h)} - \frac{4\pi}{a^2h^2}\sum_{n,m=1}^{\infty}\frac{m(1-t_1)\bar{f}(\gamma_n)T_1\,\exp[-k(\gamma_n{}^2+\lambda_m{}^2)(t-t_1)]J_0(\gamma_n r)\,\sin(\lambda_m z)}{k\cos(m\pi)[\gamma_n{}^2+\lambda_m{}^2]^2[J_0{'}(\gamma_n a)]^2} \tag{12}$$

3.2 Determination of Thermal moment $M_t$as:

Substituting the value of $T(r,z,t)$ from equation (12) to equation (2) ,one obtains the thermal moment M_t as

$$M_t = \frac{2a_tE}{a^2}\sum_{n=1}^{\infty}\frac{\bar{f}(\gamma_n)T_1J_0\big(\bar{f}(\gamma_n)r\big)}{\gamma_n{}^2\sinh(\gamma_n h)[J_0'(\gamma_n a)]^2}[h\gamma_n\cosh(\gamma_n h) - \sinh(\gamma_n h)]$$

$$+ \quad \frac{4\pi a_tE}{a^2h^2}\sum_{m,n=1}^{\infty}\frac{m(1-t_1)\bar{f}(\gamma_n)T_1\exp[-k(\gamma_n{}^2+\lambda_m{}^2)(t-t_1)]J_0(\gamma_n r)}{k\lambda_m{}^2\cos(m\pi)[\gamma_n{}^2+\lambda_m{}^2]^2[J_0'(\gamma_n a)]^2}[\sin(\lambda_m h) - \lambda_m h\cos(\lambda_m h)] \tag{13}$$

3.3 Determination of Thermal moment ω

According to boundary conditions as mentioned in equation (4), thermal deflection in a finite length solid circular cylinder is given by

$$\omega(r,t) = \frac{2a_tE}{D(1-v)a^2}\sum_{n=1}^{\infty}\frac{\bar{f}(\gamma_n)T_1[2a\gamma_n[J_0(\gamma_n r) - J_0(\gamma_n a)] + \gamma_n{}^2(r^2-a^2)J_1(\gamma_n a)][h\gamma_n\cosh(\gamma_n h) - \sinh(\gamma_n h)]}{2a\gamma_n{}^5\sinh(\gamma_n h)[J_0{'}(\gamma_n a)]^2}$$

$$+\frac{4\pi a_tE}{D(1-v)a^2h^2}\sum_{m,n=1}^{\infty}\frac{m(1-t_1)\bar{f}(\gamma_n)T_1[2a\gamma_n[J_0(\gamma_n r) - J_0(\gamma_n a)] + \gamma_n{}^2(r^2-a^2)J_1(\gamma_n a)]}{k\lambda_m{}^2\cos(m\pi)\gamma_n{}^2[\gamma_n{}^2+\lambda_m{}^2]^2[J_0{'}(\gamma_n a)]^2}$$

$$\exp[-k(\gamma_n{}^2+\lambda_m{}^2)(t-t_1)][\sin(\lambda_m h) - \lambda_m h\cos(\lambda_m h)] \tag{14}$$

IV. NUMERICAL RESULTS, DISCUSSION AND REMARKS:

Numerical calculations have been carried out for a copper solid circular cylinder with following properties.
Poisson ratio, υ = 0.35
Thermal expansion coefficient , a_t = 16.5 × 10^{-6} K^{-1}
Thermal diffusivity , k = 112.34×$10^{-6}m^2s^{-1}$

Dimensions Used:

Radius of solid circular cylinder a = 1 m
Height of solid circular cylinder h = 2 m

Constants assumed for ramp type heating:

T_1 = 1K
Internal radius r_1 = 0.5m
Fixed time t_1 = 25 sec

Special Case:

$f(r) = 1$ for $0 \le r_1 \le r$
$= 0$ for $r > r_1$

In the Equations (12) one obtains the expression for temperature distribution function as

$$T(r,z,t) = \frac{2}{a^2}\sum_{n=1}^{\infty}\frac{T_1\sinh(\gamma_n z)r_1J_1(\gamma_n r_1)J_0(\gamma_n r)}{\sinh(\gamma_n h)\mu_n[J_0{'}(\gamma_n a)]^2}$$

$$-\frac{4\pi}{a^2h^2}\sum_{n,m=1}^{\infty}\frac{m(1-t_1)r_1T_1\sin(\lambda_m z)\exp[-k(\gamma_n{}^2+\lambda_m{}^2)(t-t_1)]J_1(\gamma_n r_1)J_0(\gamma_n r)}{k\cos(m\pi)\gamma_n[\gamma_n{}^2+\lambda_m{}^2]^2[J_0{'}(\gamma_n a)]^2} \tag{15}$$

The derived numerical results from equation (12) and (14)have been illustrated graphically in figures 1 to 4 as follows.

Figure 1 represents graph of temperature **T(r,z,t)** versus **r** . It is observed that temperature T develops tensile stress from r=0 to r = 0.5 .Then temperature is zero from r = 0.5 to r = 1 .

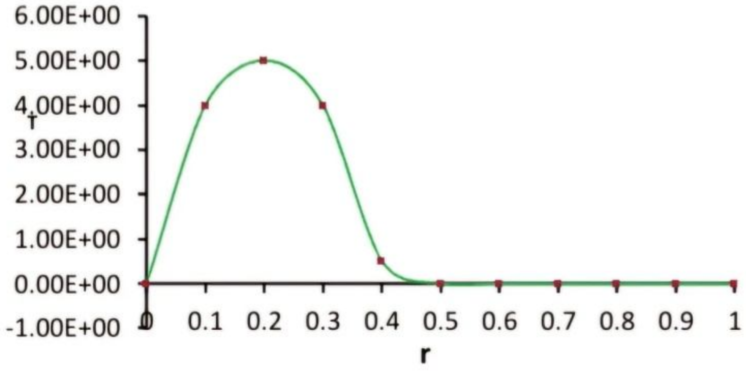

Fig 1: Graph of *T* versus r

Figure 2 : represents graph of temperature T versus **z** . It is observed that T goes on increasing uniformly from z=0 to z=2 .

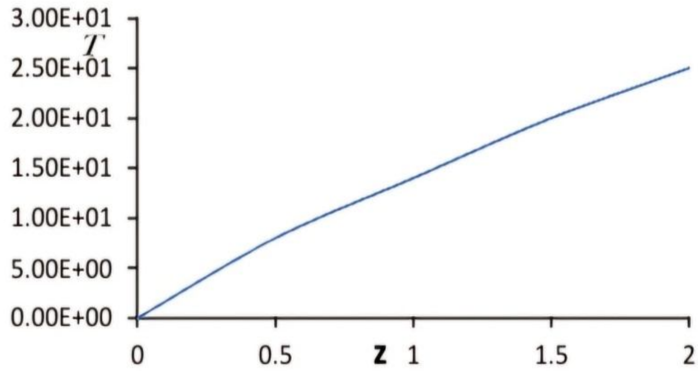

Fig 2: Graph of *T* versus z

Figure 3: represents graph of temperature ***ω*** versus **r** . It is observed that ***ω*** goes on decreasing from r=0 to r = 1.5 and then from r=1.5 to r = 2 ,***ω*** goes on goes on increasing uniformly.

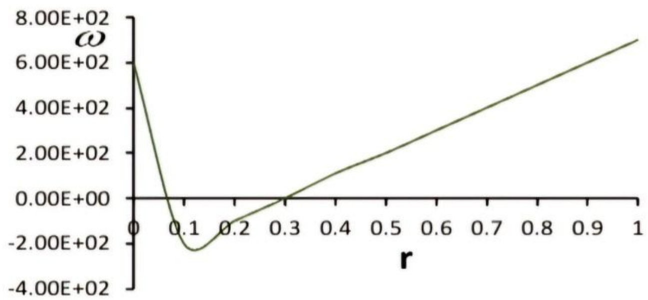

Fig 3: Graph of *ω* versus r

Figure 4: represents graph of temperature ω versus **t** . It is observed that ω is zero from t = 0 sec to t = 28 sec. and then from t = 28 sec. to t = 60 sec. goes on increasing .

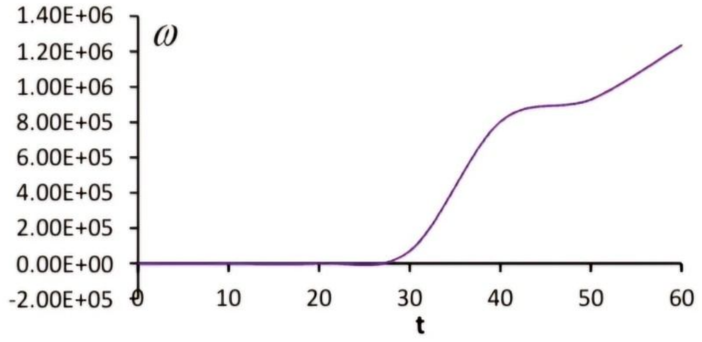

Fig 4: Graph of ω versus t

V. CONCLUSION:

In this study , one treated thermoelastic problem of the solid circular cylinder subjected to ramp type heating to the upper surface of solid circular cylinder at z=h . Under given initial and boundary conditions temperature distribution, and thermal deflection have been determined with the help of Hankel transform and Laplace transform techniques. The results are obtained as series of Bessel's functions in the form of infinite series. Moreover, assigning suitable values to the parameters and functions in the equations of temperature one may conclude that the system of equations proposed in this study can be adapted to design of useful structures or machines in engineering applications in the determination of thermoelastic behavior and illustrated graphically.

VI. REFERENCES:

[1] Dange,Warsha K. , "Thermal Stresses Of Two-Dimensional Transient Thermoelastic Problem For A Hollow Cylinder", International Journal Of Engineering And Innovative Technology (IJEIT), Volume 4, Issue 6, December 2014.

[2]Dange, Warsha K. "Two-Dimensional Transient Thermoelastic Problem Of A Annular Disc Due To Radiation " International Journal Of Mathematics Trends And Technology- Volume 17, Issue 1- January 2015.

[3]Dange,Warsha K. "Two-Dimensional Transient Thermoelastic Problem For A Hollow Cylinder With Internal Heat Generation" International Journal Of ScienceAndResearch (IJSR), Volume 4, Issue 1,January 2015.

[4]Dange Warsha K., N.W. Khobragade, M.H. Durge ,"Deflection Of Isosceles Vibrating Triangular Plate", International Journal Of Pure And Applied Mathematics**. 01/2010**; 60(3) .

[5]Dange Warsha K., N.W. Khobragade," Large Deflection Of A Thin Equilateral Triangular Plate"International Journal Of Pure And Applied Mathematics. **01/2010**; 60(3)

[6]Ghadle K.P. Gaikwad Kishor R."An Inverse Quasi-Static Thermoelastic Problem Of A Thick Circular Plate" Southern Africa Journal Of Pure And Applied Mathematics Sajpam Volume 5, (2011), 13-25.

[7]Khalsa Lalsingh ,Vinod Varghese," Two-Dimensional Transient Problem For A Thick Disc With Internal Heat Sources" Faculty Of Mechanical Engineering Transaction (2010),38,173-180

[8]Khobragade N.W. ,Jadhav C.M. Ahirrao B.R. " An Inverse Thermoelastic Problem Of Finite Length Thick Hollow Cylinder With Internal Heat Source " Advances In Applied Science Research, 4(3), (2013), 302-314.

[9]Ozisik N.M. , Boundary Value Problem Of Heat Conduction ,International Text Book Company Scranton Pennsylvania 1968.

[10]Roy Choudhary, S. K.: A Note On Quasi-Static Thermal Deflection Of A Thin Clamped Circular Plate Due To Ramp-Type Heating Of A Concentric Circular Region Of The Upper Face, J. Of The Franklin Institute, 206 (1973), 213-219.

[11]Sneddon I.N. (1972) ,The Use Of Integral Transform New York Mc. Graw Hill.

[12]Wankhede, P. C.: On The Quasi-Static Thermal Stresses In A Circular Plate, Indian J. Pure Appl. Math., Vol. 13 (8) (1982), 1273-1277.