JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

TOTALLY I$\alpha_g^\wedge$-CONTINUOUS FUNCTION IN INTUITIONISTIC TOPOLOGICAL SPACES

K. Heartlin[1]&Dr. J. Arul Jesti[2]

[1]Research Scholar, Reg No:19212212092006, [2]Assistant Professor

[2]PG and Research Department of Mathematics, St.Mary's College(Autonomous),

(Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli),

Thoothukudi.

[1]heartlingladson@gmail.com [2]aruljesti@gmail.com

ABSTRACT

The concept of intuitionistic sets and intuitionistic points in topological spaces was first introduced by Coker. The purpose of this paper is to introduce a mapping totally I$\alpha_g^\wedge$-continuous function in intuitionistic topological spaces and analyze its relations with other existing intuitionistic functions.

Keywords: totally I$\alpha_g^\wedge$-continuous and intuitionistic topological spaces.

INTRODUCTION

Coker [1] introduced the concept of intuitionistic sets and intuitionistic points in 1996. [3] J.G.Lee, P.K.Lim, J.H.Kim, K.Hur introduced intuitionistic continuous, closed and open mappings in 2017. [4] J.Arul Jesti and K.Heartlin introduced the concept of alpha^generalized closed sets in intuitionistic topological spaces and discuss some properties related to I$\alpha_g^\wedge$-closed set in intuitionistic topological spaces. [5] J.Arul Jesti and K.Heartlin On I$\alpha_g^\wedge$-continuous function In intuitionistic topological spaces.The purpose of this paper is to develop totally I$\alpha_g^\wedge$-continuous function in intuitionistic topological spaces and also study its relations with some of existing intuitionistic relations.

2 PRELIMINARIES

Definition 2.1 [1]: Let $\mathcal{H}$ be a non-empty set. An intuitionistic set (IS for short) A is an object having the form A=$<\mathcal{H}, A_1, A_2>$ Where A_1, A_2 are subsets of $\mathcal{H}$ satisfying $A_1 \cap A_2 = \varphi$. The set A_1 is called the set of members of A, while A_2 is called set of non members of A.

Definition 2.2 [1]: Let $\mathcal{H}$ be a non-empty set and A and B are intuitionistic set in the form $A = <\mathcal{H}, A_1, A_2>$, $B = <\mathcal{H}, B_1, B_2>$ respectively. Then

a) $A \subseteq B$ iff $A_1 \subseteq B_1$ and $A_2 \supseteq B_2$

b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$

c) $A^C = <\mathcal{H}, A_2, A_1>$

d) $A - B = A \cap B^C$

e) $\varphi = < \mathcal{H}, \varphi, \mathcal{H} >, \mathcal{H} = < \mathcal{H}, \mathcal{H}, \varphi >$

f) $A \cup B = < \mathcal{H}, A_1 \cup B_1, A_2 \cap B_2 >$

g) $A \cap B = < \mathcal{H}, A_1 \cap B_1, A_2 \cup B_2 >$.

Definition 2.3 [3]: A subset A of $(\mathcal{H}, I\tau_\mu)$ is called an ***intuitionistic alpha ^ generalized closed*** (briefly **I$\alpha_g^\wedge$-closed)** if Igcl(A) ⊆ U, whenever A⊆ U and U is Iα-open in $\mathcal{H}$. We denote the family of all I$\alpha_g^\wedge$-closed sets in space $\mathcal{H}$ by I$\alpha_g^\wedge$C($\mathcal{H}$).

Definition 2.4 [6]: Let $(\mathcal{H}, I\tau_\mu)$ and $(Y, I\tau_\theta)$ be two ITS's and $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ be a function. Then f is said to be intuitionistic continuous iff the pre image of each Is in $I\tau_\theta$ is an Is in $I\tau_\mu$.

3.Totally I$\alpha_g^\wedge$-continuous function

In this section we investigate the concepts of totally I$\alpha_g^\wedge$-continuous function and also we studied some additional properties and give some examples.

Definition 3.1: A function $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is called ***totally I$\alpha_g^\wedge$-continuous function*** if the inverse image of every I-open set of $(Y, I\tau_\theta)$ is both I$\alpha_g^\wedge$-open and I$\alpha_g^\wedge$-closed subset of $(\mathcal{H}, I\tau_\mu)$.

Example 3.2: Let $\mathcal{H} = \{a, b\}$, and $I\tau_\mu = \{\mathcal{H}, \varphi, \mathcal{A}_1, \mathcal{A}_2\}$ where $\mathcal{A}_1 = < \mathcal{H}, \{b\}, \{a\} >$, $\mathcal{A}_2 = < \mathcal{H}, \varphi, \{b\} >$ then I$\alpha_g^\wedge O(\mathcal{H}) = \{\mathcal{H}, \varphi, < \mathcal{H}, \varphi, \varphi >, < \mathcal{H}, \varphi, \{a\} >, < \mathcal{H}, \{b\}, \varphi >, < \mathcal{H}, \varphi, \{b\} >, < \mathcal{H}, \{b\}, \{a\} >\}$ and I$\alpha_g^\wedge C(\mathcal{H}) = \{\mathcal{H}, \varphi, < \mathcal{H}, \varphi, \varphi >, < \mathcal{H}, \{a\}, \varphi >, < \mathcal{H}, \{b\}, \varphi >, < \mathcal{H}, \varphi, \{b\} >, < \mathcal{H}, \{a\}, \{b\} >\}$. Then $Y = \{a, b\}$ with $I\tau_\theta$={ Y, ϕ, $< Y, \{b\}, \varphi >, < Y, \varphi, \varphi >\}$ and I$\alpha_g^\wedge O(Y) = \{Y, \varphi, < Y, \varphi, \varphi >, < Y, \{a\}, \varphi >, < Y, \{b\}, \varphi >, < Y, \varphi, \{a\} >, < Y, \{b\}, \{a\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$.

Then, $f^{-1}(< Y, \varphi, \varphi >) = < \mathcal{H}, \varphi, \varphi, >$ and $f^{-1}(< Y, \{b\}, \varphi >) = < \mathcal{H}, \{b\}, \varphi >$ which are both I$\alpha_g^\wedge$-open and I$\alpha_g^\wedge$-closed in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is totally I$\alpha_g^\wedge$-continuous.

Theorem 3.3: If a map $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ is totally I$\alpha_g^\wedge$-continuous, then f is I$\alpha_g^\wedge$-continuous.

Proof: Let O be a I-open set of $(Y, I\tau_\theta)$. Since, f is totally I$\alpha_g^\wedge$-continuous, $f^{-1}(O)$is both I$\alpha_g^\wedge$-open and I$\alpha_g^\wedge$-closed in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is I$\alpha_g^\wedge$-continuous.

Remark 3.4: The converse of above theorem need not be true as shown in the following example.

Example 3.5: Let $\mathcal{H} = \{a, b\}, I\tau_\mu = \{\mathcal{H}, ϕ, < \mathcal{H}, \varphi, \varphi >, < \mathcal{H}, \{b\}, \varphi >\}$. Then I$\alpha_g^\wedge$-O($\mathcal{H}$) = $\{\mathcal{H}, ϕ, < \mathcal{H}, \varphi, \varphi >, < \mathcal{H}, \{b\}, \varphi >, < \mathcal{H}, \{a\}, \varphi >, < \mathcal{H}, \varphi, \{a\} >, < \mathcal{H}, \{b\}, \{a\} >\}$. Y={a,b} with IT $I\tau_\theta$={Y, ϕ, $< Y, \{b\}, \{a\} >, < Y, \varphi, \{a\} >\}$. Then I$\alpha_g^\wedge$-O(Y) = {Y, ϕ, $< Y, \varphi, \varphi >, < Y, \{b\}, \varphi >, < Y, \varphi, \{a\} >, < Y, \varphi, \{b\} >, < Y, \{b\}, \{a\} >\}$. Define $f: (\mathcal{H}, I\tau_\mu) \longrightarrow (Y, I\tau_\theta)$ as $f(a) = a, f(b) = b$. $f^{-1}(< Y, \varphi, \{a\} >) = < \mathcal{H}, \varphi, \{a\} >, f^{-1}(< Y, \{b\}, \{a\} >) = < \mathcal{H}, \{b\}, \{a\} >$. Then f is I$\alpha_g^\wedge$-continuous. Here, $f^{-1}(< Y, \{b\}, \{a\} >) = < Y, \{b\}, \{a\} >$ which is I$\alpha_g^\wedge$-open in $(\mathcal{H}, I\tau_\mu)$ but which is not I$\alpha_g^\wedge$-closed in $(\mathcal{H}, I\tau_\mu)$. Therefore, f is not totally I$\alpha_g^\wedge$-continuous.

Theorem 3.6: Every totally I-continuous function is totally I$\alpha_g^\wedge$-continuous.

Proof: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ be a totally continuous map and J be a I-open set of $(\mathrm{Y}, I\tau_{\theta})$. Since, f is totally continuous , $f^{-1}(J)$is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. Since every I-open set is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open and every I-closed set is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed, $f^{-1}(J)$ is both $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open and $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Remark 3.7: The converse of above theorem need not be true as shown in the following example.

Example 3.8: In example 3.2, f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous. But, $f^{-1}(< Y, \{b\}, \varphi >) =< \mathcal{H}, \{b\}, \varphi >$ which is not both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is not totally I-continuous.

Theorem 3.9: Every perfectly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous map is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Proof: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ be a perfectly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous map. Let D be a I-open set of $(\mathrm{Y}, I\tau_{\theta})$. Then D is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open in $(\mathrm{Y}, I\tau_{\theta})$. Since f is perfectly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous, $f^{-1}(D)$is both I-open and I-closed in $(\mathcal{H}, I\tau_{\mu})$, implies $f^{-1}(D)$is both $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open and $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Remark 3.10: The converse of above theorem need not be true as shown in the following example.

Example 3.11: In example 3.2, f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous. Here, $f^{-1}(< Y, \varphi, \{a\} >) =< \mathcal{H}, \varphi, \{a\} >$ which is not both I-open but not I-closed in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is not perfectly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Remark 3.12: The concept of totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous and strongly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous are independent of each other.

Example 3.13: In example 3.2, $f^{-1}(< Y, \varphi, \varphi >) =< \mathcal{H}, \varphi, \ \varphi, >$ and $f^{-1}(< Y, \{b\}, \varphi >) =< \mathcal{H}, \{b\}, \varphi >$ which are both $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open and $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous. Here, $f^{-1}(< Y, \varphi, \varphi >) =< \mathcal{H}, \varphi, \ \varphi, >$ and $f^{-1}(< Y, \{b\}, \varphi >) =< \mathcal{H}, \{b\}, \varphi >$ which are not I-open in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is not strongly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Example 3.14: Let $\mathcal{H} = \{\mathrm{a}, \mathrm{b}\}$ and family $I\tau_{\mu} = \{\mathcal{H}, \varphi, < \mathcal{H}, \varphi, \ \{b\} >, < \mathcal{H}, \{a\}, \ \{b\} >, < X, \{a\}, \ \varphi >, < X, \ \varphi, \varphi >\} = \mathrm{I}\alpha^{\wedge}_{\mathrm{g}}O(X)$ and $Y = \{a, b\}$. with $I\tau_{\theta}$=$\{\mathrm{Y}, \phi, < Y, \varphi, \ \{b\} >, < Y, \{a\}, \{b\} >, < Y, \{a\}, \varphi >\}$. Define $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ as $f(a) = a, f(b) = b$. Then f is strongly $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous. Here, $f^{-1}(< Y, \{a\}, \{b\} >) =< \mathcal{H}, \ \{a\}, \{b\} >$, which is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open but not an $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed set in $(\mathcal{H}, I\tau_{\mu})$. Therefore, f is not totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Theorem 3.15: Let $f: (\mathcal{H}, I\tau_{\mu}) \longrightarrow (\mathrm{Y}, I\tau_{\theta})$ and $g : (\mathrm{Y}, I\tau_{\theta}) \rightarrow (\mathrm{Z}, I\tau_{\rho})$ be two functions. Then,

(i) If f is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-irresolute and g is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous then $g \circ f$ is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous

(ii) If f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous and g is continuous then $g \circ f$ is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

Proof:

(i) Let E be a I-open set in Z. Since g is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous, $g^{-1}(E)$ is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-clopen in Y. Since f is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-irresolute, $f^{-1}(g^{-1}(E))$ is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-open and $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-closed in $(\mathcal{H}, I\tau_{\mu})$. Since $(g \circ f)^{-1}(E) = f^{-1}(g^{-1}(E))$, $g \circ f$ is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

(ii) Let L be a I-open set in Z. Since g is I-continuous, $g^{-1}(L)$ is I-open in Y. Since f is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous, $f^{-1}(g^{-1}(L))$ is $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-clopen in $\mathcal{H}$. Hence, $g \circ f$ is totally $\mathrm{I}\alpha^{\wedge}_{\mathrm{g}}$-continuous.

References:

[1] D. Coker, A Note On Intuitionistic Sets and Intuitionistic Points, Turkish Journal of Mathematics 20(3) (1996), 343-351

[2] D. Coker, An introduction to intuitionistic topological spaces. Busefal. 2000; 81: 51-56

[3] J .Younis Yaseen, G.Asmaa Raouf(2009) On Generalized Closed Sets and Generalized Continuity On Intuitionistic Topological spaces, Journal of al-anbar University for pure Science, 3(1), (2009).

[4] J.Arul Jesti and K.Heartlin A New Notion of Closed Sets in Intuitionistic Topological Spaces Journal of Algebraic Statistics Vol 13,ISSN 1309-3452, P.no 3188-3196 (2022)

[5] J.Arul Jesti and K.Heartlin On $I\alpha_g^{\wedge}$- Continuous Function In Intuitionistic Topological Spaces Gis Science Vol 40, ISSN 1869-9391 P.no 1709-1715, (2022)

[6] H.Taha Jassim , R. Samer Yaseen and S. Luma Abdual Baqi, some Generalized sets and Mappings in intuitionistic topological space, Journal of AL. Qadisiyah for computer science and Mathematices 7(2)(2015), 80-96.

[7] J.Arul Jesti and K.Heartlin strongly and perfectly $I\alpha_g^{\wedge}$-continuous function in intuitionistic topological spaces Journal of Namibian Studies (communicated).