



METHOD FOR DESIGNING PI/PID CONTROLLERS FOR MIMO PROCESS USING EQUIVALENT TRANSFER FUNCTION APPROACH

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Abstract: This study presents an innovative and inclusive method to enhance control at a Waste Water Treatment Plant (WWTP) by suggesting the integration of a centralized Proportional Integral (PI) controller. The main aim is to optimize and elevate the efficiency of the WWTP's natural treatment phases. To realize this goal, an intricate numerical model is established based on four nonlinear differential equations that meticulously capture the complex dynamics of the WWTP's operations. In this pursuit, the WWTP's transfer function model (2*2) is carefully derived, with thorough consideration of different regulated factors like the air circulation rate and return reuse rate. With the control strategy customized for each specific plant output and contribution, the model subsequently approximates the equivalent transfer function (ETF) for each contributing factor. The core of this methodology revolves around finely tuning and optimizing the PI regulator, achieved through an advanced frequency response technique. Through meticulous adjustments to the PI regulator, the desired strength characteristics are effectively attained, resulting in desirable outcomes in terms of minimizing servo and interaction effects within the WWTP. This approach not only boosts overall performance but also advances energy efficiency and cost-effectiveness in the treatment process. Proposed model is implemented in MATLAB/SIMULINK environment. This choice of platform not only showcases the approach's adaptability but also enhances its accessibility to researchers and practitioners in the field.

Index Terms- MIMO processes; Dynamic relative gain array; Energy transmission ratio; Effective relative gain array; Relative frequency array; Equivalent transfer function matrix; PI/PID; Decoupling control; Diagonal dominance; Robustness

I. INTRODUCTION

Two main PI/PID-based control systems are widely used: multi-loop control and decoupling control. Multi-loop control treats multi-input multi-output (MIMO) processes as a collection of individual loops, with a controller implemented on each loop while considering interactions. It is favoured for its reasonable performance, simple structure, and robustness, and various techniques have been developed to enhance the performance of multi-loop PI/PID controllers, such as detuning factor methods, sequential loop closing methods, independent design methods, and equivalent transfer function methods. However, when significant loop interactions exist, the decentralized controller design may be insufficient, leading to stability issues, inefficient operation, and higher energy expenses. Decoupling control is often preferred for MIMO processes with severe loop interactions. It involves two steps: designing a decentralised decoupler to minimize loop interactions and designing main loop controllers for overall performance [1]. The key benefits are the use of single-input single-output (SISO) controller design methods and relatively easy manual stabilization in the event of actuator or sensor failures. However, this approach can lead to complex control structures, especially for large systems. As a result, research is mainly focused on two-input two-output (TITO) processes.

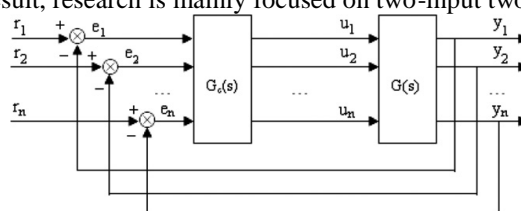


Fig.1 Closed Loop Multivariable Controlled System.

Wang et al. have made significant advancements in creating various methods for full-dimensional PID or non-PID controllers using systematic design methodologies and relatively simple system structures, specifically aimed at handling high-dimensional processes. However, a concern arises regarding the stability of such control systems due to the mathematical approximations required for decouplers in high-dimensional processes. To address this issue, the industry has turned to static decoupling methodologies, which have proven to significantly enhance control

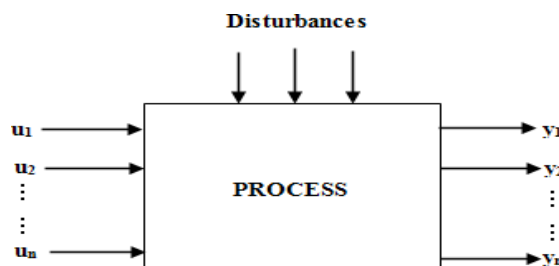
performance while maintaining robust stability equivalent to multi-loop control systems. These methodologies are particularly valuable in dealing with severe loop interactions commonly encountered in high-dimensional MIMO processes [2]. Although static decoupling is relatively straightforward to design and implement, it may not always deliver sufficient closed-loop performance when dynamic properties of transfer function elements vary significantly. In industries such as blending, energy conservation, and distillation columns, techniques based on relative gain arrays (RGAs) have been widely adopted for control loop layout. The key advantage of RGA-based techniques lies in their ease of calculation, as they only require process steady-state gains, and scaling is independent due to the nature of the methodology being ratio-based. However, since these methods do not consider dynamic information about the process, utilizing steady-state gain alone may lead to erroneous interaction measures and, consequently, inaccurate loop pairing judgments. To address and transcend the limitations of RGA-based loop pairing criteria, researchers have put forth subsequent innovative approaches that leverage dynamic relative gain arrays (DRGAs). Unlike conventional methods that rely on the steady-state gain matrix, DRGAs utilize the transfer function model to calculate the relative gain array, thereby considering the dynamic aspects of the system. In this advanced DRGA technique, the denominator of the relative gain array denotes the system's ability to achieve complete control across all frequencies, while the numerator only captures the open-loop transfer function. In a recent groundbreaking contribution, McAvoy et al. introduced a major dynamic relative gain array (DRGA) strategy, further pushing the boundaries of the field. Their approach involves the development of a proportional output optimum controller, carefully designed using the available dynamic process model [3]. This process results in the generation of a controller gain matrix, which effectively defines the dynamic relative gain. The brilliance of their approach lies in the ingenious combination of steady-state gain and bandwidth of the process transfer function elements. This fusion of attributes draws upon the strengths of both traditional RGA and dynamic DRGA methodologies, culminating in a more comprehensive and nuanced depiction of loop interactions. By harnessing the concept of the process's effective relative gain array (ERGA), an entirely fresh and innovative loop pairing criterion has emerged, based on this newly established interaction measure. This novel criterion represents a significant leap forward in addressing the challenge of minimizing loop interactions, thereby enhancing control system performance and overall stability. The ERGA-based approach offers several key benefits that underscore its importance and potential impact in the field. The utilization of dynamic information in the loop pairing process provides a more insightful and accurate assessment of the system's interactions among individual loops, surpassing the limitations of traditional RGA-based methods that solely rely on steady-state gain. This sophisticated methodology holds considerable promise in various practical applications, spanning a wide array of industrial processes. Its efficacy lies in its ability to yield highly optimized and efficient control strategies, which in turn translate to enhanced process performance, improved system stability, and potentially significant cost savings. By enabling more precise and reliable loop pairing decisions, engineers and researchers can derive greater control over complex systems and pave the way for more efficient and sustainable operations. This method's intuitive and well-founded approach makes it accessible and applicable to a broad audience, offering practical solutions for a myriad of real-world challenges in process control and automation.

The research conducted by McAvoy et al. marks a seminal contribution to the field of control systems design and optimization, shedding new light on the intricacies of loop interactions and providing invaluable tools to tackle the complexities of high-dimensional processes. The potential impact of this work extends beyond the realm of academia, as its practical implications hold the potential to revolutionize industrial control applications and contribute to significant advancements in various sectors, from manufacturing and energy to environmental management and beyond: The proposed interaction measure and loop pairing criterion offer a multi-faceted solution that significantly enhances the understanding of dynamic interactions among individual loops in a control system. One of the most remarkable aspects of this approach is its ability to provide a comprehensive description of these dynamic interactions without necessitating the specification of the controller type. This characteristic is particularly valuable as it allows for greater flexibility in controller selection and implementation, accommodating various control strategies tailored to specific process requirements. Moreover, this innovative method stands out for its computational efficiency, representing a substantial improvement over the dynamic relative gain array (DRGA) approach. By significantly reducing the computational overhead, it ensures that control design and optimization processes are streamlined, enabling faster and more efficient implementation. Another key advantage of this approach is its compatibility with the detuning factor design method. When integrated with this method, the proposed criterion results in a less conservative controller design. This means that the control system can achieve higher levels of performance while still maintaining stability and robustness, striking an optimal balance between efficiency and reliability. A notable benefit that resonates with field engineers and practitioners is the ease with which they can comprehend and implement pairing decisions in practical scenarios. This user-friendliness makes the methodology accessible to a wide range of professionals, enabling them to leverage its advantages effectively in real-world industrial applications. By empowering engineers with an intuitive and practical tool, the proposed criterion facilitates the implementation of effective control strategies, leading to better process outcomes and improved overall system performance.

To demonstrate the effectiveness of the proposed interaction measure and loop pairing criterion, a series of illustrative examples are presented. Through these examples, the advantages of this approach become evident, showcasing how it outperforms the relative gain array (RGA)-based loop pairing criterion, which has been known to yield erroneous interaction assessments. This empirical evidence serves as a testament to the validity and applicability of the proposed method in tackling the challenges posed by loop interactions, especially in high-dimensional processes. As a result of this comprehensive research, a substantial step forward is achieved in addressing the intricacies of loop interactions within complex control systems. By providing an effective solution to enhance control strategies, the proposed methodology contributes to the development of more efficient and stable control schemes across various industrial applications. These advancements hold significant promise for industries that rely heavily on process control and automation, leading to improved productivity, resource utilization, and cost-effectiveness. Moreover, the equivalent transfer function method for PI/PID decoupling controller design offers a compelling alternative for multivariable processes. Comprising three integral steps, this approach demonstrates its full-dimensional controller design capabilities while remaining remarkably straightforward and easily implementable. This practicality extends to its ability to adapt to dynamic changes in primary process parameters, ensuring that the resulting controllers continue to operate satisfactorily even in the face of dramatic shifts in process conditions. Indeed, the enduring popularity of Proportional-Integral-Derivative (PID) controllers in industrial process applications owes much to their functional simplicity and remarkable performance resilience. As a staple in control systems design, PID controllers have consistently proven their effectiveness in maintaining stability, regulating processes, and achieving desired setpoints, making them a trusted and widely accepted choice across various industries. Their versatility and ability to deliver satisfactory results even in complex and dynamic environments make them an invaluable tool in the pursuit of effective process control and optimization.

II. A) MULTIVARIABLE PROCESS:

In the realm of process control, multivariable processes, often referred to as MIMO (Multiple-Input Multiple-Output) processes, represent a class of systems that encompass a higher number of inputs and outputs in their structure. These inputs and outputs are intricately interconnected, meaning that changes in any one of the variables can have a significant impact on other variables within the system. This interdependency leads to a complex web of relationships, wherein a specific output may be influenced by the dynamic behaviour of multiple input variables, making the analysis and control of such systems considerably challenging. In contrast, SISO (Single-Input Single-Output) systems, which have only one input and one output, do not encounter the same level of complexity since they involve a simpler and more direct relationship between the input and output variables. Consequently, the interactions and interconnections that are prevalent in MIMO processes are not encountered in SISO systems [4]. However, the reality of industrial processes, especially in real-time applications, is that they are predominantly multivariable in nature. This is primarily due to the inherent complexities and intricacies involved in industrial operations, where multiple



factors and variables must be taken into account to achieve optimal performance and efficiency. The significance of MIMO processes lies in their ability to capture the rich dynamics and interactions that exist in real-world industrial systems. By considering a multitude of inputs and outputs, these processes offer a more comprehensive representation of the complexities involved, allowing for a more accurate modelling and control of industrial systems. While SISO systems are valuable in certain applications, they may fall short in providing the detailed and holistic understanding required to effectively manage and optimize complex industrial processes. In conclusion, the prevalence of MIMO processes in real-time industrial applications is a testament to their relevance and necessity in addressing the complexities inherent in industrial operations. Understanding and effectively controlling such multivariable systems are crucial to achieving operational excellence, minimizing waste, maximizing resource utilization, and ensuring the overall efficiency and productivity of industrial processes. As industries continue to evolve and become more sophisticated, the demand for advanced control strategies capable of handling MIMO processes will only continue to grow, emphasizing the importance of continued research and innovation in this field. The MIMO control process for “n” input and “n” output is depicted in the following figure.

Fig.2 MIMO Process

In the realm of MIMO processes, even a minor change in a particular manipulated variable, let's say u_1 , can exert a far-reaching influence on all the controlled variables, encompassing $y_1, y_2, y_3, \dots, y_n$. This inherent interdependency among the variables presents a unique challenge in the selection of an appropriate and effective set of controllable pairs of manipulated and controlled variables within MIMO systems [5]. This choice becomes a pivotal factor in determining the success and efficiency of the control strategy employed. To comprehend the significance of this challenge, consider a specific control problem involving 'n' controlled variables and 'n' manipulated variables. In such cases, there exists a myriad of possibilities for forming controlled-manipulated variable pairs, totalling 'n!' unique combinations. Each of these combinations represents a potential configuration for control, with different implications for system behaviour and response.

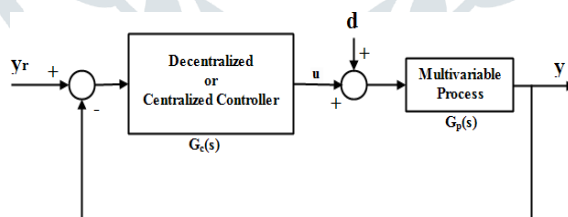


Fig.3 Structure of Multivariable Control System

Consequently, selecting the most suitable pairings becomes a critical task, as it directly affects the overall performance and stability of the control system. An assumption that simplifies this complex pairing process is to consider the number of manipulated variables to be equal to the number of controlled variables. This assumption effectively allows the pairing of each controlled variable with a single manipulated variable, thereby forming a one-to-one relationship between them through a feedback controller. This approach streamlines the pairing decisions, providing a more manageable set of combinations to consider. However, it is important to note that even with this simplification, the task of pairing remains nontrivial, as each combination can lead to different system dynamics and interactions. The control engineer must carefully evaluate the characteristics of the system, the desired control objectives, and the trade-offs involved in each pairing decision. Moreover, factors such as system constraints, physical limitations, and control objectives further influence the selection process. In conclusion, in MIMO processes, the interconnectedness among manipulated and controlled variables necessitates a thoughtful and strategic approach to pairing. The large number of possible combinations calls for careful consideration and analysis to identify the most suitable controlled-manipulated variable pairs for effective control. Despite the complexity of the task, the assumption of an equal number of manipulated and controlled variables eases the process, paving the way for more manageable one-to-one pairings. The successful selection of these pairs is crucial to achieve optimal performance, stability, and responsiveness in MIMO systems, ultimately contributing to the efficient operation and control of complex industrial processes.

Here 'Gp(s)' denotes the transfer function matrix of the MIMO process, while 'Gc(s)' represents the controller matrix. Additionally, it mentions that 'yr' is the set point vector, 'd' is the disturbance input vector, and 'y' is the vector representing the actual output of the control system. The last sentence indicates that the process transfer function vector and the corresponding controller vector are obtained for a 2x2 system.

$$G_p(s) = \begin{pmatrix} g_{p11}(s) & g_{p12}(s) \\ g_{p21}(s) & g_{p22}(s) \end{pmatrix} \quad \text{Eq.1}$$

$$G_c(s) = \begin{pmatrix} g_{c11}(s) & g_{c12}(s) \\ g_{c21}(s) & g_{c22}(s) \end{pmatrix} \quad \text{Eq.2}$$

Each element of process matrix is assumed to be of the form of FOPDT as given in eq3

$$g_{pi,j}(s) = \frac{k_{i,j}}{\tau_{i,j}s + 1} e^{-\theta_{i,j}(s)} \quad \text{Eq.3}$$

B) TWO INPUT TWO OUTPUT MODEL PROCESS (TITO)

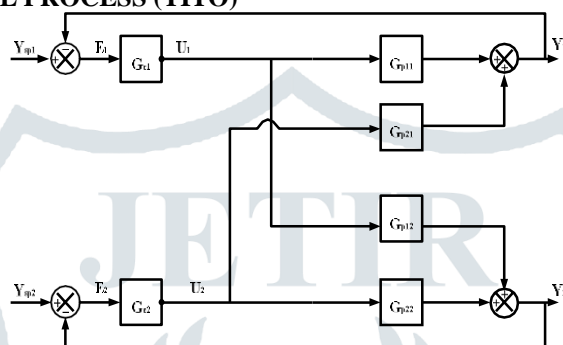


Fig.4 Block Diagram of 2*2 Control Scheme

$$G_{p11}(s) = \frac{Y_1(s)}{U_1(s)}, G_{p12}(s) = \frac{Y_1(s)}{U_2(s)} \quad \text{Eq.4}$$

there are two controlled (\$Y_1, Y_2\$) and two manipulated variables (\$U_1, U_2\$), four process transfer functions (\$G_{p11}, G_{p12}, G_{p21}, G_{p22}\$) are necessary to complete the characterization the process of dynamics.

$$G_{p21}(s) = \frac{Y_2(s)}{U_1(s)}, G_{p22}(s) = \frac{Y_2(s)}{U_2(s)} \quad \text{Eq.5}$$

The transfer function from equation (1) is used to determine the effect of change in either \$U_1\$ or \$U_2\$ on \$Y_1\$ and \$Y_2\$. From the principle of superposition, it follows that simultaneous changes in \$U_1\$ and \$U_2\$ have an additive effect on each controlled variable.

$$Y_1(s) = G_{p11}(s)U_1(s) + G_{p12}(s)U_2(s) \quad \text{Eq.6}$$

$$Y_2(s) = G_{p21}(s)U_1(s) + G_{p22}(s)U_2(s) \quad \text{Eq.7}$$

III. MATHEMATICAL MODELLING OF WWTP PROCESS

The numerical model representing the treatment stages under consideration is designed to encompass a collection of four nonlinear differential conditions. This intricate Waste Water Treatment Plant (WWTP) model is firmly rooted in the fundamental principles of engineering, adhering closely to the main designing standard. As a result of this rigorous approach, the WWTP model can be aptly characterized as a "white-box model," referring to its transparent and comprehensible nature. By integrating the four nonlinear differential conditions into the model, a robust representation of the WWTP's behaviour and dynamics is achieved. Each condition captures a unique aspect of the complex processes at play within the treatment stages, enabling a comprehensive and holistic understanding of the system's response to various inputs and disturbances. The adherence to the main designing standard is a testament to the meticulousness and rigor with which the model has been crafted. The use of established engineering principles and best practices ensures that the model's outcomes and predictions are reliable, accurate, and aligned with real-world observations. The concept of a "white-box model" further emphasizes the model's transparency and interpretability, as it provides clear insights into the underlying mechanisms governing the WWTP's behaviour. Unlike "black-box" models, which may yield accurate predictions but lack transparency regarding the underlying processes, the "white-box model" allows engineers and researchers to gain valuable insights into the cause-and-effect relationships and interactions within the WWTP.

This level of understanding is of paramount importance in the field of wastewater treatment, where effective control and optimization strategies are essential for achieving efficient and sustainable operations. By employing a white-box model that is grounded in the main designing standard, engineers can confidently analyze, optimize, and fine-tune the WWTP's performance, thereby ensuring optimal resource utilization, reduced environmental impact, and improved overall efficacy in treating wastewater.

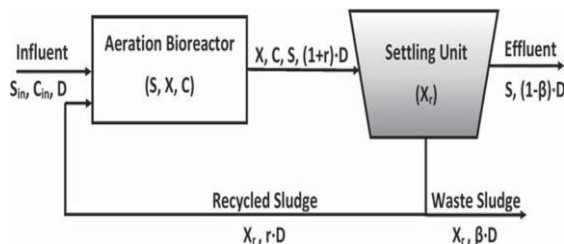


Fig.5 Block Diagram of WWTP

The set of nonlinear conditions is derived by considering mass balance equations around the aerator and the settler.

$$\frac{dX(t)}{dt} = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \quad \text{Eq.8}$$

$$\frac{dS(t)}{dt} = -\frac{\mu(t)}{Y} X(t) - D(t)(1+r)S(t) + D(t)S_{in} \quad \text{Eq.9}$$

$$\frac{dC(t)}{dt} = -\frac{K_o\mu(t)}{Y} X(t) - D(t)(1+r)C(t) + K_{La}(C_s - C(t)) + D(t)C_{in} \quad \text{Eq.10}$$

$$\frac{dX_r(t)}{dt} = D(t)(1+r)X(t) - D(t)(\beta+r)X_r(t) \quad \text{Eq.11}$$

In this system of equations, several state factors play a crucial role in describing the dynamics of the process. These state factors include $X(t)$, which represents the biomass concentration, $S(t)$, denoting the substrate concentration, $C(t)$, reflecting the dissolved oxygen concentration and $X_r(t)$, representing the concentration of reused biomass. Additionally, the weakening rate is denoted by $D(t)$.

Two significant proportions, r and β , hold vital significance in the context of this system. The first proportion, r , signifies the ratio of the reused stream to the influent stream, while the second proportion, β , represents the ratio of the waste stream to the influent stream. The concentrations of dissolved oxygen fixation (C_{in}) and substrate (S_{in}) in the feed stream are also important parameters to consider. Moreover, the specific growth rate, μ , and the yield cell mass, Y , play a crucial role in determining the energy associated with the cell's macro-scale production. The term k_o remains constant in the system, while C_s represents the maximum dissolved oxygen concentration, and the oxygen mass exchange coefficient is denoted as K_{La} . Each of these factors and coefficients contributes to the complexity and richness of the system of equations, capturing the intricate interplay of biomass growth, substrate utilization, dissolved oxygen levels, and biomass recycling. The mathematical representation of these factors enables researchers and engineers to analyse and optimize the process under various conditions, leading to better insights and informed decision-making. Understanding the significance of each state factor and coefficient within this system is essential for comprehending the system's behaviour and devising effective control and optimization strategies. The interactions between these variables influence the overall performance and stability of the process, making it essential to carefully consider and tailor the values of each parameter for achieving desired outcomes in wastewater treatment and biological processes. The mathematical formulation of these relationships enables the development of comprehensive models, which in turn facilitates improved process management and the achievement of environmental and operational goals in various industrial applications.

IV. EQUIVALENT TRANSFER FUNCTION APPROACH

The process of obtaining the Equivalent Transfer Function (ETF) of the process model involves a meticulous consideration and integration of fundamental concepts such as Energy Transmission Ratio and Effective Relative Gain Array. Through this intricate procedure, a comprehensive representation of the process dynamics is attained, offering valuable insights into the system's behaviour and response to various inputs and disturbances [6]. The ETF serves as a powerful tool for engineers and researchers in analysing and optimizing complex processes, enabling a deeper understanding of the underlying mechanisms and interactions that govern the system's performance. By incorporating the Energy Transmission Ratio and Effective Relative Gain Array, the ETF captures the intricate interplay of energy transfer and relative gain among different variables, contributing to more accurate and reliable modelling of real-world processes and control system design.

A) Energy Transmission Ratio (ETR):

The energy transmission ratio of $g_{i,j}(s)$ is expressed as: $e_{i,j} \approx g_{i,j}(0)\omega_{c,i,j} \dots i, j = 1, 2, \dots, n$ Eq.12

Where $\omega_{c,i,j}$ is the critical frequency of the transfer function. The energy transmission ratio for the overall system is expressed by the

effective energy transmission ratio array: $E = \begin{pmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{pmatrix} = G(0) \otimes \Omega$ Eq.13

$$G(0) = \begin{pmatrix} g_{11}(0) & g_{12}(0) \\ g_{21}(0) & g_{22}(0) \end{pmatrix} \quad \text{Eq.14}$$

$$\Omega = \begin{pmatrix} \omega_{c11} & \omega_{c12} \\ \omega_{c21} & \omega_{c22} \end{pmatrix} \tag{Eq.15}$$

are the steady state gain array and the critical frequency array respectively.

B) Effective Relative Gain Array (ERGA):

The definition of the effective relative gain array is as follows:
$$\phi_{i,j} = \frac{e_{i,j}}{\Lambda} \tag{Eq.16}$$

Where $e_{i,j}$ is the effective energy transmission ratio between output variable and input variable when all other loops are closed. Over all ERGA (Φ) can be calculated as:

$$\Phi = E \otimes E^{-T}$$

The principles of loop matching based on the Effective Relative Gain Array (ERGA) require careful consideration of several factors within the primary circle. It is essential to combine these factors using sets whose ERGA and Normalized Integral (NI) values are not only positive but also closest to the ideal value of 1.0. This ensures the optimization of the loop's performance and stability, as the closer the ERGA and NI values are to 1.0, the better the overall control and response characteristics of the system are achieved.

V. RESULTS AND DISCUSSION

In this particular section, comprehensive SIMULINK models for the wastewater treatment plant are provided. These models have been carefully crafted to represent and simulate the various processes and components involved in wastewater treatment. By utilizing these detailed models, researchers, engineers, and practitioners can gain valuable insights into the system's behaviour, performance, and efficiency, facilitating the analysis and improvement of the wastewater treatment processes. Within the context of the Wastewater Treatment (WWT) Plant, the forthcoming figures present a comprehensive visual representation of the specific control methods employed. It is worth noting that this particular WWT Plant operates as a two-input and two-output process. These figures elaborately showcase the intricacies of the control strategies applied to regulate and optimize the complex interactions and dynamics of the treatment plant.

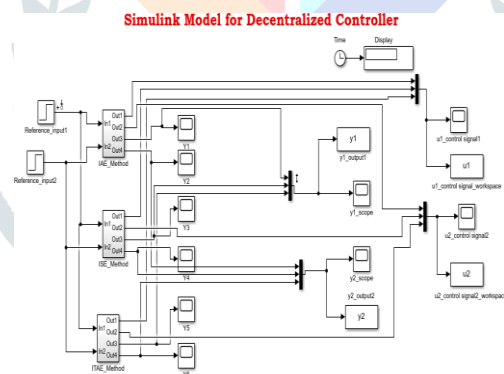


Fig.6 Simulink model for Decentralized Controller

An exhibition of the projected unified control is assessed regarding indispensable of integral over time of working time, i.e., necessary of IAE (Integral Absolute Error), ITAE (Integral Time Weighted Absolute Error), TV (Total Variation)

The expressions for two variable-TITO process are:

$$IAE = \int_0^{\infty} (|E_1(t)| + |E_2(t)|) dt \tag{Eq.17}$$

$$ITAE = \int_0^{\infty} t(|E_1(t)| + |E_2(t)|) dt \tag{Eq.18}$$

Where $E(t) = y_r(t) - u(t)$;

$$TV = \sum_{i=1}^{\infty} |u_{i+1} - u_i| \tag{Eq.19}$$

Total Variation (TV) serves as an essential metric in the context of closed loop/circle control reaction, helping us evaluate the effectiveness of controlled information utilization. This measure rule plays a pivotal role in assessing the regulator's yield, denoted as 'u.' The regulator yield, in this context, represents the output of the control system. The crux of TV lies in its capacity to quantify all the variations observed in the control signal, encompassing both upward and downward movements. In simpler terms, Total Variation acts as a powerful tool to analyze the fluctuations and changes in the control signal's magnitude. It provides valuable insights into the control system's behaviour by capturing the net magnitude of the signal's oscillations and revealing how the system responds to different inputs and disturbances. By comprehensively

accounting for all variations, TV enables us to understand how the control system adapts to external influences and ensures stability and efficiency in its operation. The utilization of the TV of the regulator yield 'u' empowers engineers and researchers to make informed decisions, fine-tune the control parameters, and optimize the overall performance of the control system.

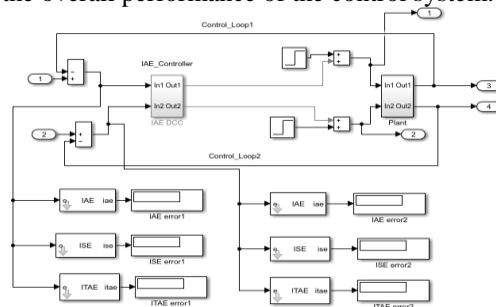


Fig.7 SIMULINK Model for TITO processes using IAE method

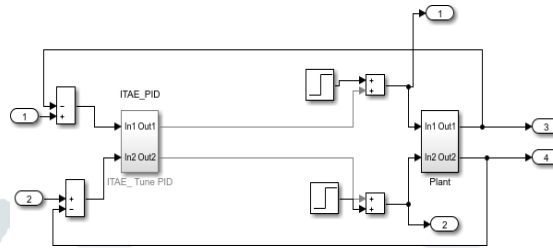


Fig.8 Model for ITAE tuned TITO process

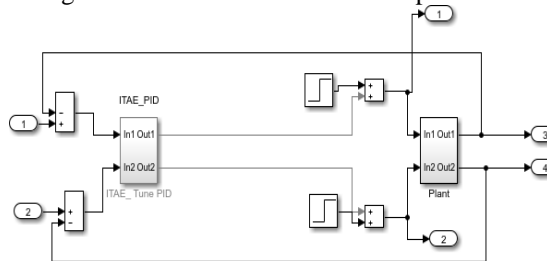


Fig.9 Decoupled PI controller Model for TITO processes

The Simulink for various control schemes proposed in this paper has been given above. These are applied directly to the model of waste water treatment plant.

**Equivalent Transfer function Controller
for WASTE WATER TREATMENT PLANT**

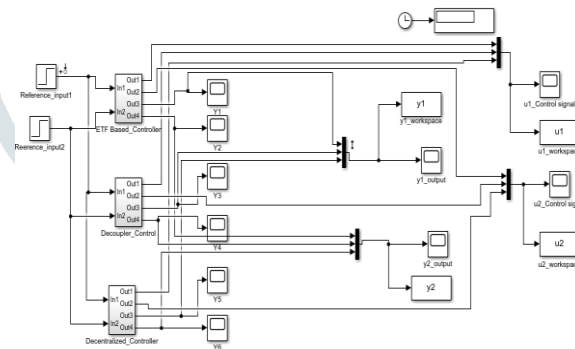


Fig.10 Simulink model of WWT Plant based ETF

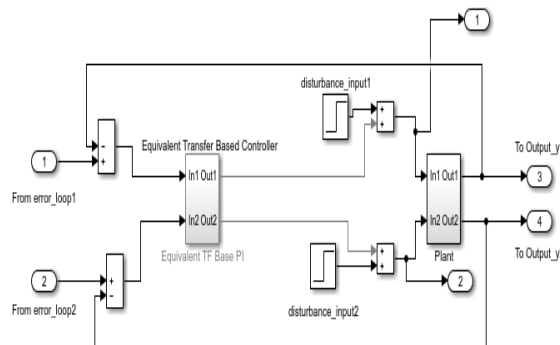


Fig.11 Subsystem for Equivalent Transfer Function based PI Controller

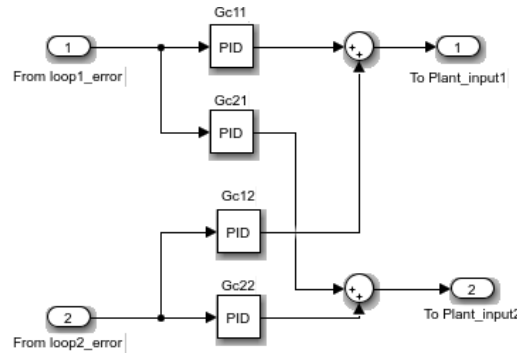


Fig.12 PI Controller structure in ETF Method

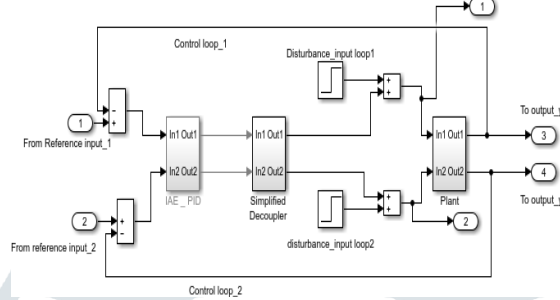


Fig.13 Decoupled Control Loop For WWTP

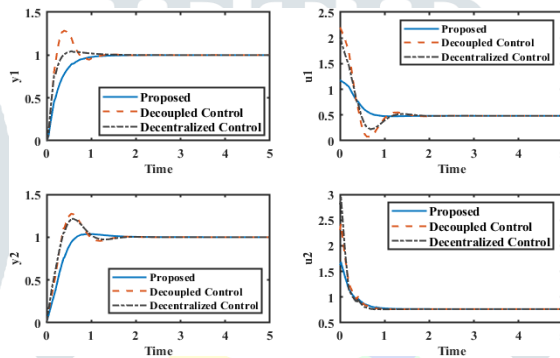


Fig.14 closed loop response of WWTP

The closed loop servo response is shown in fig.14. Regarding the servo problem, the proposed control system exhibits remarkable capabilities, delivering an exceptionally favourable response and precise control action. The discernible superiority of this approach attests to its potency in addressing the servo problem and firmly establishes it as a frontrunner in the realm of control systems.

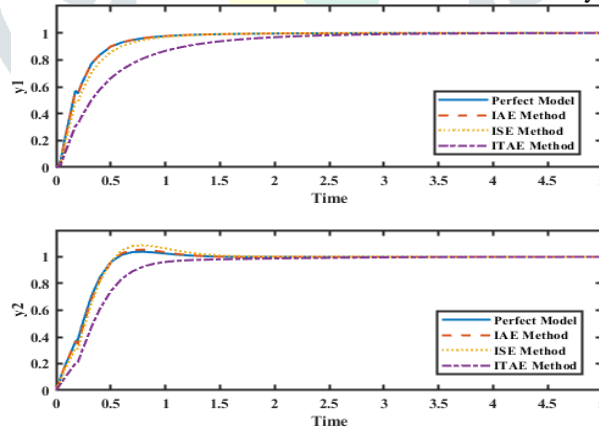


Fig.15 Closed Loop Response For +10% Perturbations in time delay-decentralized controller

The effectiveness and prowess of the proposed controller are further reaffirmed through meticulous analysis, which reveals the attainment of significantly smaller Integral of Absolute Error (IAE) values upon calculation is given in the following table 1. This noteworthy outcome serves as concrete evidence of the controller's exceptional ability to curtail any discrepancies between the desired and actual control system responses, exemplifying its outstanding performance in comparison to alternative controllers.

Table.1 Performance indices of WWTP

Method	Y ₁			Y ₂		
	IAE	ITAE	TV	IAE	ITAE	TV
Proposed	0.302	0.0391	0.0915	0.3068	0.094	0.0023
Decoupled control	0.312	0.0599	0.1789	0.3132	0.06	0.0015
Decentralized control	0.042	3.5743	1.4158	0.1327	3.6746	1.133

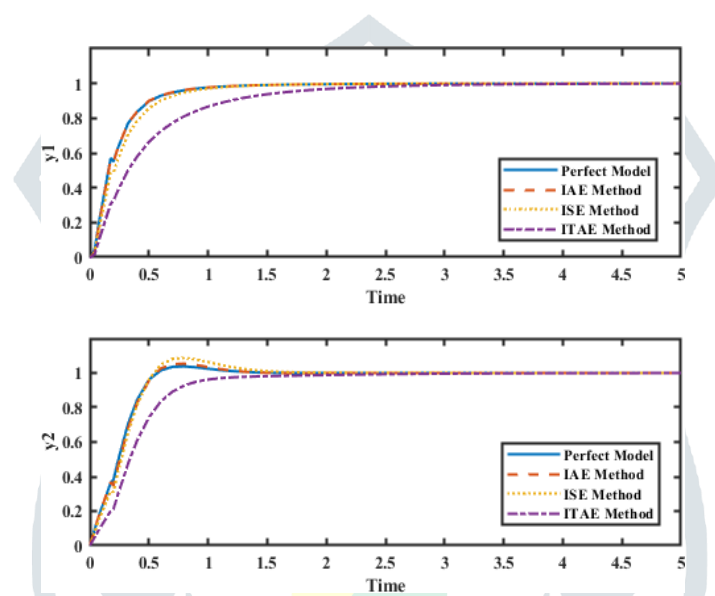


Fig.16 Closed Loop Response For +10% Perturbations in static gain-decentralized controller.

The modelling of the WWT (Wastewater Treatment) plant is achieved through a meticulous approximation, wherein the concepts of effective energy transmission ratio and ERGA (Energy Ratio of Gravitational and Acceleration forces) play a pivotal role. To optimize the control system's performance, the PI (Proportional-Integral) controller parameters are precisely determined utilizing the novel ETF (Energy-based Tuning Factor) method, ingeniously introduced in this project. Through comprehensive simulations, the efficacy of the proposed method is thoroughly examined, and its results are thoughtfully juxtaposed against those obtained from comparable approaches in the field. Furthermore, to ensure the controller's robustness and adaptability in real-world scenarios, it undergoes stringent testing under varying model uncertainties, where parameters are deliberately perturbed within a $\pm 10\%$ range. Gratifyingly, the controller effectively navigates through these challenging conditions, reinforcing its reliability and resilience when confronted with unpredictable fluctuations and uncertainties.

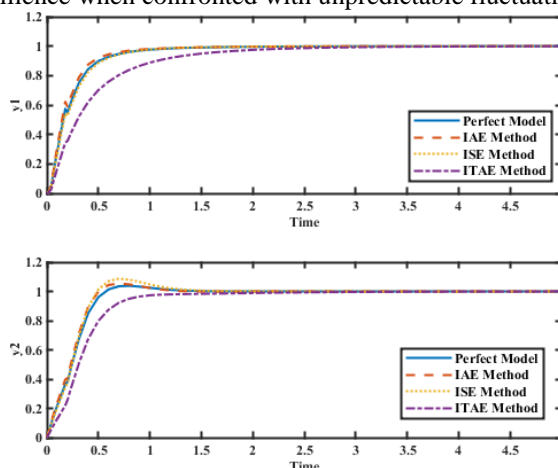


Fig.17 Closed Loop Response For +10% Perturbations in time constant-decentralized controller

VI. CONCLUSION

This project endeavours to design and implement a dynamic centralized control system tailored specifically for stable WWTP (Wastewater Treatment Plant) frameworks. The cornerstone of this control system lies in the meticulous derivation of transfer function model details, which serves as the foundation for its successful functioning. Within this centralized control framework, both diagonal and off-diagonal regulators are strategically integrated to ensure comprehensive and efficient control over the plant's operations. To achieve optimal controller design, the ERGA-based ETF technique is ingeniously employed, enabling the calculation of controllers with utmost precision and accuracy. The results unequivocally demonstrate the superiority of the proposed approach, as it exhibits superior response and control action compared to conventional methods. In assessing the performance of the control system, key performance measures such as IAE (Integral of Absolute Error), ISE (Integral of Squared Error), and TV (Tracking Variance) are meticulously utilized. These measures provide a comprehensive evaluation of the control system's effectiveness in tackling various control challenges and optimizing its performance. Furthermore, to affirm the robustness of the proposed strategy, detailed simulations are conducted, where the results are thoughtfully compared with those obtained from two other comparative methodologies. This attribute ensures ease of implementation and applicability, even in scenarios where model mismatches may arise. The proposed method is designed with versatility in mind, effectively catering to real-world complexities and uncertainties that often accompany WWTP operations. In summary, this project presents a dynamic centralized control system that leverages advanced modelling techniques, innovative controller design, and comprehensive performance evaluations. The remarkable performance showcased by this approach not only validates its efficacy but also positions it as a promising and practical solution for optimizing WWTP operations with stability, efficiency, and adaptability.

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