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# NUMERICAL APPROXIMATION OF SECOND ORDER LINEAR FREDHOLM INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract

In this paper a numerical scheme has been developed for solving second order linear integro-differential equations subject to mixed conditions. We adopted the standard collocation points and the integro-differential equations is transformed into a system of linear equations. The linear system is then solved using MATLAB programming. The proposed scheme exhibits convergence, while the efficiency and applicability of the scheme has been demonstrated using two examples. The results were compared with an existing method that used Laguerre polynomials. The proposed method is computationally reliable.

Keywords: Fredholm Integro-Differential Equation, Mixed conditions, Collocation Method, Convergence.

## 1. INTRODUCTION

In this study, we considered the system of linear integro-differential difference equation of the form

$$\sum_{n=0}^{N}\sum_{j=1}^{k} P_{ij}^{n}(x)y_j^{(n)}(x)+\sum_{r=0}^{R}\sum_{j=1}^{k} q_{ij}^{r}(x)y_j^{(r)}(x+\tau)=g_j(x)$$
$$+\int_0^x\sum_{j=1}^{k} K_{i,j}(x,t)y_j(t)dt+\int_0^1\sum_{j=1}^{k}\omega_{i,j}(x,t)y_j(t)dt \qquad (1)$$

subject to the mixed condition

$$\sum a_{i,j}^{(n)}y_n^{(i)}(a)+b_{i,j}^{(n)}y_n^{(j)}(b)=\lambda_{n.i}, i,j=1,2,\ldots,k, n=0,1,\ldots,m-1 \qquad (2)$$

where $y_j : J \to \Re^N, J \in [0,1]$. The solution to (1) and (2) is a continuous function to be determined, that is, $y \in C[J,\Re^N), P_{i,j}^n(x), q_{i,j}^r(x), g_j : J \to \Re^N$ are given continuous functions, $K,\omega : J \times J \to \Re^N$ are Lipschitz continuous, $a_{i,j}, b_{i,j}, \lambda_{n,i}$ are real constants.

The resolution of many problems in physics and engineering leads to (1) and (2) which is an important branch in modern day mathematics. Integro-differential equation arises frequently in mechanics, astronomy, biology, economics, chemistry, etc. [1]. It has attracted much attention and solving this equation has been one of the interesting tasks for mathematicians, because it can usually be reduced to a system of integral and integro-differential equations. Integro-differential equation can be classified into Fredholm, Volterra or a combination of both. It is termed Fredholm integro-differential equation if the upper part of the integral is a constant and Volterra if the upper part of the integral is a variable.

Fredholm integro-differential equation has applicability in Nano-hydrodynamics [2], glass-forming process [3], dropwise condensation [4], wind ripples in the desert [5], modelling the competition between tumour cell and the immune system [6], and also examining the noise term phenomenon [7]. Since analytical solutions of such types of problems are not easily determined, we therefore sought for numerical methods.

Several numerical methods were presented for solving of Fredholm integro-differential equations, such as Fibonacci matrix method [8], differential transform method [9], Bessel matrix method [10], Laguerre polynomials method [11] and Bell polynomials by [12]. Equation (1) is a combination of differential equation, difference equation, Fredholm integro-differential equation and Volterra integro-differential equation obtained by extending the work of [13], [14] and [15].

The aim of this study is to obtain an approximate solution of (1) and (2) using polynomial collocation method.

## 2. MATHEMATICAL REPRESENTATION OF SYSTEM AND METHODS

Suppose we write (1) in the for

$$I_1(x)+I_2(x)=g_j(x)+I_3(x)+I_4(x), \tag{3}$$

then

$$I_1=\sum_{n=0}^{N}\mathbf{P}_n(x)\mathbf{y}^{(n)}(x) \tag{4}$$

where $\mathbf{P}_n(x)=\begin{bmatrix} p_{11}^n & p_{12}^n & \cdots & p_{1k}^n \\ p_{21}^n & p_{22}^n & \cdots & p_{2k}^n \\ \vdots & \vdots & \cdots & \vdots \\ p_{k1}^n & p_{k1}^n & \cdots & p_{kk}^n \end{bmatrix}$, $\mathbf{y}^{(n)}(x)=\begin{bmatrix} y_1^{(n)} & y_1^{(n)} & \cdots & y_k^{(n)} \end{bmatrix}^T$

similarly,

$$I_2=\sum_{r=0}^{R}\mathbf{q}_r(x)\mathbf{y}^{(r)}(x)\mathbf{M} \tag{5}$$

where $\mathbf{q}_r(x)=\begin{bmatrix} q_{11}^r & q_{12}^r & \cdots & q_{1k}^r \\ q_{21}^r & q_{22}^r & \cdots & q_{2k}^r \\ \vdots & \vdots & \cdots & \vdots \\ q_{k1}^r & q_{k2}^r & \cdots & q_{kk}^r \end{bmatrix}$, $\mathbf{M}=\begin{bmatrix} \mathbf{M}_{-\mathbf{r}} & 0 & \cdots & 0 \\ 0 & \mathbf{M}_{-\mathbf{r}} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \mathbf{M}_{-\mathbf{r}} \end{bmatrix}$

$$\mathbf{M}_{-\mathbf{r}} = \begin{bmatrix} \binom{0}{0}(-\tau)^0 & \binom{1}{0}(-\tau)^1 & \binom{2}{0}(-\tau)^2 & \cdots & \binom{N}{0}(-\tau)^N \\ 0 & \binom{1}{1}(-\tau)^0 & \binom{2}{1}(-\tau)^1 & \cdots & \binom{N}{1}(-\tau)^{N-1} \\ 0 & 0 & \binom{2}{2}(-\tau)^0 & \cdots & \binom{N}{2}(-\tau)^{N-2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \binom{N}{n}(-\tau)^0 \end{bmatrix}$$

$$I_3 = \int_0^x \mathbf{K}(x,t)\mathbf{y}(t)dt \tag{6}$$

where $\mathbf{K}(x,t) = \begin{bmatrix} k_{11}(x,t) & k_{12}(x,t) & \cdots & k_{1k}(x,t) \\ k_{21}(x,t) & k_{22}(x,t) & \cdots & k_{2k}(x,t) \\ \vdots & \vdots & \cdots & \vdots \\ k_{k1}(x,t) & k_{k1}(x,t) & \cdots & k_{kk}(x,t) \end{bmatrix}$

$$I_4 = \int_0^1 \boldsymbol{\omega}(x,t)\mathbf{y}(t)dt \tag{7}$$

where $\boldsymbol{\omega}(x,t) = \begin{bmatrix} \omega_{11}(x,t) & \omega_{12}(x,t) & \cdots & \omega_{1k}(x,t) \\ \omega_{21}(x,t) & \omega_{21}(x,t) & \cdots & \omega_{2k}(x,t) \\ \vdots & \vdots & \cdots & \vdots \\ \omega_{k1}(x,t) & \omega_{k1}(x,t) & \cdots & \omega_{kk}(x,t) \end{bmatrix}$

$$g_j(x) = \mathbf{G}(x) \tag{8}$$

where $\mathbf{G}(x) = [g_1(x) \quad g_2(x) \quad \cdots \quad g_k(x)]^T$

Thus, (1) reduces to

$$\sum_{n=0}^{N} \mathbf{P}_n(x)\mathbf{y}^{(n)}(x) = -\sum_{r=0}^{R} \mathbf{q}_r(x)\mathbf{y}^{(r)}(x)\mathbf{M} + \mathbf{G}(x) + \int_0^x \mathbf{K}(x,t)\mathbf{y}(t)dt + \int_0^1 \boldsymbol{\omega}(x,t)\mathbf{y}(t)dt \tag{9}$$

with mixed conditions

$$\sum_{j=0}^{m-1}\left[\mathbf{a}_i\mathbf{y}^{(i)}(a) + \mathbf{b}_i\mathbf{y}^{(j)}(b)\right] = \boldsymbol{\lambda}_i \tag{10}$$

where $\mathbf{a}_i = \begin{bmatrix} a_{i,j}{}^1 & 0 & \cdots & 0 \\ 0 & a_{i,j}{}^2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & a_{i,j}{}^k \end{bmatrix}$, $\mathbf{y}^{(i)}(a) = \begin{bmatrix} y_1{}^{(j)}(a) \\ y_1{}^{(j)}(a) \\ \vdots \\ y_k{}^{(j)}(a) \end{bmatrix}$, $\mathbf{b}_i = \begin{bmatrix} b_{i,j}{}^1 & 0 & \cdots & 0 \\ 0 & b_{i,j}{}^2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & b_{i,j}{}^k \end{bmatrix}$,

$$\mathbf{y}^{(j)}(b)=\begin{bmatrix} y_1^{(j)}(b) \\ y_1^{(j)}(b) \\ \vdots \\ y_k^{(j)}(b) \end{bmatrix}$$

2.1 Methodology

Let the approximate solution of (9) be

$$\mathbf{y}_j(x)=\mathbf{X}_j(x)\mathbf{C}_j \tag{11}$$

where $\mathbf{X}_j=\begin{bmatrix}1 & x_j & x_j^2 & \cdots & x_j^H\end{bmatrix}$ and $\mathbf{C}_j=\begin{bmatrix}c_{j0} & c_{j1} & c_{j2} & \cdots & c_j^H\end{bmatrix}^T$ are constants to be determined.

The standard collocation

$$x_i=a+\frac{(b-a)i}{N} \tag{12}$$

is given by (12).

Collocating and substituting (11) into (9) yields

$$\mathbf{U}(x_i)\mathbf{C}=\mathbf{G}(x_i) \tag{13}$$

where $\mathbf{U}(x_i)=\sum_{n=0}^{N}\mathbf{P}_n(x)\mathbf{X}^{(n)}(x_i)+\sum_{r=0}^{R}\mathbf{q}_r(x_i)\mathbf{y}^{(r)}(x_i)\mathbf{M}-\int_0^x\mathbf{K}(x_i,t)\mathbf{X}(t)dt-\int_0^1\boldsymbol{\omega}(x_i,t)\mathbf{X}(t)dt$

Substituting (13) into the mixed condition (10)

$$\mathbf{U}_i\mathbf{C}=\sum_{j=0}^{m-1}a_{i,j}X^{(i)}(a)+b_{i,j}X^{(j)}(b) \tag{14}$$

Augmenting (13) and (14) gives

$$\begin{bmatrix}\mathbf{U}(x_i) \\ \mathbf{U}_i\end{bmatrix}\mathbf{C}=\begin{bmatrix}\mathbf{G}(x_i) \\ \boldsymbol{\lambda}_i\end{bmatrix} \tag{15}$$

The unknown constants $\mathbf{C}$ in (15) yields

$$\mathbf{C}=\mu^{-1}(x_i)\mathbf{A}(x_i) \tag{16}$$

where $$\mu(x_i)=\begin{bmatrix}\mathbf{C}(x_i) \\ \mathbf{C}_i\end{bmatrix}^T\begin{bmatrix}\mathbf{C}(x_i) \\ \mathbf{C}_i\end{bmatrix},\quad \mathbf{A}(x_i)=\begin{bmatrix}\mathbf{C}(x_i) \\ \mathbf{C}_i\end{bmatrix}^T\begin{bmatrix}\mathbf{C}(x_i) \\ \boldsymbol{\lambda}_i\end{bmatrix}$$

Equation (16) is then substituted into the approximate solution (11) to give the desired numerical solutions.

## 3. NUMERICAL EXAMPLES

In this section, some numerical examples of (1) are given to illustrate the accuracy and simplicity of the method. Let $y_n(x)$ and $y(x)$ be the approximate and exact solutions respectively, $e_N$ is the error function, therefore $abs-e_N=|y_n-y|$ is the absolute error for $N$. All results are presented in tables except where $abs-e_N=0$.

Example 1: Let us consider

$$y_1^{(2)}(x) = \frac{8}{9} + \int_0^1 \{\frac{1}{3} y_1(t) + \frac{1}{4} y_2(t)\} dt$$

$$y_2^{(2)}(x) = 6x + \frac{x^2}{18} + \int_0^1 \{\frac{x^2}{6} y_1(t) - \frac{x^2}{3} y_2(t)\} dt$$

$0 \le x \le 1$ subject to the conditions $y_1(0) = 0,\ y_1^{(1)}(0) = \frac{1}{3},\ y_2(0) = 0,\ y_2^{(1)}(0) = -\frac{1}{2}.$

The exact solution are: $y_1(x) = \frac{x^2}{2} + \frac{x}{3}, y_2(x) = x^3 - \frac{x}{2}.$

Equation (9) gives

$$\mathbf{P}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \mathbf{P}_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{P}_0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \frac{8}{9} \\ 6x + \frac{x^2}{18} \end{bmatrix}, \mathbf{K} = 0, q_r = 0, \boldsymbol{\omega} = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} \\ \frac{x^2}{6} & -\frac{x^2}{3} \end{bmatrix}.$$

For $N = 3, x_i = \begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} & 1 \end{bmatrix}.$

Equation (15) gives

$$\mathbf{U}(x_i) = \begin{bmatrix} -0.333 & -0.167 & 1.89 & =0.0833 & -0.25 & -0.125 & -0.0833 & -0.0625 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ -0.333 & -0.167 & 1.89 & 1.92 & -0.25 & -0.125 & -0.0833 & -0.0625 \\ -0.0185 & -0.00926 & -0.00617 & -0.00463 & -0.037 & -0.0185 & 1.99 & 1.99 \\ -0.333 & -0.167 & =0.0247 & -0.0185 & -0.148 & -0.0741` & 1.95 & -0.0625 \\ -0.0741 & -0.037 & -0.0247 & -0.0185 & -0.148 & -0.0741 & 1.95 & 3.96 \\ -0.333 & -0.167 & 1.89 & 5.92 & -0.25 & -0.125 & -0.0833 & -0.0625 \\ -0.167 & -0.0833 & -0.0556 & -0.0417 & -0.333 & -0.167 & 1.89 & 5.92 \end{bmatrix}$$

$$\mathbf{U}_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \boldsymbol{\lambda} = \begin{bmatrix} 0 & \frac{1}{3} & 0 & -\frac{1}{2} \end{bmatrix}^T$$

$$\mathbf{G}(x_i) = \begin{bmatrix} \frac{8}{9} & 0 & \frac{8}{9} & \frac{323}{162} & \frac{8}{9} & \frac{322}{81} & \frac{8}{9} & \frac{107}{18} \end{bmatrix}^T.$$

Equation (16) gives $\mathbf{A} = \begin{bmatrix} -1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \mu = \begin{bmatrix} \frac{8}{9} & 0 & \frac{8}{9} & \frac{323}{162} & -\frac{1}{2} & 0 & \frac{1}{3} & 0 \end{bmatrix}^T$

and the coefficient is $\mathbf{C}=\begin{bmatrix}0 & \frac{1}{3} & \frac{1}{2} & 0 & \frac{-1}{2} & 0 & 1\end{bmatrix}^T$. The scheme gives the exact solution of the system.

Example 2: Let us consider

$$y_1^{(2)}(x)-xy_2^{(1)}(x)-y_1(x)=(x-2)\sin(x)+\int_0^1\{x\cos(t)y_1(t)-x\sin(t)y_2(t)dt\}$$

$$-2xy_1^{(1)}(x)+y_2^{(2)}(x)+y_2(x)=-2x\cos(x)+\int_0^1\{\sin x(x)\cos(t)y_1(t)-\sin(s)\sin(t)y_2(t)dt\}$$

$0\le x\le 1$ subject to the conditions $y_1(0)=0,\ y_1^{(1)}(0)=1,\ y_2(0)=1,\ y_2^{(1)}(0)=0.$

The exact solution are: $y_1(x)=\sin(x), y_2(x)=\cos(x).$

From (9), $\mathbf{P}_2=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \mathbf{P}_1=\begin{bmatrix}0 & -x\\ -2x & 0\end{bmatrix}, \mathbf{P}_0=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix},$

$$\mathbf{G}=\begin{bmatrix}(x-2)\sin(x)\\ -2x\cos(x)\end{bmatrix}, \mathbf{K}=0, \mathbf{q}_r=0, \boldsymbol{\omega}=\begin{bmatrix}x\cos(t) & -x\sin t\\ \sin(x)\cos(t) & -\sin(x)\sin(t)\end{bmatrix}.$$

For $N=3, x_i=\begin{bmatrix}0 & \frac{1}{3} & \frac{2}{3} & 1\end{bmatrix}.$

Therefore equation (15) yields

$$\mathbf{U}(x_i)=\begin{bmatrix}-1.0 & 0 & 2.0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 2.0 & 0\\ -1.28 & -0.461 & 1.81 & 1.91 & 0.153 & 0.233 & -0.148 & -0.0521\\ -0.275 & -0.792 & -0.523 & -0.278 & 1.15 & 0.432 & 2.18 & 2.09\\ -1.56 & -0.921 & 1.4 & 3.59 & -0.306 & -0.466` & -0.74 & -0.771\\ -0.52 & -1.57 & -1.93 & -1.88 & 1.28 & 0.853 & 2.58 & 4.41\\ -1.84 & -1.38 & 0,761 & 4.83 & 0.46 & -0.699 & -1.78 & -2.82\\ -0.708 & -2.32 & -4.2 & -6.14 & 1.39 & 1.25 & 3.19 & 7.15\end{bmatrix}$$

$$\mathbf{U}_i=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}, \boldsymbol{\lambda}=\begin{bmatrix}0 & 1 & 1 & 0\end{bmatrix}^T$$

$\mathbf{G}(x_i)=\begin{bmatrix}0 & 0 & -\frac{1143}{2096} & -\frac{1093}{1735} & -\frac{1057}{1282} & -\frac{1730}{1651} & -\frac{1327}{1577} & -\frac{429}{397}\end{bmatrix}^T$ and equation (16) yields

$$A=\begin{bmatrix}-1 & 0 & 2 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0\\ -1.28 & -0.461 & 1.81 & 1.91 & 0.153 & -0.233 & -0.148 & -0.0521\\ -0.275 & -0.792 & -0.523 & -0.278 & 1.15 & 0.432 & 2.18 & 2.09\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\mu=\begin{bmatrix}0 & 0 & -\frac{1143}{2096} & -\frac{1093}{1735} & 0 & 1 & 1 & 0\end{bmatrix}^T.$$

Thus, $\mathbf{C}=\begin{bmatrix}0 & 1 & 0 & -0.1628939579523864345 & 1 & 0 & 0 & 0.027649239691159409692\end{bmatrix}^T$.

The numerical solution for $N=3,4$ and $5$ are:

$y_{3,1}(x)=0.1628939579523864345x^3+x$

$y_{3,2}(x)=0..027649239691159409692x^3-0.5x^2+1$

$y_{4,1}(x)=0.010226988504326779 16x^4+0.17008547279508796702x^3+x$

$y_{4,2}(x)=0.040255632236374217066x^4+0.00051640306121486844655x^3-0.5x^2+1$

$y_{5,1}(x)=0.0079132635578672084641x^5+0.00033884230364393938108 95x^4$

$-0.16675042709869618849x^3+1.0x$

$y_{5,2}(x)=0.0024564847462581126709x^5+0.043157735028387156898x^4$

$-0.00032486277552391093697x^3-0.5x^2+1.0$

Table 1: Comparison of Absolute Error with Existing Method for Example 2

| | | Laguerre Polynomials, [11] | | | Presented Method | | |
|---|---|---|---|---|---|---|---|
| $x_i$ | $y$ | $abs-e_3$ | $abs-e_4$ | $abs-e_5$ | $abs-e_3$ | $abs-e_4$ | $abs-e_5$ |
| 0.0 | $y_1$ | 1.99e-15 | 1.33e-15 | 2.38e-15 | 0.00 | 0.00 | 0.00 |
| | $y_2$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.2 | $y_1$ | 3.00e-5 | 1.43e-4 | 9.00e-6 | 3.70e-5 | 4.00e-6 | 9.00e-6 |
| | $y_2$ | 1.47e-4 | 1.43e-3 | 1.00e-6 | 1.54e-4 | 2.00e-6 | 0.00 |
| 0.4 | $y_1$ | 9.80e-4 | 7.80e-4 | 1.00e-6 | 1.57e-4 | 4.20e-5 | 0.00 |
| | $y_2$ | 6.50e-4 | 4.00e-3 | 1.00e-6 | 7.09e-4 | 3.00e-4 | 2.00e-6 |
| 0.6 | $y_1$ | 2.40e-5 | 1.29e-3 | 4.00e-6 | 1.73e-4 | 4.20e-5 | 1.00e-6 |
| | $y_2$ | 4.39e-4 | 1.16e-2 | 1.00e-6 | 7.09e-3 | 7.00e-4 | 4.00e-6 |
| 0.8 | $y_1$ | 1.22e-3 | 2.15e-4 | 1.30e-6 | 7.58e-4 | 2.51e-4 | 0.00 |
| | $y_2$ | 3.02e-3 | 8.69e-2 | 1.20e-6 | 7.09e-4 | 4.60e-5 | 1.00e-6 |
| 1.0 | $y_1$ | 5.28e-3 | 4.54e-3 | 5.80e-5 | 4.36e-3 | 1.33e-3 | 3.10e-5 |
| | $y_2$ | 1.30e-2 | 2.86e-4 | 1.02e-4 | 1.27e-2 | 4.70e-4 | 7.40e-5 |

## 4. CONCLUSION

In this paper we presented a new method that efficiently solved a system of second order linear Fredholm integro-differential equations. [11] used collocation method which transformed the system of Fredholm integro-differential equations into a system of equations in unknown Laguerre coefficients. Our new method used standard collocation points to transform the system of equations into linear algebraic equations. These algebraic equations are then solved by polynomial collocation method and substituted into the approximate solution to obtain the desired numerical results. The scheme gave the exact solution for $N=3$ in example 1. For example 2, as $N$ increases the absolute error is decreasing, thereby showing accuracy of the results. The absolute errors tends to zero which shows that the solution is converging. For the same value of $N$ the presented method gave better results in few computations. Furthermore, the results gave better approximations than the method used in [14]. Both problems were solved using a program written using MATLAB 2016a software.

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