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OBERBECK CONVECTION OF A CASSON NANOFLUID IN A VERTICAL CHANNEL

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Abstract

Mixed convective flow of a Casson nanofluid in a vertical channel with the effect of heat source/ sink, velocity slip, thermal conductivity variation and wall concentration variation is investigated. Exact solutions are obtained for dimensionless temperature, nanoparticle fraction and the velocity field. It is noticed that the wall temperature ratio enhances the rate of heat transfer and the wall concentration ratio decreases the rate of nanoparticle volume fraction. When the temperature ratio parameter (m_2), wall concentration ratio parameter (m_1), slip parameter (L), Casson parameter (β) tends to zero our results are good in agreement with the result of Michael O. Oni and Bagant K. Jha [10].

Keywords: Casson nanofluid, vertical channel, nanoparticle fraction.

Nomenclature

μ	dynamic viscosity	Gr	Grashof number
β	Casson parameter	h	distance between two parallel walls
ρ	Density	Nb	brownian motion parameter
υ	kinematic viscosity	Nr	buoyancy ratio parameter
T	dimensional temperature	Q_0	dimensional heat source/ sink parameter
θ	dimensionales temperature	Re	Reynolds number
K	thermal conductivity	S	dimensionless heat source/ sink parameter
ϕ	rescaled nanoparticle fraction	V	dimensional velocity vector
β_0	thermal expansion coefficient	u,v	velocity components
L	slip parameter	U	dimensionless velocity

p	Pressure	x, y	cartesian coordinates
c	heat capacity at constant pressure	X, Y	Dimensionless cartesian coordinates
C	nanoparticles volume fraction	Subscript f	Fluid
g	acceleration due to gravity	Subscript p	solid particle
D_B	brownian diffusion coefficient	m_2	wall temperature ratio
D_T	thermophoretic diffusion coefficient	m_1	wall concentration ratio

Introduction

Analysis of nanofluids has gained much importance from the last few years due to its enormous applications in technical and industrial fields. The nanofluids are used in medicines, nuclear reactors, agriculture and food industries. Choi [1] was the first who explored the heat conductivity enhancement of fluids with nanoparticles. Later wide spread research is going on nanofluid flows. Among them, Nagasasikala and Lavanya [2] investigated the operative of radiation and dissipation on steady convective heat transfer flow of a nanofluid in a vertical channel and they found the significance of various parameters on the nanofluid velocity and temperature distributions. Lalrinpuia and Surender [3] studied the generation of entropy in the presence of a heat source/sink in a sloping channel filled with porous medium in hydrodynamic nanofuid flow. They noticed that entropy generation was found to be minimum just above the centre of the channel throughout the study. Aicha Bouhezza et al. [4] investigated the mixed convection of nanofluids in an asymmetric heated vertical channel numerically. They analyzed that Cu-water nanofluid showed better thermal performance than that of Al_2O_3-water. Mehta et al. [5] studied the effects of radiation and heat generation in unsteady MHD mixed convective flow of nanofluids along a upstanding channel through porous medium. Sindhu and Gireesha [6] implemented a mixture model for better describing the characteristics of nanoparticles in a vertical microchannel.

Non-Newtonian fluids are fluids that disregard Newton's law of viscosity. Casson fluid is a particular case of these fluids that illustrates the stress and deformation characteristics of the flow of fluids like human blood, soup, and coolant oil. Jamshed et al. [7] evaluated "the unsteady Casson nanofluid with solar thermal radiation. Thamaraikannan and Karthikeyan [8] investigated the pulsating unsteady heat and mass transfer of MHD Casson nanofluid with radiation and slip boundary conditions past a porous channel. Hameed Khan et al. [9] studied the Casson nanoflfluids flow theoretically over a vertical riga plate. Their results indicate that adding 4% clay nanoparticles, enhanced the skin friction up to 7.04% in instance of ramped wall temperature (RWT) and 11.13% in isothermal wall temperature (IWT). Recently, Mallikarjuna et al. [11] investigated the Soret and Dofour effects on Casson nanofluid flowing in a vertical channel with the impact of thermal radiation. They noticed that Casson fluid parameter intensifies both velocity field and thermal field.

In view of the above studies, in this article, we investigated the effect of heat source/ sink with dependent temperature and concentration ratio parameters on the mixed convective flow of a Casson nanofluid in a vertical channel. The effect of above said parameters on the Casson nanofluid flow are analyzed through graphs.

Mathematical formulation

Consider the mixed convective flow of a Casson nanofluid with the influence of dependent wall temperature, concentration and slip effect in a vertical channel. The channel is of width h and the slip condition is applied on fluid velocity. T_1 and C_1 are the temperature and nanoparticle concentration at the wall $y=0$ whereas T_2 and C_2 are the temperature and nanoparticle concentration at the wall $y=h$ with the condition $T_2 > T_1$ and $C_2 > C_1$. This temperature difference results to density difference at the walls which setups natural convection in the vertical channel. The constitutive equation for Casson fluid is [12]

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + {P_y}/{\sqrt{2\pi}}\right)e_{ij}, & \pi > \pi_C \\ 2\left(\mu_B + {P_y}/{\sqrt{2\pi_C}}\right)e_{ij}, & \pi < \pi_C \end{cases}$$

where $\pi = e_{ij}e_{ij}$ and e_{ij} is the $(i,\ j)^{th}$ component of the deformation rate, π is the product of the component of the deformation rate with itself, π_C a critical value of this product based on the non-Newtonian model, μ_B the plastic dynamic viscosity of non-Newtonian fluid and P_y the yield stress of the fluid.

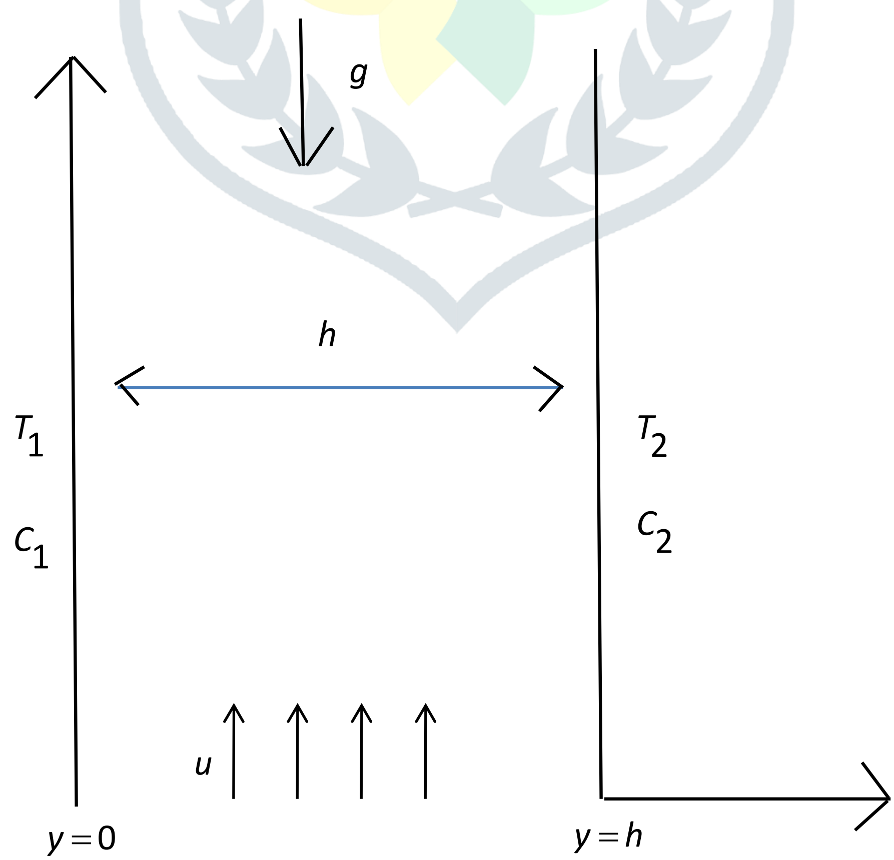

Fig. 1. Physical model

The fluid is heat conducting i.e., either heat generating(source) or absorbing(sink) fluid , viscous and Oberbeck-Boussinesq approximation is valid. The equations governing the fluid flow with above assumptions are given below:

$$\mu\left(1+\frac{1}{\beta}\right)\frac{d^2u}{dy^2}+\left\{\rho_{f_0}\beta_0\left(C-C_0\right)\left(T-T_0\right)-\left(\rho_f-\rho_{f_0}\right)\left(C-C_0\right)\right\}g=\frac{dp}{dx} \quad (1)$$

$$K\frac{d^2T}{dy^2}+\left(\rho c\right)_p\left[D_B\frac{dc}{dy}\frac{dT}{dy}+\left(\frac{D_T}{T_0}\right)\left(\frac{dT}{dy}\right)^2\right]+Q_0=.0 \quad (2)$$

$$D_B\frac{d^2C}{dy^2}+\left(\frac{D_T}{T_0}\right)\frac{d^2T}{dy^2}=0 \quad (3)$$

Boundary conditions for velocity, temperature and concentration fields are given as

$$\begin{aligned} &u-L\frac{du}{dy}=0,\quad T=T_1,\quad C=C_1\quad at\ \ y=0\\ &u+L\frac{du}{dy}=0,\quad T=T_2,\quad C=C_2\quad at\ \ y=h \end{aligned} \quad (4)$$

Non-dimensional quantities are given as

$$Y=\frac{y}{h},\ X=\frac{x}{h},\ U=\frac{u}{u_0},\ P=\frac{p}{\rho u_0^2},\ \ \theta=\frac{T-T_0}{T_2-T_0},\ \phi=\frac{C-C_0}{C_2-C_0}$$

$$T_0=\frac{T_1+T_2}{2},\ \ C_0=\frac{C_1+C_2}{2},\ \ Gr=\frac{\left(1-C_0\right)g\beta_0\left(T_2-T_0\right)h^3}{\nu^2},\ \ \mathrm{Re}=\frac{u_0h}{\nu} \quad (5)$$

$$Nb=\frac{D_B\left(C_2-C_0\right)\left(\rho c\right)_p}{K},\quad Nt=\frac{D_B\left(T_2-T_0\right)\left(\rho c\right)_p}{KT_0},\quad S_0=\frac{Q_0h^2\left(\rho c\right)_p}{K\left(T_2-T_1\right)},\quad S=\frac{Q_0h^2\left(\rho c\right)_p}{K}$$

Using the above parameters equations (1) to (3) are reduced to non-dimensional form

$$\left(1+\frac{1}{\beta}\right)\frac{d^2U}{dy^2}=\frac{dp}{dx}+Nr\phi-\frac{Gr}{\mathrm{Re}}\theta \quad (6)$$

$$\frac{d^2\theta}{dy^2}+Nb\frac{d\theta}{dy}\frac{d\phi}{dy}+Nt\left(\frac{d\theta}{dy}\right)^2+S\theta=0 \quad (7)$$

$$\frac{d^2\phi}{dy^2}+\frac{Nt}{Nb}\frac{d^2\theta}{dy^2}=0 \quad (8)$$

The corresponding boundary conditions are given below

$$U - L\frac{dU}{dy} = 0, \quad \theta = -1, \quad \phi = -1 \quad at \quad Y = 0$$
$$U + L\frac{dU}{dy} = 0, \quad \theta = 1 + m_2, \quad \phi = 1 + m_1 \quad at \quad Y = 1 \tag{9}$$

where $m_2 = \frac{2T_0 - T_1 - T_2}{T_2 - T_0}$ is the wall temperature ratio and $m_1 = \frac{2C_0 - C_1 - C_2}{C_2 - C_0}$ is the wall concentration ratio.

Solution

The exact solution for the equations (6) to (8) using the boundary conditions given in (9) are given below

$$\phi(y) = C_1 y + C_2 - \frac{Nt}{Nb}\theta(y) \tag{10}$$

$$\theta = C_3 e^{\lambda_1 y} + C_4 e^{\lambda_2 y} \tag{11}$$

$$U = \alpha \frac{dp}{dx}\frac{y^2}{2} + \frac{a_2}{6}\left(2y^3 - 3y^2\right) + \frac{a_3 y^3}{6} - \frac{a_1}{\lambda_1^2 \lambda_2^2}\left(C_3 \lambda_2^2 e^{\lambda_1 y} + C_4 e^{\lambda_2 y}\lambda_1^2\right) + C_5 y + C_6 \tag{12}$$

Constants involved in (10) to (12) are given below

$$\alpha = \frac{\beta}{\beta + 1}; \quad a_1 = \alpha\left(Nr\frac{Nt}{Nb} + \frac{Gr}{\mathrm{Re}}\right); \quad a_2 = \alpha\left(Nr\left(1 + \frac{Nt}{Nb}\right)\right); \quad a_3 = \alpha Nr\left(m_2\frac{Nt}{Nb} + m_1\right)$$

$$a_4 = \frac{a_1 C_3}{\lambda_1^2} + \frac{a_1 C_4}{\lambda_2^2} - \frac{La_1 C_3}{\lambda_1} - \frac{La_1 C_4}{\lambda_2}; \quad a_5 = \frac{a_1 C_3}{\lambda_1^2}e^{\lambda_1} + \frac{a_1 C_4}{\lambda_2^2}e^{\lambda_2} + \frac{La_1 C_3}{\lambda_1}e^{\lambda_1} + \frac{La_1 C_4}{\lambda_2}e^{\lambda_2}$$

$$C_5 = \frac{1}{1 + 2L}\left(-\alpha\frac{dp}{dx}\left(L + \frac{1}{2}\right) + \frac{a_2}{6} - \frac{a_3}{6}\left(1 + 3L\right) + a_5 - a_4\right); \quad C_6 = a_4 + LC_5$$

Results and discussion

In a vertical channel, we have obtained the dimensional expressions for the velocity U, temperature θ and nanoparticle volume fraction ϕ under the effect of velocity slip and heat source/ sink with dependent wall temperature and concentration. The physical variation in the above mentioned fields are analyzed with respect to different parameters through graphs.

Fig. 2 shows the effect of wall temperature ratio m_2 on temperature field. For values of m_2 < 0 the temperature of the left wall is less than the temperature of the right wall, for m = 0 both walls are maintained at equal temperatures, and for values of m_2 > 0 the left wall is at higher temperature. Therefore as m_2 increases, rate of heat transfer also increases. Fig. 3 presents temperature distribution for heat source/ sink parameter in two cases i.e., when the wall temperature ratio is zero and it is increased to one. In both the cases, temperature increases with the increase in S. This is because, the additional heat to fluid flow leads to enhancement of temperature.

Fig. 4 shows that the Nanoparticle volume fraction increases at the right with the increasing values of m_1 but no change is observed at the left wall. From Fig. 5, it is observed that the Nanoparticle volume fraction decreases with the increasing values of heat source/ sink parameter S. Fig. 6 shows that an increase in the velocity is noted along with the rising values of slip parameter L. Fig . 7 depicts that the velocity distribution field is augmented by growing valuations of the Casson fluid parameter β. Fig. 8 shows the effect of buoyancy-ratio parameter and heat source/ sink parameter on velocity field. It is observed that the velocity increases at the left wall and decreases at the right wall.

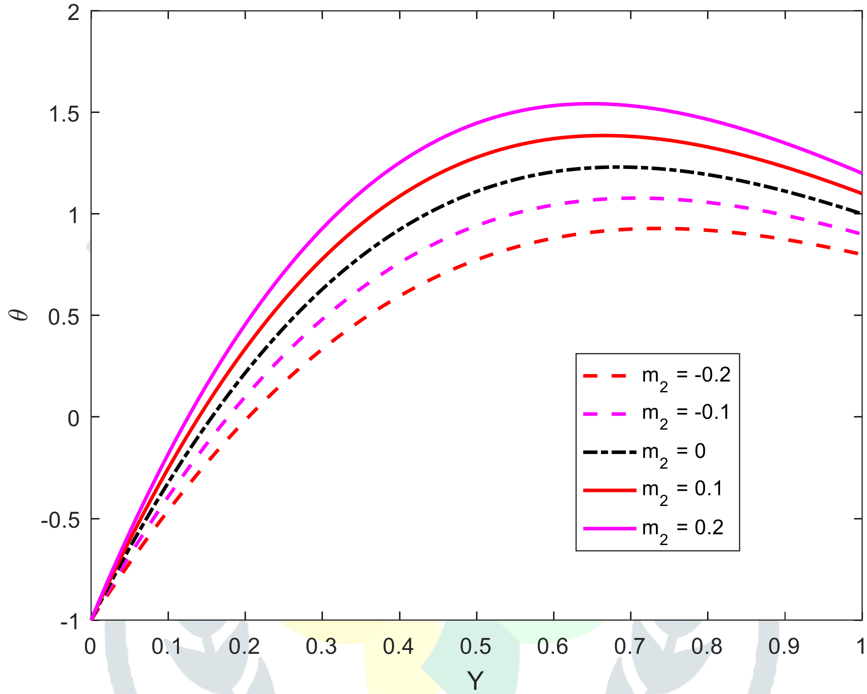

Fig. 2: Temperature profile for different values of wall temperature ratio m_2

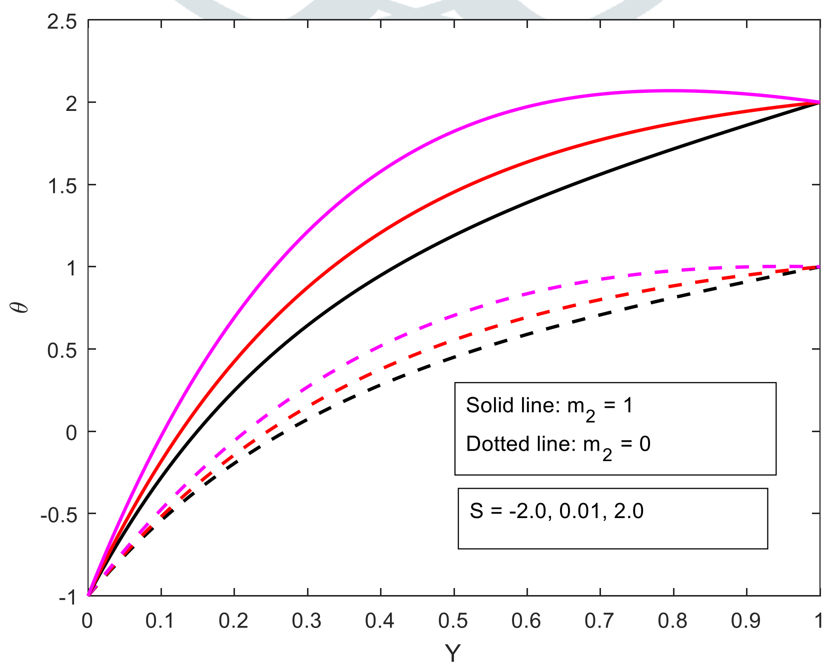

Fig. 3: Temperature profile for different values of heat source/ sink parameter S

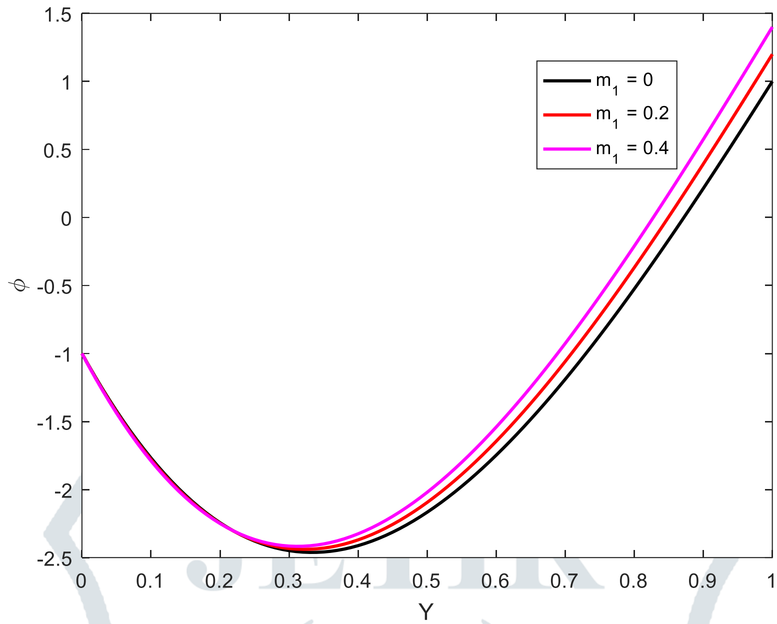

Fig. 4: Nanoparticle volume fraction for different values of wall concentration ratio m_1.

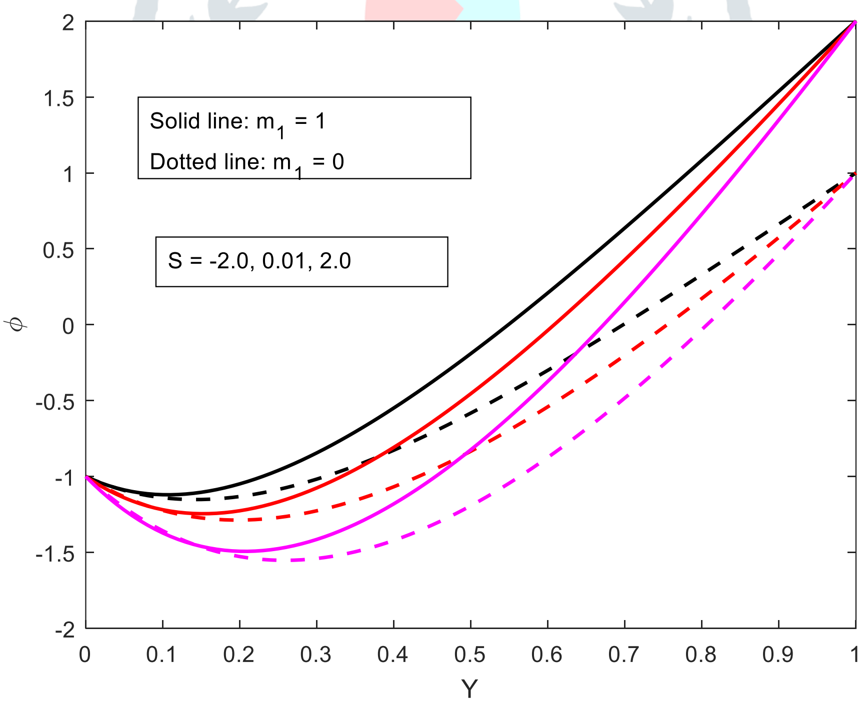

Fig. 5: Nanoparticle volume fraction for different values of heat source/ sink parameter S.

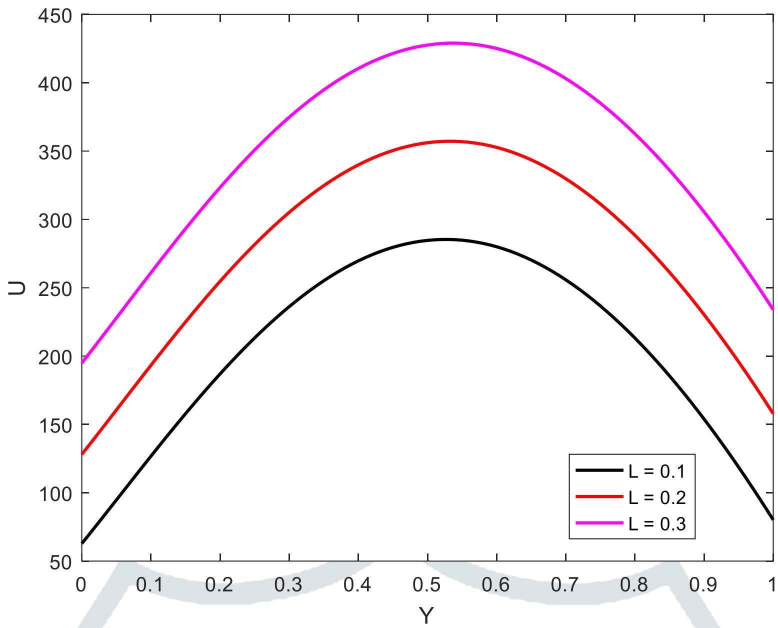

Fig. 6: Velocity profile for different values of slip parameter L

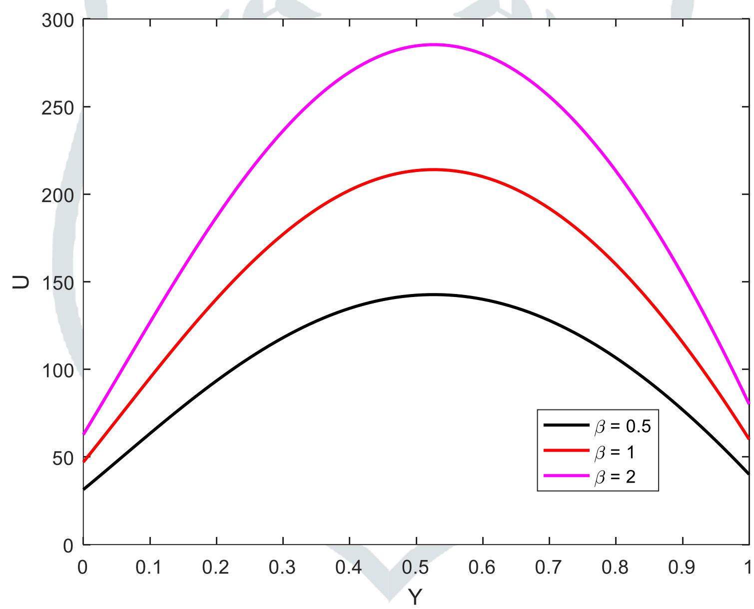

Fig. 7: Velocity profile for different values of Casson parameter β .

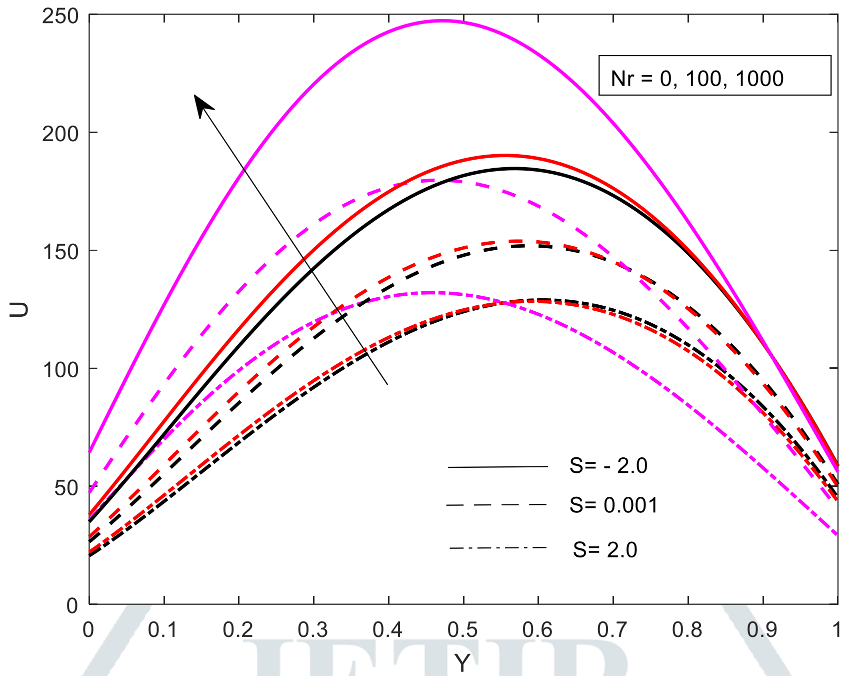

Fig. 8: Velocity profile for different values of heat source/ sink parameter S and buoyancy-ratio parameter Nr.

Conclusions

The effect of heat source/ sink with dependent temperature and concentration ratio parameters on mixed convective flow of a Casson nanofluid in a vertical channel has been analyzed. The velocity filed with slip effect, temperature and concentration distributions are discussed analytically. The analysis lead to some of the following conclusions:

(i) The wall temperature ratio enhances the rate of heat transfer.

(ii) The wall concentration ratio decreases the rate of nanoparticle volume fraction.

(iii) An increase in the buoyancy-ratio parameter enhances the flow at the left wall and opposite behavior is noticed at the right wall.

References

[1] Choi, S. (1995). “Enhancing thermal conductivity of fluids with nanoparticles” Proceedings of the ASME Int. Mech. Eng. Cong. & Exp., San Francisco, USA, Vol. 105, Issue 66, pp. 99–105.

[2] Nagasasikala, M., Lavanya, B. (2018). “Effects of dissipation and radiation on heat transfer flow of a convective rotating Cuo-Water nanofluid in a vertical channel” Journal of Advanced Research in Fluid Mechanics and Thermal Sciences, Vol. 50, Issue 2, pp. 108-117.

[3] Lalrinpuia, T., and Surender, O. (2019). “Entropy generation in MHD nanofuid fow with heat source/sink” SN Applied Sciences, doi:1:1672 https://doi.org/10.1007/s42452-019-1733-4.

[4] Aicha Bouhezza, Omar Kholai and Mohamed Teggar (2019). “Numerical investigation of nanofluids mixed convection in a vertical channel”, Mathematical Modelling of Engineering Problems, Vol. 6, Issue. 4, pp. 575-580.

[5] Mehta, R., Chouhan, VS. and Mehta, T. (2020). “MHD flow of nanofluids in the presence of porous media, radiation and heat generation through a vertical channel” , Journal of Physics: Conference Series 1504012008.

[6] Sindhu, S. and Gireesha, BJ. (2020). “Irreversibility analysis of nanofluid flow in a vertical microchannel with the influence of particle shape”, Journal of Process Mechanical Engineering pp. 1–9, DOI: 10.1177/0954408920958110.

[7] Jamshed, W., Deevi, S.U., Goodrarzi, M., Prakash, M., Nisar, K.S., Zakarya, M. and Abdel, A.H. (2021). “Evaluating the unsteady Casson nanofluid over a stretching sheet with solar thermal radiation: An optimal case study”, Case Stud. Ther. Eng. Vol. 26.

[8] Thamaraikannan, N and Karthikeyan, S. (2022). “The slip boundary impacts of Casson nanofluid past a porous channel with radiation”, AIP Conference Proceedings 2519, 020002.

[9] Khan, H., Alif Khan, N., Khan, I. and Mohamed, A. (2022). “Electromagnetic flow of Casson nanofluid over a vertical riga plate with ramped wall conditions, Front. Phys. 10:1005447. doi: 10.3389/fphy.2022.1005447.

[10] Oni, M.O. and Jha, B.K. (2023). “Generation/Absorption effect on mixed convection flow in a vertical channel filled with a nanofluid:s Exact solution, “Journal of Oil and Gas Research Reviews”, Vol. 3, Issue 1, pp.01-14.

[11] Patil Mallikarjuna, B., Shobha, K.C., Bhattacharyya, S., Zafar, S. (2023). “Soret and dufour effects in the flow of Casson nanofluid in a vertical channel with thermal radiation: entropy analysis, Journal of Thermal Analysis and Calorimetry, Vol. 148, Issue 3, pp.1-11.

[12] Pramanik, S. (2014). “Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation”, Ain Shams Engineering Journal, Vol. 5, pp. 205–212.