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A MODEL FOR DISTRIBUTION OF DISORDERED RADIATION WITH CYLINDRICAL SYMMETRY

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ABSTRACT :

In this paper, we have found a static cylindrically symmetric solution of Einstein's field equation for a distribution of disordered radiation. The expressions for pressure, density and non-vanishing components of conformal curvature tensor have been obtained. We find that the space-time is non-degenerate Petrov Type I. We have also investiaged Newtonian analogue of force in the model obtained.

Key Words :

Cylindrical, radiation, comoving coordinates, curvature tensor, metric.

1. INTRODUCTION

Various researchers have paid their attension towards the study of Einstein's field equations for distribution of disordered radian. In fact an interesting application to an investigation of a state in which a radiation is concentrated around a star is found in general relativity. In this line, the pioneering work is that of Klein (7). He has obtained an approximate solution to Einstein's field equations in spherical symmetry for a distribution of diffuse which he has presented as a set of series expansions. Hargreaves [5] has discussed the stability of static spherically symmetric fluid spheres which consists of a core of ideal gas and radiation in which the ratio of gas pressure to the total pressure is a small constant and an envelope consisting of an a disabetic gas. Singh and Abdussattar [15] have obtained an exact static spherically symmetric solution of Binstain field equations for disordered radiation and to overcome the difficulty of infinite density at the centre, they assumed that the distribution has a core of finite radius v_0 and constant density $\in_0$ which is fitted on to the solution of disordered radiation. Recently Singh and Kumar [16] have obtained some exact static spherical solution of Einstein's field equation with cosmological constant $\wedge = 0$ and equation of state $p = \alpha\rho$ with a suitable choice of metric potential g_{11} and g_{44}. Exact solutions of Einstein's field equation have been also obtained by various all Thors [9, 10, 19, 26] using equation of state $p = \rho$. Singh and Yadav [11] have investigated static fluid sphere with equation of state $p = \rho$. Further study in this line has has been made by Yadav and Saini [17] which is more general than one due to Singh and Yadav [17]. Solutions to Einstein's field equations with simple equation of state have been found by many workers in different cases e.g. for $\rho + 3p$ = const (Whittakar [24]) for $\rho = 3$ [Klain [6]), and for $p = \rho$ + constant (Buchdahl and land [2], Allnut [1]. But if we consider, e.g. polytropic fluid sphere $\rho = ap^{1+1/n}$ (Klein [8] or a mixture of ideal gas radiation (Suhohen [18]. We have to use numerical methods. Davidson [3] has provided a solution for non-stationary analog to the static case when $p = \frac{1}{3\rho}$. Some other workers in this line are Thomas [22], Yadav and Sharma [27] and Tolman [23] etc.

A non-static plane symmetric space-time filled with disordered radiation has been obtained by Roy and Singh [14]. Recently an exact solution of physically analogous systems with plane symmetry has been obtained by Teixeira, Wolk and Som [20]. Their solution tends asymptotically to the plane vacuum solution of Levi-Civita [11] and presents a larger condensation in the innermost regions which dilutes outwards. Teixeira, Wolk and Son [21] have also considered a source free disordered distribution of electromagnetic radiation in general relativity and obtained time independent exact solution with cylindrical symmetry. They considered the metric in the form

(1.1) $ds^2 = e^{2\alpha}dx_0^2 - e^{2\beta}dr^2 - e^{\beta-\alpha}(dz^2 + r^2 d\phi^2)$

where α and β are functions of r – alone. They found a finite maximum concentration on the axis of symmetry which decreases monotonically to zero outwards.

In this paper, we have obtained a static cylindrically symmetric solution of Einstein's field equation for a distribution of disordered radiation. The expressions for pressure, density and non-vanishing components of conformal curvature tensor have been obtained. We find that the space-time is non-degenerate Petrov Type I. We have also investiaged Newtonian analogue of force in the model obtained.

2. The Field Equations

We consider the most general cylindrically symmetric metric in the form given by Marder [12]

(2.1) $ds^2 = \alpha^2(dx^2 - dt^2) + \beta^2 dy^2 + \gamma^2 dz^2$

where α, β and γ are functions of x-alone. The non-vanishing components of Ricci tensor R^i_j and Weyl conformal curvature tensor for this metric have been given in the appendix of this chapter.

A disordered distribution of radiation can be regarded as a perfect fluid having energy momentum tensor.

(2.2) $T^i_j = (\rho + p)u^i u_j - p\delta^i_j$

Characterized by the equation of state

(2.3) $\rho = 3p$

where ρ is density, p the pressure and u^i the flow vector satisfying

(2.4) $g_{ij}u^i u^j = -1$

(2.5) $R^i_j - \perp_2 R\delta^i_j + \wedge\delta^i_j = 8\pi T^i_j$

Where T^i_j is given by (2.2). Also we assume the coordinates to be comoving so that (2.6) $u^1 = u^2 = u^3 = 0$ and $u^4 \frac{1}{\alpha}$.

The field equation given above with $\wedge = 0$ for the metric (2.1) given

(2.7) $\frac{1}{\alpha^2}\left[\frac{\alpha_1\beta_1}{\alpha\beta} + \frac{\alpha_1\gamma_1}{A\gamma} + \frac{\beta_1\gamma_1}{\beta\gamma}\right] = 8\pi p$

(2.8) $\frac{1}{\alpha^2}\left[\frac{\gamma_{11}}{\gamma} + \frac{\alpha_{11}}{\alpha} - \frac{\alpha_1^2}{\alpha_2}\right] = 8\pi p$

(2.9) $\frac{1}{\alpha^2}\left[\frac{\beta_{11}}{P} + \frac{\alpha_{11}}{\alpha} - \frac{\alpha_1^2}{\alpha^2}\right] = 8\pi p$

(2.10) $\frac{1}{\alpha^2}\left[\frac{\alpha_1\beta_1}{\alpha\beta} + \frac{\alpha_1\gamma_1}{\alpha\gamma} - \frac{\beta_1\gamma_1}{\beta\gamma} - \frac{\beta_{11}}{\beta} - \frac{\gamma_{11}}{\gamma}\right] = 8\pi\rho$

He suffix indicate ordinary differentiation with respect to x.

3. Solution of the Field Equations :

Equations (2.7) – (2.10) are four equations in five unknowns A, B, C, ρ and p. Thus the system in indeterminate. For complete determination of the system, we use the extra condition given for disordered radiation. Therefore from equations (2.7) and (2.10) when ρ = 3p, we have

(3.1) $$\frac{\alpha_1\beta_1}{\alpha\beta}+\frac{\alpha_1\gamma_1}{\alpha\gamma}+\frac{2\beta_1 r_1}{\beta\gamma}+\frac{r_{11}}{2\gamma}+\frac{\beta_{11}}{2\beta}=0$$

From equations (2.8) and (2.9), we have

(3.2) $$\frac{\beta_{11}}{\beta}=\frac{\gamma_{11}}{\gamma}$$

Equations (2.7) and (2.8) lead to

(3.3) $$\frac{\alpha_1\beta_1}{\alpha\beta}+\frac{\alpha_1\gamma_1}{\alpha\gamma}+\frac{\beta_1\gamma_1}{\beta\gamma}-\frac{\gamma_{11}}{\gamma}-\frac{\alpha_{11}}{\alpha}+\frac{\alpha_1^2}{\alpha_2}=0$$

From equations (3.1) and (3.2), we have

(3.4) $$\frac{\alpha\beta}{\alpha\beta}+\frac{\alpha_1\gamma_1}{\alpha\gamma}+\frac{2\beta_1\gamma_1}{\beta\gamma}+\frac{\gamma_{11}}{\gamma}=0$$

Subtracting (3.3) from (3.4), we have

(2.3.5) $$\left(\alpha_1/\alpha\right)_1+\frac{\beta_1\gamma_1}{\beta\gamma}+\frac{2\gamma_{11}}{\gamma}=0$$

From equations (3.3) and (3.5) we have

(3.6) $$\frac{\alpha_1}{\alpha}(\beta\gamma)=-(2\beta_1\gamma l1+\beta\gamma_{11})$$

Equations (3.5) and (3.6) lead to

(3.7) $\left\{\frac{\alpha_1}{\alpha}(Br)\right\}_1 = 3(\beta_1\gamma_1 + \beta\gamma_{11})$

Equation (3.7) on integration leads to

(3.8) $\frac{\alpha_1}{\alpha}(\beta\gamma) = -3\beta\gamma_1 + A$

Where A is a constant of integration. Equation (3.8) leads to

(3.9) $\frac{\alpha_1}{\alpha} = \frac{A}{\beta r} - \frac{3\gamma_1}{\gamma}$

From equations (3.4) and (3.9), we have

(3.10) $\frac{A(\beta\gamma)_1}{\beta} - \frac{\beta_1\gamma_1}{\beta\gamma} - \frac{3\gamma_1^2}{\gamma^2} + \frac{\gamma_{11}}{\gamma} = 0$

Putting $\frac{\beta}{\gamma} = \gamma$ and BC = μ in equations (3.2) and (3.10), we have

(3.11) $\frac{A\mu_1}{\mu^2} + \frac{\lambda_1^2}{4\lambda^2} - \frac{5\mu^2}{4\mu^2} + \frac{\lambda_1\mu_1}{\lambda\mu} + \frac{\mu_{11}}{2\mu} - \frac{\lambda_{11}}{2\lambda} = 0$

where D is a constant of integration. Equation (3.11) and (3.12) lead to

(3.13) $2\mu\mu_1 + 6D\mu_1 - 5\mu_1^2 + 4A\mu_1 = D^2$

Letting $\mu_1 = f(\mu)$ in equation (2.13), we have

(3.14) $\frac{2\phi d\phi}{D^2 + 5\phi^2 - \phi(6D + 4A)} = \frac{d\mu}{\mu}$

which on integration leads to

(3.15) $\mu = \frac{(\phi - \eta - C)^{\frac{\eta+C}{5C}}}{K(\phi - \eta + C)^{\frac{\eta+C}{5C}}}$

where $\eta = \frac{3D+2A}{5}, \frac{D^2}{5} - \eta^2 = -C^2$

and k is a constant of integration. From (3.11) and (3.19), we have

$$(3.16)\ \lambda = \frac{1}{M}\left\{\frac{\phi-\eta-C}{\phi-\eta+C}\right\}^{\frac{D}{5C}}$$

Where M is a constant of integration. Hence

$$(3.17)\ \beta^2 = \lambda\mu = \frac{(\phi-\eta-C)}{MK(\phi-\eta+C)^{\frac{\eta+d-C}{5C}}}$$

and

$$(3.18)\ \gamma^2 = \frac{\mu}{\lambda} = \frac{(M(\phi-\eta-C)^{\frac{(\eta-D+C)}{5C}}}{K(\phi-\eta+C)^{\frac{(\eta-D+C)}{5C}}}$$

From (3.9), (3.17) and (3.18), we have

$$(3.19)\ \alpha^2 = \frac{N^2K^3(\phi-\eta-C)^{\frac{(2\eta-3C)}{5C}}}{M^3(\phi-\eta+C)^{\frac{(2\eta+3C)}{5C}}}$$

where N is a constant of integration. Hence the matric reduces to the form

$$(3.20)\ dS^2 = \frac{N^2K^3(\phi-\eta-C)^{\frac{(2\eta-3C)}{5C}}}{M^2(\phi-\eta-C)^{\frac{(2\eta+3C)}{5C}}}$$

$$+\frac{(\phi-\eta-C)^{\frac{(\eta+D+C)}{5C}}}{MK(\phi-\eta+C)^{\frac{(\eta+D-C)}{5C}}}dy^2$$

$$+\frac{M(\phi-\eta-C)^{\frac{(\eta-D+C)}{5C}}}{K(\phi-\eta+C)^{\frac{(\eta-D-C)}{5C}}}dy^2$$

By the following transformations

(3.21a) $\phi - \eta = X,$

(3.21b) $\frac{y}{\sqrt{MK}} = Y,$

(3.21 c) $\frac{\sqrt{M}}{K} Z = Z,$

(3.21 d) $\frac{N\sqrt{K^3}}{\sqrt{M^3}} = T,$

The metric (3.20) goes to the form

$$(3.22)\ ds^2 = \zeta \frac{X - C)^{\frac{(4\eta-11C)}{5C}}}{(X + C)^{\frac{(4\eta+11C)}{5C}}} dX^2 \frac{(X - C)^{\frac{\eta+D+C}{5C}}}{(X - C)^{\frac{\eta+D-C}{5C}}} y^2$$

$$+ \frac{(X - C)^{\frac{\eta-D+C}{5C}}}{(X + C)^{\frac{\eta-D-C}{5C}}} dZ^2$$

$$- \frac{(X - C)^{\frac{2\eta-3C}{5C}}}{(X + C)^{\frac{2\eta+3C}{5C}}} dT^2$$

where

$\zeta = \frac{4N^2K}{25M^3}, \eta, C$, K and M are +ve constants.

4. Some Physical and Geometrical Features

The pressure for the model (3.22) is given by

$$(4.1)\quad 8\pi p = \frac{(X+C)^{\frac{4\eta+C}{5C}}}{5\zeta(X-C)^{\frac{4\eta-C}{5C}}}\{(C-X)(C+X)\}$$

The model has to satisfy the reality condition (Ellois [4]

(4.2) (i) $(\rho + p) > 0$

(ii) $(\rho + 3p) > 0$

Which are equivalent to the condition $p > 0$. This requires that

$$0 < X < C$$

The non-vanishing comoonehts of conformal curvature tensor are

$$(4.3)\quad C_{12}^{12} = -\frac{(X+C)^{\frac{4\eta+6C}{5C}}}{10\zeta(X-C)^{\frac{4\eta-6C}{5C}}}\left[1+\frac{D^2-(X+\eta)^2}{5(X^2-C^2)}+\frac{2D(x-2^\eta)}{5(X^2-C^2)}\right]$$

$$(4.4)\quad C_{13}^{13} = -\frac{(X+C)^{\frac{(4\eta+6C)}{5C}}}{10\zeta(X-C)^{\frac{4\eta-6C}{5C}}}\left[1+\frac{D^2-(X+\eta)^2}{5(X^2-C^2)}-\frac{2D(x-2^\eta)}{5(X^2-C^2)}\right]$$

$$(4.5)\quad C_{23}^{23} = \frac{(X+C)^{\frac{4\eta+6C}{5C}}}{5\zeta(X-C)^{\frac{4\eta-6C}{5C}}}\left[1+\frac{D^2-(X+\eta)^2}{5(X^2-C^2)}\right]$$

Hence the space-time is non-degenerate petrov Type I $\left(C_{12}^{12} \neq C_{13}^{13}\right)$. It is found that flow vector is non-expanding, non-shearing and non-rotating.

5. Newtonian Analogue of force in the Model

The vector R_i and S_i representing the Newtonian analogue of force (Narlikay and Singh [13]) are depend as

$$(5.1)\ R_i = \Delta^{i}_{ji} = \frac{\zeta,i}{\zeta}$$

$$(5.2)\ S_i = \Delta^{\mu}_{jk} g^{ik} g_{\mu i} = g^{ij} g_{ji,k} - \frac{\zeta,i}{\zeta}$$

where

$$\zeta = \sqrt{\frac{g}{Y}}$$

$$(5.3)\ g_{ij} = d_{iag}\left[\frac{\zeta(X-C)^{\frac{4\eta-\mu C}{5C}}}{(X-C)^{\frac{4\eta-\mu C}{5C}}} \cdot \frac{(X-C)^{\frac{(\eta+D+C)}{5C}}}{(X+C)^{\frac{(\eta+D-C)}{5C}}} + \frac{(X-C)^{\frac{(\eta-D+C)}{5C}}}{(X+C)^{\frac{(\eta+D-C)}{5C}}} - \frac{(X-C)^{\frac{2\eta 3C)}{5C}}}{(X+C)^{\frac{2\eta+3C}{5C}}}\right]$$

$$(5.4)\ g = -\zeta \frac{(X-C)^{\frac{(8\eta+2C)}{5C}}}{(X-C)^{\frac{(8\eta+2C)}{5C}}},$$

$$(5.5)\ g^{ij} = d_{iag}\left[\frac{(X+C)^{\frac{4\eta+11C)}{5C}}}{\zeta(X-C)^{\frac{4\eta-11C)}{5C}}} \cdot \frac{(X+C)^{\frac{\eta+D-C}{5C}}}{(X-C)^{\frac{\eta+D+C}{5C}}}\right]$$

$$\left[\frac{(X+C)^{\frac{\eta-D-C}{5C}}}{(X-C)^{\frac{\eta-D+C}{5C}}} - \frac{(X+C)^{\frac{(2\eta+3C)}{5C}}}{(X-C)^{\frac{(2\eta-3C)}{5C}}}\right]$$

we take the corresponding flat metric Y_{11} to be that of special retativity given by

$$(5.6)\ dS^2 = dX^2 - \alpha T^2 + dy^2 + dZ^2$$

For the line element (5.6), we have

$$(5.7)\ \gamma_{ij} = \mathrm{diag}(1,1,1,-1)$$

(5.8) $\gamma = -1$

Therefore from (5.1) and (5.2), we find that

(4.5) $R_i = \left[\frac{8(2\eta - 3X)}{5(X^2 - C^2)}, 0, 0, 0\right]$

(4.6) $S_i = \left[\frac{2(X - 4\eta)}{5(X^2 - C)}, 0, 0, 0\right]$

6. References

1. Allnutt, J.A. (1980), Exact Solutions of Einstein's field equations edited by D. Kramer et. al, Cambridge Univ. Press, Cambridge, 227
2. Buchdahl, H.A. and Land, W. J. (1968), J. Australian Math. Sco., 86.
3. Davidson, W. (1991), J. Math. Phys., 32, 1560-61
4. Ellis, G.F.R. (1971) : General Relativity and Cosmology, ed. Sach. R.K. Academic Press, new York, 104.
5. Hargreeves, J.C. (1972); J. Phys. A., Gen. Phys., 5, 211.
6. Klein, O.C. (1947), Arkiv. Math. Astron. Fysik, A34, 1.
7. Klein, O. (1948) : Ark. Mat. Astr. Fys., 34A, 19, 1.
8. Klein, O. (1953), Ark. Fysik, 7, 487.
9. Letelier, P.S. (1975), J. Math. Phys., 16, 293
10. Letelier, P.S. and Tabensky, R.S. (1975), Il. Nuovo Cim., 28 M, 408.
11. Levi-Civita, T. (1918) : Rend. Reale. Acad. Lincei, 27, II Sem., 240.
12. Marder, L. (1958), Proc. Roy. Soc., A 246, 133.

13. Narlikar, V.V. and Singh, K.P. (1951), Proc. Nat. Inst. Sci. (India), 17, 31

14. Roy, S.R. and Singh, P.N. (1977): J. Phys. A. math., Nucl. Gen. London; 10, 1, 49.

15. Singh, K.P. and Abdussattar (1973) : Ind. J.Pure and Appl. Math., 4, 4, 468.

16. Singh, S. and Kumar, M. (2019), Res. Rev. Jour., 4, 1684-88.

17. Singh, T and Yadav, R.B.S. (1981), J. math. Phys. Sci., 15, 283.

18. Suhonen, E. (1968), Kgl. Danske Vidensk Sels. Math. Fys. Medd, 36, 1

19. Tabensky, R. and Taub, A.H. (1973), Commun. Math. Phys., 29, 61.

20. Teixeire, A.F. Da. F., Wolk, I. and Som, M.M. (1976): Exact Relativistic Solution of Diserdered Radiation with Plane Symmetry, CBPF Preprint A 007/76.

21. Teixeira, A.F. De. F., Wolk, I. and Som, M.M. (1977): IL Nuovo Cimento della Societa Italians difisica, 41B, 2, 387.

22. Thomas, E. Kiess (2009), Class, Quantum Grav., A26, 11.

23. Tolman, R.C. (1939), Phys. Rev., 55, 364.

24. Whittaker, J.M. (1968), Proc. Roy. Soc. London, A306, 1.

25. Yadav, R.B.S. and Saini, S.L. (1991), Astrophys. And space Sci., 184, 331-36.

26. Yadav, R.B.S. et. al. (2007), Acta ciencia India, Vol. 33(2), 185.

27. Yadav, A.K. and Sharma, A. (2013), Res. Astr. Astrophys. 13, 501.

7. Appendix

The surviving components of the Ricci tensor R^i_j for the metric are

$$(7.1)\quad \begin{cases} R_1^1 = \dfrac{1}{A^2}\left[\dfrac{A_{44}}{A} + \dfrac{A_4B_4}{AB} + \dfrac{A_4C_4}{AC} - \dfrac{A_4^2}{A^2}\right] \\ R_2^2 = \dfrac{1}{A^2}\left[\dfrac{B_{44}}{B} + \dfrac{B_4C_4}{BC}\right] \\ R_3^3 = \dfrac{1}{A^2}\left[\dfrac{C_{44}}{C} + \dfrac{B_4C_4}{BC}\right] \\ R_4^4 = \dfrac{1}{A^2}\left[\dfrac{A_{44}}{A} + \dfrac{B_{44}}{B} + \dfrac{C_{44}}{C} - \dfrac{A_4C_4}{AC} - \dfrac{A_4B_4}{AB} - \dfrac{A_4^2}{A^2}\right] \end{cases}$$

And

$$(7.2)\quad R = \frac{2}{A^2}\left[\frac{A_{44}}{A} - \frac{A_4^2}{A^2} + \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC}\right]$$

The non-vanishing components of $G_j^i = R_j^i - \frac{1}{2}Rg_j^i$ are therefore given by

$$(7.3)\quad \begin{cases} G_j^i = \dfrac{1}{A^2}\left[\dfrac{B_{44}}{B} + \dfrac{C_{44}}{C} + \dfrac{B_4C_4}{BC} - \dfrac{A_4B_4}{AB} - \dfrac{A_4C_4}{AC}\right] \\ G_2^2 = \dfrac{1}{A^2}\left[\dfrac{C_{44}}{C} + \dfrac{A_{44}}{A} - \dfrac{A_4^2}{A^2}\right] \\ G_3^3 = \dfrac{1}{A^2}\left[\dfrac{B_{44}}{B} + \dfrac{A_{44}}{A} - \dfrac{A_4^2}{A^2}\right] \\ G_4^4 = \dfrac{1}{A^2}\left[\dfrac{A_4B_4}{AB} + \dfrac{B_4C_4}{BC} + \dfrac{A_4C_4}{AC}\right] \end{cases}$$

The non-vanishing components of the Weyl conformal curvature tensor C_{jk}^{hi} for the metric (2.1) are

(7.4)

$$\left[\begin{array}{l} C_{14}^{14} = C_{23}^{23} = \frac{1}{6A^2}\left[\frac{B_{44}}{B} + \frac{C_{44}}{C} - \frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{2B_4C_4}{BC}\right] \\ C_{12}^{12} = C_{34}^{34} = \frac{1}{6A^2}\left[\begin{array}{l}\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{2C_{44}}{C} + \frac{3A_4C_4}{AC} - \frac{A_4^2}{A^2} \\ -\frac{3A_4B_4}{AB} + \frac{B_4C_4}{BC}\end{array}\right] \\ C_{13}^{13} = C_{24}^{24}\ \frac{1}{6A^2}\left[\begin{array}{l}\frac{A_{44}}{A} + \frac{C_{44}}{C} - \frac{2B_{44}}{B} + \frac{3A_4B_4}{AB} - \frac{A_4^2}{A^2} \\ -\frac{3A_4C_4}{AC} + \frac{B_4C_4}{BC}\end{array}\right] \end{array}\right.$$

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