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# Transient Thermoelastic Boundary Value Problem with radiation

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**ABSTRACT**— In this paper an attempt has been made to study the thermoelastic problem of a thick annular disc occupying the space D :$a \le r \le b, -h \le z \le h$, with boundary conditions of the radiation type. One apply integral transform techniques to find the thermoelastic solution . The results are obtained as a series of Bessel's function . Numerical calculations are carried out for annular disc made of aluminum metal.

**KEY WORDS :** thermoelastic problem, thick annular disc, temperature distribution, thermal stress, integral transform

## I. INTRODUCTION :

As a result of the increased usage of industrial and construction materials the interest in isotropic thermal stress problems has grown considerably. However there are only a few studies concerned with the two-dimensional steady state thermal stress. Nowacki [1] has determined steady-state thermal stresses in a thick circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Wankhede [5] has determined the quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature. However, there aren't many investigations on transient state. Roy Choudhuri [4] has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In a recent work, Noda et al. [6] and Deshmukh et al. [7] have solved some problems. A K Shinde, et al.[8] has studied inverse transient thermoelastic problem with heat source in an annular disc. In all investigations mentioned above they have not however considered any thermoelastic problem with boundary conditions of radiation type, which satisfies the time-dependent heat conduction equation. This paper is concerned with transient thermoelastic problem of a thick annular disc occupying the space D :$a \le r \le b, -h \le z \le h$, with boundary conditions of the radiation type.

## II. STATEMENT OF THE PROBLEM :

Consider the thick annular disc whose axis is coincident with z-axis, defined by $a \le r \le b$, and $-h \le z \le h$, where $a$ and $b$ are the internal and external radii respectively and $(r, z, t)$ are cylindrical coordinates. Heat conduction problem and the prescribed boundary conditions of the radiation type are considered with symmetry with respect to the z-axis. Then the temperature $T(r, z, t)$ at any point and thermal stresses of the disc are required to be determined. The equation for heat conduction is $T(r, z, t)$, the temperature in cylindrical coordinates is:

$$\kappa\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial z^2}\right]=\frac{\partial T}{\partial t} \quad (1)$$

Subject to the initial and boundary conditions

$M_t(T,1,0,0) = 0$ for all $a \le r \le b$ , $-h \le z \le h$ (2)

$M_r(T,1,k_1,a) = 0, M_r(T,1,k_2,b) = 0$ for all $-h \le z \le h$ , $t > 0$ (3)

$M_z(T,1,k_3,h) = exp(-\omega t)\delta(r-r_0), M_z(T,1,k_4,-h) = exp(-\omega t)\delta(r-r_0)$ for all $a \le r \le b$ , $t > 0$ (4)

Where $exp(-\omega t)\delta(r-r_0)$ is the additional sectional heat available on its surface at $z = h$, $-h$; $\kappa$ is the thermal diffusivity of the material of the disc (which is assumed to be constant);$\lambda$ being the thermal conductivity of the material;$\rho$ is the density; $C$ is the calorific capacity assumed to be constant and the most general expression for these conditions can be given by

$M_\vartheta(f, \bar{k}, \bar{\bar{k}}, \$) = (\bar{k}f + \bar{\bar{k}}\hat{f})_{\vartheta=\$}$

with prime ( ^ ) denoting differentiation with respect to $\vartheta$; $\delta(r-r_0)$ is the Dirac Delta function having $a \le r_0 \le b$; $\omega > 0$ is a constant; $\bar{k}$ and $\bar{\bar{k}}$ are radiation coefficients of the disc, respectively.

The Navier's equations without the body forces for the axisymmetric two-dimensional thermoelastic problem can be expressed as

$$\left.\begin{aligned}\nabla^2 u_r - \frac{u_r}{r^2}+\frac{1}{1-2\upsilon}\frac{\partial e}{\partial r}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial r}=0\\ \nabla^2 u_z - \frac{1}{1-2\upsilon}\frac{\partial e}{\partial z}-\frac{2(1+\upsilon)}{1-2\upsilon}\alpha_t\frac{\partial T}{\partial z}=0\end{aligned}\right\} \quad (5)$$

where $u_r$ and $u_z$ are the displacement components in the radial and axial directions respectively and the dilatation $e$ as

$$e=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$$

The displacement function in the cylindrical coordinate system is represented by Goodier's thermoelastic displacement potential $\phi$ and Michell's function $M$ as

$$u_r = \frac{\partial \varphi}{\partial r} - \frac{\partial^2 M}{\partial r \partial z}, \quad (6)$$

$$u_z = \frac{\partial \varphi}{\partial z} + 2(1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial^2 z} \quad (7)$$

in which Goodier's thermoelastic potential must satisfy the equation

$$\nabla^2 \varphi = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t T \quad (8)$$

and the Michell's function $M$ must satisfy the equation

$$\nabla^2(\nabla^2 M) = 0 \quad (9)$$

Where $\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2}$

The components of the stresses are represented by the use of the potential $\varphi$ and Michell's function $M$ as

$$\sigma_{rr} = 2G\left\{\left(\frac{\partial^2 \varphi}{\partial r^2} - \nabla^2 \varphi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 M - \frac{\partial^2 M}{\partial r^2}\right)\right\}, \quad (10)$$

$$\sigma_{\theta\theta} = 2G\left\{\left(\frac{1}{r}\frac{\partial \varphi}{\partial r} - \nabla^2 \varphi\right) + \frac{\partial}{\partial z}\left(\upsilon \nabla^2 M - \frac{1}{r}\frac{\partial M}{\partial r}\right)\right\}, \quad (11)$$

$$\sigma_{zz} = 2G\left\{\left(\frac{\partial^2 \varphi}{\partial r^2} - \nabla^2 \varphi\right) + \frac{\partial}{\partial z}\left((2-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\}, \quad (12)$$

and

$$\sigma_{rz} = 2G\left\{\frac{\partial^2 \varphi}{\partial r \partial z} + \frac{\partial}{\partial r}\left((1-\upsilon)\nabla^2 M - \frac{\partial^2 M}{\partial z^2}\right)\right\} \quad (13)$$

where $G$ and $\upsilon$ are the shear modulus and Poisson's ratio respectively

## III. SOLUTION OF THE PROBLEM:

By applying integral transform to the equations (1) to (4) and making use of respective inversion one obtains the expression for temperature distribution function as

$$T(r,z,t) = \sum_{n=1}^{\infty} \frac{1}{C_n}\left\{\sum_{m=1}^{\infty} \frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]P_m(z)\right\} S_0(k_1,k_2,\mu_n r) \quad (14)$$

where

$$\wp_{n,m} = k\left[\frac{P_m(h)}{k_3} - \frac{P_m(-h)}{k_4}\right] r_0 S_0(k_1,k_2,\mu_n r_0)$$

Referring to the fundamental equation (1) and its solution (14) for the heat conduction problem, the solution for the displacement function is represented by Goodier's thermoelastic displacement potential $\varphi$ governed by equation (8) are represented by

$$\varphi(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]P_m(z)\right\} S_0(k_1,k_2,\mu_n r_0) \quad (15)$$

Similarly, the solution for Michell's function $M$ are assumed so as to satisfy the governed condition of equation (9) a

$$M(r,z,t) = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\} \times [sinh(\mu_n z) + z\,cosh(\mu_n z)] \times S_0(k_1,k_2,\mu_n r) \quad (16)$$

Now substitute the values of thermoelastic displacement potential $\varphi$ and Michell's function $M$ in equations (6) and (7), one obtains

$$u_r = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}[P_m(z) - (\mu_n+1)\,cosh(\mu_n z) - \mu_n z\,sinh(\mu_n z)]\mu_n S_0'(k_1,k_2,\mu_n r) \quad (17)$$

$$u_z = -\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\} \times [-a_m(Q_m\,sin(a_m z) - W_m\,cos(a_m z)) + 2(1-2\upsilon)\mu_n\,sinh(\mu_n z) - \mu_n^2(sinh(\mu_n z) + z\,cosh(\mu_n z))] \times S_0(k_1,k_2,\mu_n r) \quad (18)$$

Thus, making use of the two displacement components the dilation is established as

$$e = \left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}[(\mu_n^2+a_m^2)P_m(z) - 2(1-2\upsilon)\mu_n^2\,cosh(\mu_n z)] S_0(k_1,k_2,\mu_n r) \quad (19)$$

Then, the stress components can be evaluated by substituting the values of thermoelastic displacement potential $\varphi$ from equation (15) and Michell's function $M$ from equation (16) in equations (10), (11), (12) and (13), one obtains

$$\sigma_{rr} = -2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\} \times \{[(\mu_n^2+a_m^2)^{-1}\mu_n^2[P_m(z) - (\mu_n+1)\,cosh(\mu_n z) - z\mu_n\,sinh(\mu_n z)]]S_0''(k_1,k_2,\mu_n r) - [P_m(z) - 2(\mu_n^2+a_m^2)^{-1}\upsilon\mu_n^2 cosh(\mu_n z)]\}S_0(k_1,k_2,\mu_n r)] \quad (20)$$

$$\sigma_{\theta\theta} = -2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t) - exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}[\mu_n(\mu_n^2+a_m^2)^{-1}r^{-1}[P_m(z) - (\mu_n+1)\,cosh(\mu_n z\mu_n\,sinh(\mu_n z))]S_0'(k_1,k_2,\mu_n r) - [P_m(z) + 2(\mu_n^2+a_m^2)^{-1}\upsilon\mu_n^2\,cosh(\mu_n z)]S_0(k_1,k_2,\mu_n r) \quad (21)$$

$$\sigma_{zz} = -2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t)-exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$
$$[P_m(z)\mu_n^2(\mu_n^2+a_m^2)^{-1}S_0''(k_1,k_2,\mu_n r)-[P_m(z)-(\mu_n^2+a_m^2)^{-1}[2(2-\upsilon)\mu_n^2\cosh(\mu_n z)-$$
$$\mu_n^2\sinh(\mu_n z)+2\mu_n\sinh(\mu_n z)+z\mu_n^2\cosh(\mu_n z)]S_0(k_1,k_2,\mu_n r) \quad (22)$$

$$\sigma_{rz} = -2G\left(\frac{1+\upsilon}{1-\upsilon}\right)\alpha_t \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(\kappa(\mu_n^2+a_m^2)-\omega)(\mu_n^2+a_m^2)}[exp(-\omega t)-exp(-\kappa(\mu_n^2+a_m^2)t)]\right\}$$
$$\times\{-a_m\mu_n[Q_m\sin(a_m z)+W_m\cos(a_m z)]-2\upsilon{\mu_n}^2\sinh(\mu_n z)-\mu_n^3[\sinh(\mu_n z)+z\cosh(\mu_n z)]\}S_0'(k_1,k_2,\mu_n r) \quad (23)$$

## IV. NUMERICAL RESULTS, DISCUSSION AND REMARKS:

Numerical calculations are carried out for the thin annular disc made up of aluminium metal and results are explained graphically. This analysis is performed by setting the radiation coefficient constants, $k_1 = k_2 = 1$ ,$r_0 = 1.5$, $a = 1\text{cm}, b = 4\text{cm}, h = 2\text{cm}$ and $\kappa = 0.86$ $\text{cm}^2/\text{sec}$ $and\ \omega = 1$ . Equation (14) and (15)and (16) becomes.

$$T(r,z,t) = \sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(0.86(\mu_n^2+a_m^2)-1)}[exp(-t)-exp(-0.86(\mu_n^2+a_m^2)t)]P_m(z)\right\}\times S_0(1,1,\mu_n r)$$

$$\varphi(r,z,t) = -\left(\frac{1+0.281}{1-o.281}\right)25.5\times10^{-6}\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(0.86(\mu_n^2+a_m^2)-1)(\mu_n^2+a_m^2)}[exp(-t)-exp(-0.86(\mu_n^2+a_m^2)t)]P_m(z)\right\}S_0(1,1,\mu_n r)$$

$$M(r,z,t) = -\left(\frac{1+0.281}{1-o.281}\right)25.510^{-6}\sum_{n=1}^{\infty}\frac{1}{C_n}\left\{\sum_{m=1}^{\infty}\frac{\wp_{n,m}}{\lambda_m(0.86(\mu_n^2+a_m^2)-1)(\mu_n^2+a_m^2)}[exp(-t)-exp(-0.86(\mu_n^2+a_m^2)t)]\right\}[\sinh(\mu_n z)+z\cosh(\mu_n z)]\times S_0(1,1,\mu_n r)$$

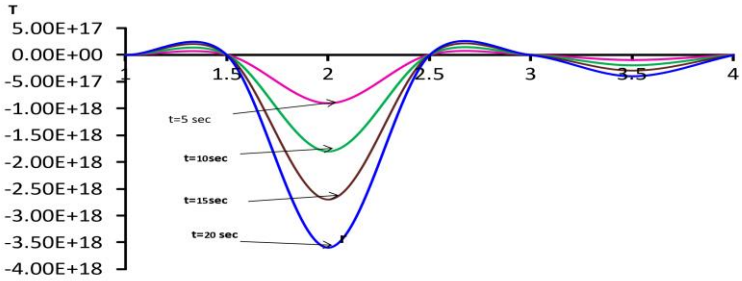


**Fig 1: Graph of *T* versus r**

**Figure 2** shows the graph of $T$ versus $r$ for different values of t .It is observed that $T$ is approximately equal to zero for $r = 1$ to $r = 1.5$ and $r = 2.5$ and $r = 4$ and compressive stress from $r = 1.5\ to\ r = 2.5$ within circular region of annular disc .

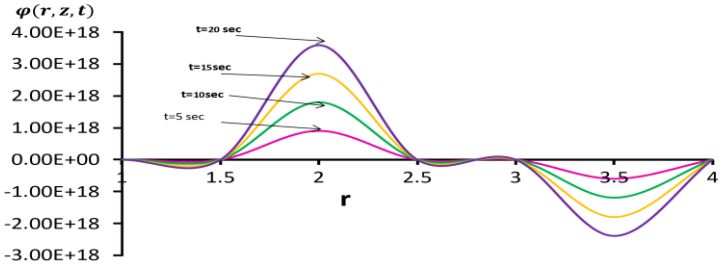


**Fig 2: Graph of $\varphi$ versus r**

**Figure 2** shows the graph of $\varphi$ versus $r$ for different values of t . It is observed that $\varphi$ develops tensile stress from $r = 1.5\ to\ r = 2.5$ and compressive stress from $r = 3\ to\ r = 4$ within circular region of annular disc . Also $\varphi$ is approximately equal to zero for $r = 1$ to $r = 1.5$ and $r = 2.5$ to $r = 3$

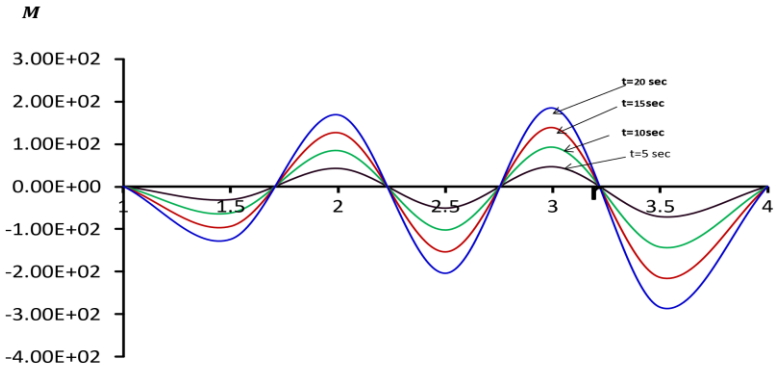


**Fig 3: Graph of $M$ versus r**

**Figure 3** shows the graph of $M$ versus $r$ for different values of t . It is observed that $M$ develops tensile stress from $r = 1.7\ to\ r = 2.3\ and\ r = 2.7\ to\ r = 3.3$ and compressive stress from $r = 1\ to\ r = 1.7\ , r = 2.3\ to\ r = 2.7\ and\ r = 3.3\ to\ r = 4$ within the circular region of the annular disc.

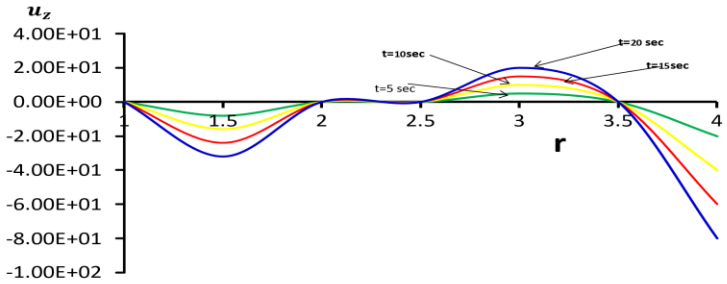


**Fig 4: Graph of $u_z$ versus r**

**Figure 4** shows the graph of $u_z$ versus $r$ for different values of t . It is observed that $u_z$ develops tensile stresses from $r = 2.5\ to\ r = 3.5$ and compressive stress from $r = 1\ to\ r = 2$ . Also $u_z$ is approximately equal to zero for $r = 2$ to $r = 2.5$ . Then $u_z$ goes on decreasing from $r = 3.5\ to\ r = 4$ in the circular region of the annular disc.

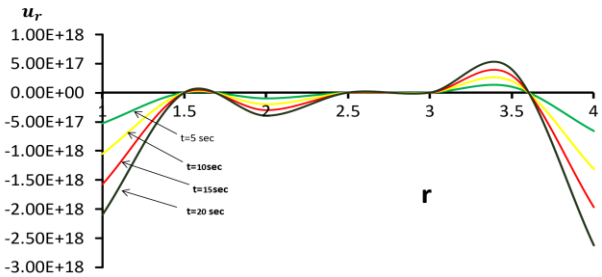


**Fig 5: Graph of $u_r$ versus r**

**Figure 5** shows the graph of $u_r$ versus $r$ for different values of t . It is observed that $u_r$ develops tensile stresses from $r = 3\ to\ r = 3.6$ and compressive stress from $r = 1.6\ to\ r = 2.5$ . Also $u_r$ is approximately equal to zero for $r = 2.5$ to $r = 3$ . Then $u_z$ goes on increasing from $r = 1\ to\ r = 1.5\ and$ decreasing from $r = 3.6\ to\ r = 4$ in the circular region of the annular disc.

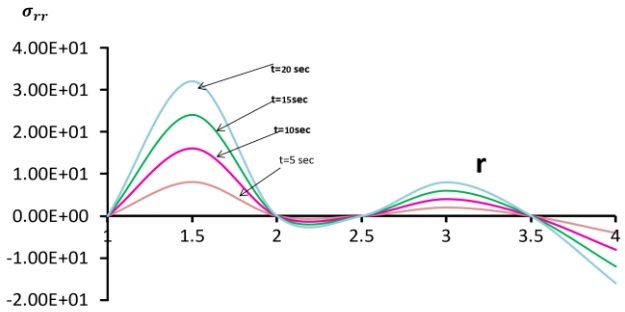


**Fig 6: Graph of $\sigma_{rr}$ versus r**

**Figure 6** shows the graph of $\boldsymbol{\sigma_{rr}}$ versus $r$ for different values of t . It is observed that $\boldsymbol{\sigma_{rr}}$ develops tensile stresses from $r = 1\ to\ r = 2$ and from $r = 2.5\ to\ r = 3.5$ . $\boldsymbol{\sigma_{rr}}$ is approximately equal to zero for $r = 2$ to $r = 2.5$ . Also $\boldsymbol{\sigma_{rr}}$ decreasing from $r = 3.5\ to\ r = 4$ in the circular region of the annular disc .

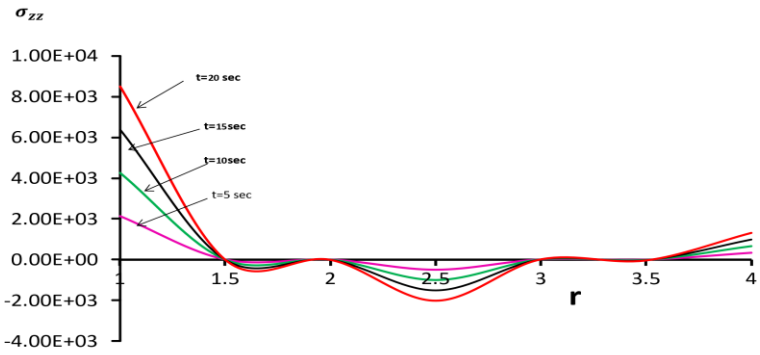


**Fig 7: Graph of $\sigma_{zz}$ versus r**

**Figure 7** shows the graph of $\boldsymbol{\sigma_{zz}}$ versus $r$ for different values of t . It is observed that $\boldsymbol{\sigma_{zz}}$ decreases from $r = 1\ to\ r = 1.5$ . $\boldsymbol{\sigma_{zz}}$ is approximately equal to zero for $r = 1.5$ to $r = 2$ and $r = 3$to $r = 3.5$ . Also $\boldsymbol{\sigma_{zz}}$ develops compressive stress from $r = 2\ to\ r = 3\ and\ then\ goes\ on\ increasing\ from\ r = 3.5$ to $r = 4$ in the circular region of the annular disc .

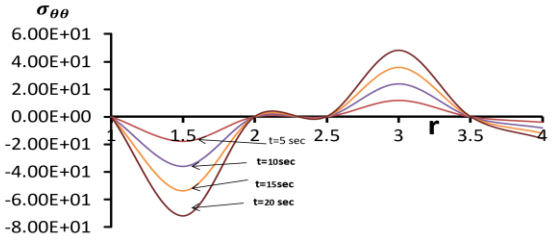


**Fig 8: Graph of $\sigma_{\theta\theta}$ versus r**

**Figure 8** shows the graph of $\sigma_{\theta\theta}$ versus $r$ for different values of t . It is observed that $\sigma_{\theta\theta}$ develops tensile stresses from $r = 2.5\ to\ r = 3.5$ and compressive stress from $r = 1\ to\ r = 2$. Also $\sigma_{\theta\theta}$ is approximately equal to zero for $r = 2$ to $r = 2.5\ and$ decreasing from $r = 3.5\ to\ r = 4$ in the circular region of the annular disc.

## *V. CONCLUSION:*

In this study, we treated the two-dimensional thermoelastic problem of a annular disc with additional sectional heat, $exp(-\omega t)\delta(r - r_0)$ available on its surfaces *z* = h,-h. Numerical calculations were carried out. Moreover, by assigning suitable values to the parameters and functions in the equations of temperature and displacements respectively, expressions of special interest can be derived for any particular case. We may conclude that the system of equations proposed in this study can be adapted to the design of useful structures or machines in engineering applications to determine thermoelastic behavior with radiation.

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