JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

A BATCH ARRIVING QUEUING SYSTEM WITH FOUR TYPES OF OPTIONAL SERVICES AND VACATIONS

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Abstract:

This paper focuses on the batch arriving and batch service queuing system where customers arriving according to the general bulk rule ($a_{1,}$ b_1) and follows the Poisson distribution. Customers are served in bulk manner according to the different four types of optional distribution. If there are not sufficient number of customers for receiving the service, then may or may not go for the vacations until the available numbers of customers are lies between a_1 and b_1. Server comes back for the vacation if arrived numbers of customers are sufficient for giving the service. Here we have also included the balking and reneging behaviors of customers due to have insufficient amount of time. Here we have determined the dependent probability generating function by using the corresponding steady static equations and number of customer waiting in the system.

Keywords: bulk rule, probability generation function, Poisson distribution.

Introduction:

Delay queueing problems are most common in our daily life activities also at a bank, a supermarket and public transport, also considered in the technical conditions such as in computer networking, telecommunication and manufacturing etc. Queueing models with multi-server have wide range of applications such as railway station , supermarket, airport etc. in the study of multi server queueing system, we assumes that server should be reliable in which individual service rates are same for all the servers in the service system. In the queueing system in which human servers are included is more reliable than others. the multi server queueing model leads to two or more than two servers.

In this work, we have considered here that customers receive the service in multi phase system. There are four types of services in which three types of optional services and one essential service. Customers come to the service system for receiving the optional service, after getting it they have the choice to go for any types of optional service or leave the service system.

Model description:

Here, we consider the batch Markovian queueing System with bulk arrival and bulk departure. Arrival rate of customer follows the Poisson distribution with rate λ. Here we consider the four optional services and one essential service which follows the exponential distribution with service rate μ. In this model server provide the service to customers in bulk manner with (A, B) rule. Server provides the service to at most b customers and at least a customer. The batch size of the serving batch will lie between a and b. customers arrive to receive the service at the service station but after arriving, first they will go to the essential service after that they will decide to one of them optional services from the available services or leave the service station. If customer does not join any of the service then he/she may leave the service station with probability p or may join the any service with probability q Such that **p+q = 1**

Customer comes to service station for receiving the service, if server is busy to providing the service to other customers. Then customers have to wait in the service centre till the server gets free.

We consider some assumptions to explain the mathematical queueing model.

(i) Customer come to the service centre in group/bulk manner and rate of arrival follows the Poisson distribution.

(ii) We consider the mathematical model in which service pattern follows the bulk service rate (a,b), under this rule server provide the service to at least a customers and at most b customers. If number of customer present at service station lies between a, and b, (i.e greater than a and less than b) then server will provide the service to all customers, again if number of customers present, at service station are greater than b, then server will provide the service to at most b, customers. Left customers will be served in next round of service.

(iii) Suppose that ξ_i is the probability of arriving 'i' length of batch at any interval of time t. Assume that the total probability of customers

$$\sum_{i=1}^{\infty} \xi_i = 1$$

$$for \qquad 0 \le \xi_i \le 1$$

We also consider that λ is the arrival rate of customers and μ is the service rate of customers at any instant of time.

(iv) We consider a_j (j=1,2) be the optional service rate. It follows the exponential distribution with distribution function F(a) for i=1, 2 be the probability density.

(v) Let $\mu_j(x)$denots the conditional probability distribution function at a time t for any x

$\mu_j(x)$is defined by

$$\mu_j(x) = \frac{f_j(x)}{1 - f_j(x)} \qquad\qquad for\, j = 1, 2, 3, 4$$

(vi) Customer will go any of the optional service. Customer chooses the first optional service with rate d_1, if he/she does not choose the first optional service than rate will be 1-d_1 If whole batch of customers arrives the service at a time then the size of batch will lie between $a_1$and b_1.

(vii) If no one customer is present at the service station then server can go for the vacations for same time after that arriving customers will have to wait until server will come for giving the service to customers. After coming from vacation period server starts the service if there is availability of batch of size $\boldsymbol{a_1}$and

$\boldsymbol{b_1}$ i.e customers are present in the service system are greater than a_1 and less than b_1. Server will serve to at most b_1 customers at one time. Remaining customers will serve in the next round.

(viii) Server will go for the vacation with the Poisson distribution function with vacation rate G.

(ix) If customers come for the service and server is busy to providing service to another batch. Then arriving batch may or may not leave the service centre, it depends on the time he/she or whole batch have. If they have less time for receiving the service then they leave the system with probability $\boldsymbol{(1-c_1)}$ and stay at the service system with probability c_1. Another way, we consider our second assumption is that, if whole batch come for receiving the service and server is not available at service centre, he is on vacation period. Then customers may leave the service system with probability $(1-c_1)$ and stay at the service system with probability (c_1).

(x) Here we consider these parameters which are not dependent to each other.

Definitions and Notations

i. $\int_0^\infty \mathrm{P_{mi}(x,t)\,dx}$ is the Probability that m customers in the service system are present at a time t with not accepting the batch of size $l\ (a_1 \leq l \leq b_1)$ is receiving service for any i =1,2

ii. $\mathrm{M}(t)$ will be the probability that number of customers is less than a_1 which is not sufficient for giving the batch service at that time. We are required to at least a_1 customers should be present in the service system for receiving the service at any instant of time.

iii. $\mathrm{v_m}\ (t)$ represents the probability there are m customers are present in the system for receiving the service when server is on the vacation and not available to giving the service.

Mathematical formulations

By considering above description, we have given this system of the equations-

$$\frac{\partial}{\partial x}\mathrm{P_{0,1}(x,t)}+\frac{\partial}{\partial t}\mathrm{P_{0,1}(x,t)} = -\lambda\mathrm{P_{0,1}(x,t)}-\sum_{\mathrm{m=a_1}}^{\mathrm{b_1}}\mu_1\mathrm{(x)P_{0,1}(x,t)}+\lambda(1-\mathrm{c_1})\,\mathrm{P_{0,1}(x,t)} + \lambda\mathrm{c_1P_{0,1}(x,t)} \quad \dots(3)$$

$$\frac{\partial}{\partial x}\mathrm{P_{m,1}(x,t)}+\frac{\partial}{\partial t}\mathrm{P_{m,1}(x,t)} = -\lambda\frac{\partial}{\partial x}\mathrm{P_{m,1}(x,t)}-\sum_{\mathrm{m=a_1}}^{\mathrm{b_1}}\mu_1\mathrm{(x)P_{m,1}(x,t)}+\sum_{\mathrm{m=a_1}}^{\mathrm{b_1}}\mathrm{g_q(x)P_{m-q,1}(x,t)}+\lambda(1-\mathrm{c_1})\,\mathrm{P_{m,1}(x,t)} + \lambda\mathrm{c_1P_{m,1}(x,t)} \quad \dots(4)$$

$$\frac{\partial}{\partial x}\mathrm{P_{0,2}(x,t)}+\frac{\partial}{\partial t}\mathrm{P_{0,2}(x,t)} = -\lambda\mathrm{P_{0,2}(x,t)}-\sum_{\mathrm{m=a_1}}^{\mathrm{b_1}}\mu_2\mathrm{(x)P_{0,2}(x,t)}+\lambda(1-\mathrm{c_1})\,\mathrm{P_{0,2}(x,t)} + \lambda\mathrm{c_1P_{0,2}(x,t)} \quad \dots(5)$$

$$\frac{\partial}{\partial x} \mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) + \frac{\partial}{\partial t} \mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) = -\lambda \frac{\partial}{\partial x} \mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) - \sum_{\mathrm{m}=\mathrm{a}_1}^{\mathrm{b}_1} \mu_2(\mathrm{x})\mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) + \sum_{\mathrm{m}=\mathrm{a}_1}^{\mathrm{b}_1} \mathrm{g}_{\mathrm{q}}(\mathrm{x})\mathrm{P}_{\mathrm{m-q},2}(\mathrm{x},\mathrm{t}) + \lambda(1-\mathrm{c}_1)\,\mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) + \lambda \mathrm{c}_1 \mathrm{P}_{\mathrm{m},2}(\mathrm{x},\mathrm{t}) \quad \ldots(6)$$

$$\frac{d}{dx} P_{0,3}(x,t) + \frac{d}{dt} P_{0,3}(x,t) = -\lambda P_{0,3}(x,t) - \mu_3(x) P_{0,3}(x,t) + \lambda(1 - c_1) P_{0,3}(x,t) + \lambda c_1 P_{0,3}(x,t) \quad \ldots.(7)$$

$$\frac{d}{dx} P_{m,3}(x,t) + \frac{d}{dt} P_{m,3}(x,t) = -\lambda c_1 P_{m,3}(x,t) - \sum_{m=a_1}^{b_1} \mu_3(x) P_{m,3}(x,t) + c_1 \lambda \sum_{q=1}^{m} g_q P_{m-q,3}(x,t) + \lambda(1-c_1) P_{m,3}(x,t) + \lambda c_1 P_{m,3}(x,t) \quad \ldots(8)$$

$$\frac{d}{dx} P_{0,4}(x,t) + \frac{d}{dt} P_{0,4}(x,t) = -\lambda P_{0,4}(x,t) - \mu_3(x) P_{0,4}(x,t) + \lambda(1 - c_1) P_{0,3}(x,t) + \lambda c_1 P_{0,4}(x,t) \quad \ldots(9)$$

$$\frac{d}{dx} P_{m,4}(x,t) + \frac{d}{dx} P_{m,4}(x,t) = -\lambda P_{m,4}(x,t) - \sum_{m=a_1}^{b_1} \mu_4(x) P_{m,4}(x,t) + c_1 \lambda \sum_{q=1}^{m} g_q P_{m-q,3}(x,t) + \lambda(1-c_1) P_{m,3}(x,t) + \lambda c_1 P_{m,3}(x) \quad \ldots(10)$$

$$\frac{d}{dx} v_m(t) + (\lambda + G) v_m(t) = \lambda c_2 \sum_{q=1}^{m} \frac{\lambda q}{\mu q} v_{m-q}(t) + \lambda(1-c_2) v_m(t) + (1-d_1)(1-d_2)\Big[\int_0^\infty P_{m,1}(x,t)\mu_1(x)dx + \int_0^\infty P_{m,2}(x,t)\mu_2(x)dx + \int_0^\infty P_{m,3}(x,t)\mu_3(x)dx \int_0^\infty P_{m,4}(x,t)\mu_4(x)dx \quad \ldots(11)$$

$$\tfrac{d}{dt} V_0(t) + (\lambda + G) V_0(t) = \lambda(1 - C_2) V_0(t) + (1-d_1)(1-d_2)\Big[\int_0^\infty P_{0,1}(x,t)\mu_1(x)d_x + \int_0^\infty P_{0,2}(x,t)\mu_2(x)d_x + \int_0^\infty P_{0,3}(x,t)\mu_3(x)\Big] + \int_0^\infty P_{0,4}(x,t)\mu_4(x)d_x \quad \ldots(12)$$

$$\frac{d}{dt} m(t) + \lambda m(t) = G m(t) + \lambda(1-b_1) m(t) + \lambda(1-b_2) m(t) \quad \ldots(13)$$

With the boundary condition

$$P_{m,1}(o,t) = GV_{m+a_1}(t) + GV_{m+b_1}(t) \quad ...(14)$$

$$P_{m,1}(o,t) = G\sum_{m=a_1}^{b_1} V_m(t) + \lambda b_1 m(t) + \lambda(1-b_2)m(t) \quad ...(15)$$

$$P_{m,2}(o,t) = d_1 \int_0^\infty P_{m,1}(x,t)\,\mu_q(x)d_x + d_2 \int_0^\infty P_{m,2}(x,t)\,\mu_1(x)d_x \quad ...(16)$$

$$P_{m,3}(o,t) = (1-d_1)\int_0^\infty P_{m,1}(x,t)\,\mu_q(x)d_x + (1-d_2)\int_0^\infty P_{m,2}(x,t)\,\mu_2(x)d_x \quad ...(17)$$

$$P_{m,4}(0,t) = (1-C_1)\int_0^\infty P_{m,1}(x,t)\mu_q(x)dx + (1-c_{2)}\int_0^\infty P_{m,2}(x,t)\mu_3(x)dx \quad ...(18)$$

By considering assumptions, server will be idle if number of customers will be less than as

$$V_{m(0)} = 0$$

$$V_{0(0)} = 0$$

$$m(0) = 1$$

$$P_{m,i(0)} = 0$$

$$for\ m = 0,1,2,3, \dots \dots$$

$$i = 1,2,3,4 \quad ...(19)$$

Here, we are considering some generating functions.

$$P_i(x,z,t) = \sum_{m=0}^{\infty} P_{m,i}(x,t)z^m \quad for\ i = 1,2,3,4$$

$$P_i(z,t) = \sum_{m=0}^{\infty} P_{m,i}(t)z^m \quad for\ i = 1,2,3,4$$

$$V(z,t) = \sum_{m=0}^{\infty} V_m(t)z^m$$

$$\sigma(z) = \sum_{m=1}^{\infty} \sigma_m z^m \quad ...(20)$$

By taking the Laplace transform of Equations (3) to (13) and using the Equation (20), we have

$$\frac{d}{dx}\bar{P}_{m,1}(x,s) + \left(s + \lambda + \mu_1(s)P_{m,1}(x,s)\right) = \lambda C1 \sum_{q=1}^{m} gq\,\bar{P}_{m-q,1}(x,s) + \lambda(1-C_1)P_{m,1}(x,s) + \lambda C_1 P_{m,1}(x,s) \quad ...(21)$$

$$\frac{d}{dx}\bar{P}_{0,1}(x,s)+(s+\lambda+\sum_{m=a1}^{b1}\mu_1(x)\bar{P}_{0,1}(x,s)) = \lambda(1-C_1)P_{0,\bar{1}}(x,s)+\lambda C_1P_{0,\bar{1}}(x,s) \quad ...(22)$$

$$\frac{d}{dx}\bar{P}_{m,2}(x,s)+(s+\lambda+\mu_2(s)P_{m,2}(x,s))$$

$$=\lambda C_1\sum_{q=1}^{m}g_q\,P_{m-\overline{q,2}}(x,s)\;+\lambda(1-C_1)P_{m,2}\lambda(s,x)+\lambda C_1P_{m,2}(x,s) \quad ...(23)$$

$$\frac{d}{dx}\bar{P}_{0,2}(x,s)+(s+\lambda+\sum_{m=a1}^{b1}\mu 2(x))P_{0,2}(x,s))=\lambda(1-C_1)\bar{P}_{0,2}(x,s)+\lambda C_1\bar{P}_{0,2}\ (x,s) ...(24)$$

$$\frac{d}{dx}P_{\overline{m,3}}(x,s)+(S+\lambda+\mu_3(x)P_{\overline{m,3}}(x,s)\;=\lambda C_1\sum_{q=1}^{m}g_q\,P_{\overline{m,q,2}}(x,s)+\lambda(1-C_1)P_{\overline{m,3}}(x,s)+\lambda C_1P_{\overline{m,3}}(x,s) \quad ...(25)$$

$$\frac{d}{dx}P_{0,3}(x,s)+(s+\lambda+\sum_{m=a1}^{b1}\mu_3(x)P_{\overline{0,3}}(x,s)=\lambda(1-C_1)P_{0,3}(x,s)+\lambda C_1P_{0,3}(x,s) \quad ...(26)$$

$$\frac{d}{dx}P_{\overline{m,4}}(x,s)+(S+\lambda+\mu_4(x)P_{\overline{m,4}}(x,s)\;=\lambda C_1\sum_{q=1}^{m}g_q\,P_{\overline{m,q,3}}(x,s)+\lambda(1-C_1)P_{\overline{m,4}}(x,s)+\lambda C_1P_{\overline{m,4}}(x,s) \quad ...(27)$$

$$\frac{d}{dx}P_{0,4}(x,s)+(s+\lambda+\sum_{m=a1}^{b1}\mu_4(x)P_{\overline{0,4}}(x,s)=\lambda(1-C_1)P_{0,4}(x,s)+\lambda C_1P_{0,4}(x,s) \quad ...(28)$$

$$(s+\lambda\;+G)V_m(s)=\lambda C_2\sum_{q=1}^{m}g_q\,V_{m-q}(s)+\lambda(1-C_1)V_m(s)+(1-d_1)(1-d_2)[\![\int_0^\infty P_{m,1}(x,s)\mu_1(x)dx+\int_0^\infty P_{m,2}(x,s)\mu_2(x)dx+\int_0^\infty P_{m,3}(x,s)\mu_3(x)dx+\int_0^\infty P_{m,4}(x,s)\mu_4(x)dx]\!] \quad ...(29)$$

$$(s+\;\lambda+G)V_0(s)=\lambda(1-C_2)V_0(s))+(1-d_1)(1-d_2)[\![\int_0^\infty P_{0,1}(x,s)\mu_1(x)dx+\int_0^\infty P_{0,2}(x,s)\mu_2(x)dx+\int_0^\infty P_{0,3}(x,s)\mu_3(x)dx+\int_0^\infty P_{0,4}(x,s)\mu_4(x)dx]\!] \quad ...(30)$$

$$(G+s+\lambda)m(t)=\lambda(1-C_1)m(s) \quad ...(31)$$

Also taking the Laplace transform of boundary conditions

$$P_{m,1}(0,t)=GV_{m+a1}(s)+GV_{m+b1}(s) \quad ...(32)$$

$$P_{m,2}(0,s)=G\sum_{m=a1}^{b1}V_m\,(s)+\lambda C_{1,}m(s)+\lambda\big(1-C_{1,}\big)m(s) \quad ...(33)$$

$$P_{m,3}(0,s)=(1-d_1)\int_0^\infty P_{m,1}(s,x)\mu_q(x)dx+(1-d_2)\int_0^\infty P_{m,2}(s,x)\mu_q(x)dx \quad ...(34)$$

$$P_{m,4}(0,s)=(1-C_1)\int_0^\infty P_{m,1}(x,s)\mu_q(x)dx+(1-C_2)\int_0^\infty P_{m,2}(x,s)\mu_q(x)dx \quad ...(35)$$

Now By Multiply z^m in Equation (21) and summing from 1 to ∞ and adding Equation (22), We have obtained

$$\sum_{m=1}^{\infty}\frac{d}{dx}P_{m,1}(\mathrm{x},\mathrm{s})z^m+\sum_{m=1}^{\infty}P_{m,1}(\mathrm{x},\mathrm{s})\left(s+\lambda+\sum_{m=a1}^{b1}\mu_1(\mathrm{x})\right)z^m+\frac{d}{dx}P_{0,1}(\mathrm{x},\mathrm{s})+((s+\lambda+\mu_1(\mathrm{x}))P_{0,1}(\mathrm{x},\mathrm{s})$$
$$=\sum_{\mathrm{m}=1}^{\infty}\lambda\mathrm{C}_1\sum_{\mathrm{q}=1}^{\mathrm{m}}\mathrm{g}_{(\mathrm{q},1)}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{m,1}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{0,1}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{P}\,(x,z,s)$$
$$+\lambda\mathrm{C}_1P_{0,1}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{P}\,(x,z,s)$$

...(36)

Similarly we have other equations, by doing the same operation

$$\sum_{m=1}^{\infty}\frac{d}{dx}P_{m,2}(\mathrm{x},\mathrm{s})z^m+\sum_{m=1}^{\infty}P_{m,2}(\mathrm{x},\mathrm{s})\left(s+\lambda+\sum_{m=a1}^{b1}\mu_2(\mathrm{x})\right)z^m+\frac{d}{dx}P_{0,2}(\mathrm{x},\mathrm{s})+((s+\lambda+\mu_1(\mathrm{x}))P_{0,2}(\mathrm{x},\mathrm{s})$$
$$=\sum_{\mathrm{m}=1}^{\infty}\lambda\mathrm{C}_1\sum_{\mathrm{q}=1}^{\mathrm{m}}\mathrm{g}_{(\mathrm{q},2)}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{m,2}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{0,2}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{P}\,(x,z,s)$$
$$+\lambda\mathrm{C}_1P_{0,2}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{P}\,(x,z,s)$$

...(37)

$\sum_{m=1}^{\infty}\frac{d}{dx}P_{m,3}(\mathrm{x},\mathrm{s})z^m+\sum_{m=a1}^{\infty}\left(s+\lambda+\mu_3(\mathrm{x})+\ P_{0,3}(\mathrm{x},\mathrm{s})\right)\ P_{m,3}(\mathrm{x},\mathrm{s})z^m+\frac{d}{dx}P_{0,3}(\mathrm{x},\mathrm{s})+\Big(s+\lambda+\mu_3(\mathrm{x})+$ $P_{0,3}(\mathrm{x},\mathrm{s})\Big)=\sum_{m=1}^{\infty}\lambda C_1\sum_{q=1}^{m}g_{q,3}\,(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{m,3}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{0,3}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{\mathrm{P}}\,(x,z,s)+$ $\lambda(\mathrm{C}_1)P_{0,3}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{\mathrm{P}}\,(x,z,s)$ ...(38)

$\sum_{m=1}^{\infty}\frac{d}{dx}P_{m,4}(\mathrm{x},\mathrm{s})z^m+\sum_{m=1}^{\infty}(s+\lambda+\sum_{m=c4}^{b1}\mu_4(\mathrm{x}))\ +\ P_{m,4}(\mathrm{x},\mathrm{s})z^m+\frac{d}{dx}P_{0,4}(\mathrm{x},\mathrm{s})+\Big(s+\lambda+\mu_4(\mathrm{x})\ P_{0,4}(\mathrm{x},\mathrm{s})\Big)=$ $\sum_{m=1}^{\infty}\lambda C_1\sum_{q=1}^{m}g_{q,4}\,(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{m,4}(\mathrm{x},\mathrm{s})+\lambda(1-\mathrm{C}_1)P_{0,4}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{\mathrm{P}}\,(x,z,s)+\lambda C_1P_{0,4}(\mathrm{x},\mathrm{s})\frac{\mathrm{d}}{\mathrm{dx}}\overline{\mathrm{P}}\,(x,z,s)$ ...(39)

Now by solving the equations (36),(37),(38),(39), we have

$\sum_{m=1}^{\infty}P_1\ (x,z,s)+\{s+\lambda C_1(1-g)(z)+\sum_{m=a1}^{b1}\lambda C_1\mu_1(\mathrm{x})\}xP_1\ (x,z.s)=0$ ...(40)

$\sum_{m=1}^{\infty}P_2\ (x,z,s)+\{s+\lambda C_1(1-g)(z)+\sum_{m=a1}^{b1}\lambda C_1\mu_2(\mathrm{x})\}xP_2\ (x,z.s)=0$ ...(41)

$\sum_{m=1}^{\infty}P_3\,(x,z,s)+\{s+\lambda C_1(1-g)(z)+\sum_{m=a1}^{b1}\lambda C_1\mu_3(\mathrm{x})\}xP_3\ (x,z.s)=0$ ...(42)

$\sum_{m=1}^{\infty}P_4\,(x,z,s)+\{s+\lambda C_1(1-g)(z)+\sum_{m=a1}^{b1}\lambda C_1\mu_4(\mathrm{x})\}xP_4\ (x,z.s)=0$ ...(43)

$G+s+\lambda C_2(1-g)(z)V(z,s)=(1-d_1)(1-d_2)\big[\int_0^{\infty}P_1\,(x,z,s)\mu_1\,(x)dx+\int_0^{\infty}P_2\,(x,z,s)\mu_2\,(x)dx+$ $\int_0^{\infty}P_3\,(x,z,s)\mu_3\,(x)dx+\int_0^{\infty}P_4\,(x,z,s)\mu_4\,(x)dx\ \big]$ ...(44)

Multiply by $z^{n,m}$ in equations (32) and summing over n from 0 to ∞ and adding equation (33) by multiplying z^n Also using generating function definitions we have

$$P_1(0,z,s) = [\lambda C_1 (s+\lambda) z^{-a1}] m(s) + G z^{-a1} v(z,s) + G \sum_{h=1}^{a1} V_{mq} (s) + G V_0(s) z^{-a} 1 \quad \dots (45)$$

Similarly we have other Equations

$$P_2(0,z,s) = G \sum_{m=a1}^{b1} V_m (s) \quad \dots (46)$$

$$P_3(0,z,s) = (1-d_1) \int_0^\infty P_m (x,z,s) \mu_q (x) dx + (1-d_2) \int_0^\infty P_2 (s,x) \mu_q (x) dx$$

$$P_4(0,z,s) = (1-C_1) \int_0^\infty P_{m,1} (x,z,s) \mu_q (x) dx + (1-C_2) \int_0^\infty P_{m,2} (x,z,s) \mu_4 (x) dx$$

By integrating the Equations (40),(41),(42) and (43) between the limits 0 t ∞ we get

$$P_1(x,z,s) = P_1(0,z,s) e^{-[s+\lambda c_1 \langle 1-g\langle z \rangle \rangle]} \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_1 \langle x \rangle \quad \dots (47)$$

$$P_2(x,z,s) = P_2(0,z,s) e^{-[s+\lambda c_1 \langle 1-g\langle z \rangle \rangle]} \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_2 \langle x \rangle \quad \dots (48)$$

$$P_3(x,z,s) = P_3(0,z,s) e^{-[s+\lambda c_1 \langle 1-g\langle z \rangle \rangle]} \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_3 \langle x \rangle \quad \dots (49)$$

$$P_4(x,z,s) = P_4(0,z,s) e^{-[s+\lambda c_1 \langle 1-g\langle z \rangle \rangle]} \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_4 \langle x \rangle \quad \dots (50)$$

Again integrating the Equations (49),(50),(51) and (52) between limits 0 to x

$$\int_0^x P_1(x,z,s) dx = P_1(0,z,s) \left[\left[e^{-[s+\lambda C_1(1-g(z))]} + \int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1 \mu_1 (x) - \frac{I_1\left(s+\lambda C_1(1-g(z))\right)}{[s+\lambda C_1(1-g(z))]} \right. \right. \quad \dots (51)$$

$$\int_0^x P_2(x,z,s) dx = P_2(0,z,s) \left[\left[e^{-[s+\lambda C_1(1-g(z))]} + \int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1 \mu_2 (x) - \frac{I_2\left(s+\lambda C_1(1-g(z))\right)}{[s+\lambda C_1(1-g(z))]} \right. \right. \quad \dots (52)$$

$$\int_0^x P_3(x,z,s) dx = P_3 (0,z,s) \left[\left[e^{-[s+\lambda C_1(1-g(z))]} + \int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1 \mu_3 (x) - \frac{I_3\left(s+\lambda C_1(1-g(z))\right)}{[s+\lambda C_1(1-g(z))]} \right. \right. \quad \dots (53)$$

$$\int_0^x P_4(x,z,s) dx = P_4 (0,z,s) \left[\left[e^{-[s+\lambda C_1(1-g(z))]} + \int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1 \mu_4 (x) - \frac{I_4\left(s+\lambda C_1(1-g(z))\right)}{[s+\lambda C_1(1-g(z))]} \right. \right. \quad \dots (54)$$

Now Assuming that

$$s + \lambda c_1 \langle 1 - g\langle z \rangle \rangle = E_1 \quad \dots (55)$$

$$s + \lambda c_2 \langle 1 - g\langle z \rangle \rangle = E_2 \quad \dots (56)$$

By Applying these two Conditions on the above Equations, we get

$\int_0^x P_1(x,z,s)\mathrm{dx} = P_1(0,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_1 \langle x\rangle - \frac{\mathrm{I}_{1(\mathrm{E}1)}}{\mathrm{E}_1}\right]$ ...(57)

$\int_0^x P_2(x,z,s)\mathrm{dx} = P_2(0,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_2 \langle x\rangle - \frac{\mathrm{I}_{1(\mathrm{E}1)}}{\mathrm{E}_1}\right]$ ...(58) $\int_0^x P_3(x,z,s)\mathrm{dx} =$ $P_3(0,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_3 \langle x\rangle - \frac{\mathrm{I}_{3(\mathrm{E}1)}}{\mathrm{E}_1}\right]$ ... (59)

$\int_0^x P_4(x,z,s)\mathrm{dx} = P_4(0,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a1}^{b1} \lambda c_1 \mu_4 \langle x\rangle - \frac{\mathrm{I}_{3(\mathrm{E}1)}}{\mathrm{E}_1}\right]$ ...(60)

By solving the equations (59),(60),(61) and (62), and using

$\boldsymbol{P_1(0,z,s)}$, $\boldsymbol{P_2(x,z,s)}$, $\boldsymbol{P_3(x,z,s)}$ **and** $\boldsymbol{P_4(x,z,s)}$

$P_1(z,s) = [\lambda C_1 - (S+\lambda C_1)z^{-a_1}]m(s) + Gz^{-a_1}V(z,s) + z^{-a_1} + G\sum_{n=a_1}^{b_1} Vm_q(s) +$ $GV_0(s)z^{-a_1}\left[e^{-E_1\int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1\mu_1(x)} + -\frac{I_1(E_1)}{E_1}\right]$ ...(61)

$P_2(z,s) = G\sum_{m=a_1}^{b_1} V_m(s)\left[e^{-E_1+\int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1\mu_2(x)} - \frac{I_2(E_1)}{E_1}\right]$ ... (62)

$$P_3(z,s) = \left[(1-d_1)\int_0^\infty P_1(x,z,s)\mu_q(x)dx + (1-d_2)\int_0^\infty P_2(x,z,s)\mu_3(x)dx\right]\left[e^{-E_1+\int_0^\infty \sum_{m=a_1}^{b_1} \lambda C_1\mu_1(x)} - \frac{I_1(E_1)}{E_1}\right]$$

...(63)

$$P_4(z,s) = \left[(1-C_1)\int_0^\infty P_1(x,z,s)\mu_q(x)dx + (1-C_2)\int_0^\infty P_2(x,z,s)\mu_4(x)dx\right]\left[e^{-E_1+\int_0^\infty \lambda C_1\mu_1(x)} - \frac{I_4(E_1)}{E_1}\right]$$

...(64)

How calculate the probability $\mathbf{V_n}$ (t)

i.e n customers are present in the systems when server is on vacation

$[G+S+\lambda C_2(1-g(z))]$v(z,s)= $(1-d_1)(1-d_2)\left[\int_0^\infty P_1(x,z,s)\mu_1(x)d_x + \int_0^\infty P_2(2,s,x)\mu_2(x)d_x +\right.$ $\left.\int_0^\infty P_3(x,z,s)\mu_3(x)d_x + P_4(2,s,x)\mu_4(x)\right]$ ...(65)

Values of $\boldsymbol{P_1(x,z,s), P_2(x,z,s), P_3(x,z,s), P_4(x,z,s)}$ we have obtain in the above section

V(z,s)= $\frac{1}{[G+s+\lambda C_2(1-g(z))]}\left[(1-d_1)(1-d_2)P_1(o,z,s)(e^{-E_1} + \int_0^\infty \lambda\, C_1\mu_1(x) - \frac{I_1(E_1)}{E_1})\right]\mu_1(x) +$

$P_2(o,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a_1}^{p_1} \lambda C_1\mu_2(x) - \frac{I_2(E_1)}{E_2}\right]\mu_2(x) +$

$P_3(o,z,s)\left[e^{-E_1} + \int_0^\infty \sum_{m=a_1}^{p_1} \lambda C_1\mu_3(x) - \frac{I_3(E_1)}{E_1}\right]\mu_3(x) +$

$$P_4(o,z,s)\left[e^{-E_1}+\int_0^{\infty}\sum_{m=a_1}^{p_1}\lambda C_1\mu_4(x)-\frac{I_4(E_1)}{E_1}\right]\mu_4(x) \qquad ...(66)$$

Equation (68) represents the required probability

Therefore equation (63),(64),(65),(66) represents required probabilities for one essential and three types of optional services which can be given by the service system

Waiting time for the essential services can be determined by

$$\boldsymbol{L_q(s)=\frac{d}{d2}C_q(z,s)\Big|_{z=1}} \qquad ...(67)$$

Where C_q (Z) can be determined by summing over these all probability functions

$$\boldsymbol{C_q(z,s)=P_1(z,s)+P_2(z,s)+P_3(z,s)+P_4(z,s)+v(z,s)} \qquad ...(68)$$

Utilization factor of this system can be determined by

$$\boldsymbol{U=1-C_q(z,s)} \qquad ...(69)$$

This equation represents the utilization factor for the mathematical queuing model

Conclusion

In this paper, we have studied the queueing model with bulk arrival and four types of batch services in which one of them is essential service and three of them are optional services. This model is useful to reduce the customers waiting time and increase the customer's satisfactory rate. Here we have calculated the total probability of served customer's utilization factor for the mathematical model by using the generating function techniques.

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