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Mathematical Model for Two-Dimensional Flow in Nephron

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Abstract : *The present paper provides mathematical model for two-dimensional flow in Nephron using basic equations and boundary conditions along with solution when radial velocity at wall decreases*

Key Words : *Nephron, Kidney, tubules, artery, blood.*

1. Introduction

The functional unit of the kidney is called the nephron or renal tubule, and each kidney has about 1 million of these tubules. One major part of a nephron is the glomerular tuft through which blood coming from the renal artery and afferent arterioles is filtered. The glomerular filtrate is essentially identical to plasma, and no chemical separation occurs up to this point. If the kidneys deliver this filtrate for excretion, the body loses may valuable materials, including water, at a rate faster than the one at which they can be supplied by synthesis of feeding. The rest of the nephron therefore recovers these valuable materials and returns them to the blood. Thus about 80 percent of the filtrate is reasbsorbed in the proximal tubule, and of the remaining, about 95 percent is further reabsorbed by the end of the remaining, about 95 percent is further reabsorbed by the end of the collecting ducts. [1-4]

This reabsorption or seepage create a redial component of the velocity in the cylindrical tubule, which must be considered along with the axial component of the velocity (Seen Fig. 1.1)

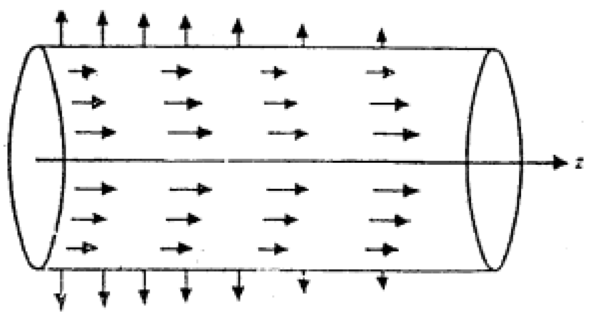

Fig. 1.1 Two-dimensional flow in renal tubule

Due to loss of fluid from the walls, both the radial and axial velocities decrease with z. Mathematically, we have to solve the problem of flow of a viscous fluid in a circular cylinder when there are axial and radial components of velocity and the radial velocity at all points on the surface of the cylinder is prescribed and is a decreasing function $\phi(z)$ of z.

2. Basic Equations and Boundary Conditions [1, 5, 6]

At the outset, we may not that the equation of motion can be simplified since the inertial term in relation to the viscous terms can be neglected. The average tubular radius is about 10^{-3} cm, the average velocity is about 10^{-1} cm/sec, and the fluid viscosity is about 7×10^{-3} dyness sec/cm^2, this gives a Reynolds number of about 10^{-3} and, since this is very much less than one, we neglect the inertial terms to get the following equations of continuity and motion

(2.1) $$\frac{I}{r}\frac{\partial}{\partial r}(rv_r)\frac{\partial v_z}{\partial z} = 0$$

(2.2) $$\frac{1}{\mu}\frac{\partial p}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_r)\right) + \frac{\partial^2 v_r}{\partial z^2}$$

(2.3) $$\frac{1}{\mu}\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2}$$

The boundary conditions are

(2.4) $$\frac{\partial v_z}{\partial r} = 0, v_r = 0, v_z = \text{finite at } r = 0$$

(2.5) $v_z = 0$, $v_r = \phi(z)$ at $r = R$

(2.6) $p = p_0$ at $z = 0$

$P = p_L$ at $z = L$

Eliminating p between (2.2) and (2.3), we get

(2.7) $$\frac{\partial^2}{\partial r \partial z}\left[\frac{1}{r}\frac{\partial}{\partial z}(rv_r)\right] + \frac{\partial^3 v_r}{\partial z^3} = \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right] + \frac{\partial^3 v_z}{\partial z^2 \partial r}$$

Taking the partial derivative of this equation with respect to z and substituting from (2.1), we get

(2.8) $$\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\right)\right)\right] + 2\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial^2}{\partial z^2}\right)\right] + \frac{1}{r}\frac{\partial^4}{\partial z^4}\right\}(rv_r) = 0$$

Alternatively, we can satisfy (2.1) by taking

(2.9) $$v_r = \frac{1}{r}\frac{\partial \psi}{\partial z}, \; v_z = -\frac{1}{r}\frac{\partial \psi}{\partial r}$$

Substituting (2.9) in (2.7), we get

(2.10) $$D^2(D^2\psi) = 0$$

where the operation D^2 is defined by

(2.11) $$D^2 = \left(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}\right)$$

if

(2.12) $v_r = f(r)\, g(z)$,

then the form of (2.8) suggests that an analytical solution may be possible if.

(2.13) $g(z) = A_0 + A_1 z$ or $g(z) = A_2 e^{-y2}$,

From (2.5) since $v_r = \phi(z)$ when r = R, we get

(2.14) $f(R)\,g(z) = \phi(z)$.

This suggests that we may get an analytical solution when the radial component of velocity on the surface of the cylinder is given by

(2.15) $\phi(z) = a_0 + a_1 z$ or $\phi(z) = ce^{-yz}$

We shall give the solutions for these two special cases in sections 3 and 4.

3. Solution When Radial Velocity at Wall Decreases Linearly with z.

From (2.10), we try the solution

(3.1) $$\psi(r,z) = f(r)a_0 z + \frac{1}{2}a_1 z^2) + G(r)$$

so that using (2.9), we get

(3.2) $$v_r = \frac{1}{r}F(r)(a_0 + a_1 z),$$

$$v_2 = -\frac{1}{r}F^*(r)\left(a_0 z + \frac{1}{2}a_1 z^2\right) - \frac{1}{r}G^*(r),$$

(3.3) $$D^2\psi = \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{r}\right)F(r)\left(a_0 z + \frac{1}{2}a_1 z^2\right) + \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)$$

$$G(r) + a_1 F(r)$$

(3.4) $$D^2(D^2\psi) = \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2 F(r)\left(a_0 z + \frac{1}{2}a_1 z^2\right) + \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2$$

$$G(r) + 2a_1\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)F(r) = 0$$

From (2.10) and (3.4), we get

(3.5) $$\left(\frac{2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2 f(r) = 0$$

(3.6) $$\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2 G(r) + 2a_1\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)F(r) = 0$$

Equation (3.5) gives

(3.7) $$\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)H(r) = 0, \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)F(r) = H(r)$$

Solving (3.7), we get

(3.8) $H(r) = Ar^2 + b$

(3.9) $$r^2\frac{d^2F}{dr^2} - r\frac{dF}{dr} = Ar^4 + Br^2$$

Integrating (3.9), we obtain

(3.10) $F(r) = C + Dr^2 + \frac{Ar^4}{8} + \frac{Br^2}{2}\ln r$

From (3.6) and (3.10)

(3.11) $\left(\frac{d^2}{dr^2} - \frac{1}{2}\frac{d}{dr}\right)\left[\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)G(r) + 2a_1F(r)\right] = 0$

Using (3.7) and (3.8), we get

(3.12) $\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)G(r) + 2a_1F(r) = Mr^2 + N$

Now from (2.4), (2..5) &(3.2)

(3.13) $\frac{d}{dr}\left[F^*(r)\right] = 0 \text{ at } r = 0,$

$\frac{d}{dr}\left[\frac{1}{r}G^*(r)\right] = 0 \text{ at } r = 0,$

(3.14) $\frac{1}{r}F(r) = 0 \text{ at } r = 0,$

(3.15) $\frac{1}{r}F^*(r) \text{ and } \frac{1}{r}G^*(r)$ are finite at r = 0,

(3.16) $F^*(r) = 0, \quad G^*(R) = 0, \; F(R) = R$

From (3.10), (3.14) & (3.15)

(3.17) C = 0, B = 0

From (3.10), (3.16) & (3.17)

(3.18) $2DR + \frac{1}{2}AR^3 = 0, \; Dr^2 + \frac{AR^4}{8} = R$

so that

(3.19) $F(r) = \frac{2r^2}{R} - \frac{r^4}{R^3} = R\left[2\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^4\right]$

Substituting (3.19) in (3.12), we get

(3.20) $\frac{d^2G}{dr^2} - \frac{1}{r}\frac{dG}{dr} = Mr^2 + N - 4a_1\frac{r^2}{R} + 2a_1\frac{r^4}{R^3}$

Integrating (3.20), we obtain

(3.21) $G(r) = M_1r^2 + N_1 + \frac{Mr^4}{8} + \frac{Nr^2\ln r}{2} - \frac{a_1}{2}\frac{r^4}{R} + \frac{a_1}{12}\frac{r^6}{R^3}$

From (3.15) and (3.21)

(3.22) N = 0

From (3.16) and (3.21)

(3.23) $2M_1R + \frac{1}{2}MR^3 - \frac{3a_1}{2}R^2 = 0$

Equation (3.23) can determine only one of the two unknown constants M and M_1, to determine both these, we need one more relation. This relation can be found in terms of Q_0 which is the total flux at z = 0. Using (3.2), we get

(3.24) $Q(z) = \int_0^R 2\sigma r v_z(r,z)dr$

$$= 2\int_0^R \left[\left(\frac{4r^3}{R^3} - \frac{4r}{R}\right)\left(a_0z + \frac{1}{2}a_1z^2\right) - 2M_1r - \frac{Mr^3}{2} - \frac{2a_1}{R}r^3 + \frac{a_1r^5}{2R^3}\right]dr$$

(3.25) $\frac{Q_0}{2\pi R^2} = \frac{MR^2}{8} - \frac{a_1}{3}R,$

(3.26) $M = \frac{8}{R^2}\left(\frac{Q_0}{2\pi R^2} + \frac{a_1R}{3}\right)$

(3.27) $M_1 = \frac{Q_0}{\pi R^2} + \frac{a_1R}{12}$

From (3.21), (3.22), (3.26) & (3.27)

(3.28) $G(r) = \left(\frac{a_1R}{12} - \frac{Q_0}{\pi R^2}\right)r^2 + N_1 + \frac{1}{R^2}\left(\frac{Q_0}{2\pi R^2} + \frac{a_1R}{3}\right)r^4$

$$-\frac{a_1}{2}\frac{r^4}{R} + \frac{a_1}{12}\frac{r^6}{R^3}$$

The constant N_1 need not be determined since $\psi(r,z)$ can always contain anarbitrary constant without affecting the velocity components.

(3.29) $v_r(r,z) = \left[2\frac{r}{R} - \left(\frac{r}{R}\right)^3\right](a_0 + a_1z),$

(3.30) $v_1(r,z) = -4\left(\frac{r}{R} - \frac{r^3}{R^3}\right)\left(a_0z + \frac{1}{2}a_1z^2\right) -$

$$2\left(\frac{a_1R}{12} - \frac{Q_0}{\pi R^2} - \frac{4}{R^2}\frac{Q_0}{2\pi R^2} + \frac{a_1R}{3}\right)r^2 + 2a_1\frac{r^2}{R} - \frac{a_1}{2}\frac{r^4}{R^3}$$

$$= \left(1 - \frac{r^2}{R^2}\right)\left[\frac{2Q_0}{\pi R^2} - \frac{2}{R}(2a_0z + a_1z^2 - \frac{a_1R}{2}\left(\frac{1}{3} - \frac{r^2}{R^2}\right)\right]$$

Differentiating (3.24), we obtain

(3.31) $\frac{dQ}{dz} = 8\pi(a_0 + a_1z)\int_0^R \left(\frac{r^3}{R^3} - \frac{r}{R}\right)dr = -2\pi R(a_0 + a_1z)$

so that the decrease of flux is equal to the amount of the fluid coming out of the cylinder per unit length per unit time. Integrating (2.3.46), we get

(3.32) $Q(z) = Q_0 - \pi R(2a_0 z + a_1 z^2),$

From (3.30) & (3.32)

(3.33) $v_z = \left(1 - \frac{r^2}{R^2}\right)\left[\frac{2Q(z)}{\pi R^2} - \frac{a_1 R}{2}\left(\frac{1}{8} - \frac{r^2}{R^2}\right)\right]$

For Hagen-Poiseuille flow in a circular tube, we have

(3.34) $v_z = \left(1 - \frac{R^2}{R^2}\right)\frac{2Q}{\pi R^2}$

Comparing (3.33) and (3.34), we find that there are two changes (i) Q is replaced by the variable Q(z), and (ii) there is further distortion due to the varying nature of the radial flow.

Using (3.32), (3.33), (3.29) & (3.37) & (3.32) we get

(3.35) $\frac{\partial p}{\partial r} = \frac{8\mu r}{R^3}(a_0 + a_1 z),$

(3.36) $\frac{\partial p}{\partial z} = -\frac{4a_1\mu}{R}\left[\frac{r^2}{R^2} + \frac{2Q(z)}{a_1 \pi R^3} + \frac{1}{3}\right]$

Integrating (3.35), we obtain

(3.37) $p(r,z) = -\frac{4\mu r^2}{R^3}(a_0 + a_1 z) + K(z)$

Differentiating (3.37) partially with respect to z and then substituting $\partial p / \partial z$ (3.36), we get

(3.38) $K^*(z) = -\frac{4a_1\mu}{R}\left(\frac{1}{3} + \frac{2Q(z)}{a_1 \pi R^3}\right)$

so that

(3.39) $K(z) = -\frac{4a_1\mu}{R}\left(\frac{1}{3}z + \frac{2\bar{Q}(z)}{a_1 \pi R^3}\right) + K_0$

where

(3.40) $Q(z) = \int_0^2 Q(z)dz$

Substituting from (3.39) in (3.38), we get

(3.41) $p(r,z) - p(0,0) = -\frac{4\mu}{R}(a_0 + a_1 z)\frac{r^2}{R^2} - \mu\left(\frac{4a_1}{3R} + \frac{8Q}{\pi R^4}\right)z$

The average pressure $\bar{p}(z)$ at any section is given by

(3.42) $\bar{p}(z) = \frac{\int_0^R p(r,z) 2\pi r \, dr}{\int_0^R 2\pi r \, dr} = -\mu\left[\frac{2a_0}{R} + \left(\frac{8Q(z)}{\pi R^4} + \frac{10a_1}{3R}\right)z\right]$

Thus the pressure drop over the tuble length L is

(3.43) $\Delta\bar{p} = \bar{p}(0) - \bar{p}(L) = \mu\left(\frac{8\bar{Q}(L)}{\pi R^4} + \frac{10a_1}{3R}\right)L$

4. Solution When Radial Velocity at Wall Decreases Exponentially with z.

We assume a solution of the form

(4.1) $\psi(r,z) = f(r)e^{-\alpha z} + g(r)$

so that the equation

(4.2) $D^2(D^2\psi) = \left[\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2 + 2\alpha^2\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right) + \alpha^4\right]$

$$f(r)e^{-\alpha z} + \left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\right)^2 g(r) = 0$$

gives

(4.3) $\left(\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} + \alpha^2\right)f(r) = 0$

(4.4) $\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right)g(r) = 0$

Those solutions of (4.3) and (4.4) which are finite at r = 0 are

(4.5) $f(r) = A_1 r J_1(\alpha r) + A_2 r^2 J_2(\alpha r)$

(4.6) $g(r) = A_3 r^4 + A_4 r^2 + A_5$

Using (2.9) in these equations, we get

(4.7) $v_r(r,z) = -\frac{f(r)}{r}\alpha e^{-\alpha z}$

$$v_2(r,z) = \frac{f^*(r)}{r}e^{-\alpha z} - \frac{g^*(r)}{r}$$

The boundary conditions (2.4) and (2.5) with $\phi(z) = v_0 e^{-\alpha z}$ give

(4.8) $\frac{d}{dr}\left[\frac{f^*(r)}{r}\right] = 0, \frac{d}{dr}\left[\frac{g^*(r)}{r}\right] = 0, \frac{f(r)}{r} = 0$

(4.9) $\frac{f^*(R)}{R} = 0, \frac{g^*(R)}{R} = 0, -\frac{f(R)}{R}\alpha = v_0$

(4.10) $A_1 J_0(\alpha R) + A_2 R J_1(\alpha R) = 0$

(4.11) $A_1 J(\alpha R) + A_2 R J_2(\alpha R) + v_0/\alpha = 0$

(4.12) $4A_3R^3 + 2A_4R = 0$

(4.13) $v_r(r,z) = \frac{J_1(\alpha R)J_1(\alpha r) - (r/r)J_0(\alpha R)J_2(\alpha r)}{J_1^2(\alpha R) - J_2(\alpha r)J_0(\alpha r)} v_{0e^{-\alpha z}},$

(4.14) $$v_z(r,z) = \frac{J_1(\alpha R)J_0(\alpha r) - (r/R)J_0(\alpha R)J_1(\alpha r)}{J_1^2(\alpha R) - J_2(\alpha R)J_0(\alpha R)}$$

$$v_0 e^{-\alpha z} + 4A_2R^2\left(1 - \frac{r^2}{R^2}\right)$$

where the constant A_3 has to be determined in terms of Q_0 which is the initial flux at z = 0, Hence, using (4.7), (4.10) and (4.12) we get

(4.15) $$Q_0 = \int_0^R 2\pi r v_z(r,0)dr = 2\pi\int_0^R [-f^*(r) - g^* r)]$$

$$r = 2\pi\left[f(0) + g(0) - f(R) - g(R)\right]$$

$$= 2\pi\left[-A_1RJ_1(\alpha R) - A_2R^2J_2(\alpha R) - A_3R^4 - A_4R^2\right]$$

$$= 2\pi\left(\frac{v_0R}{\alpha} + A_3R^4\right)$$

so that

(4.16) $$A_3R^2 = \frac{Q_0}{2\pi R^2} - \frac{v_0}{\alpha R}$$

From (2.2), (2.16) & (4.7)

(4.17) $$\frac{1}{\mu}\frac{\partial}{\partial r} = \frac{\partial}{\partial r}\left\{\frac{\partial}{\partial r}\frac{\partial}{\partial r}\left[-\alpha f(r)e^{-\alpha z}\right]\right\} - \frac{a^3 f(r)}{r}e^{-\alpha z} = -e^{-\alpha z}$$

$$\left[\alpha\frac{\partial}{\partial r}\left(\frac{f^*(r)}{r}\right) + \alpha^3\frac{r(r)}{r}\right],$$

(4.18) $$\frac{1}{\mu}\frac{\partial}{\partial z} = -\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial}{\partial r}\left[\frac{f^*(r)}{r}e^{-\alpha z} + \frac{g^*(r)}{r}\right]\right\} - \alpha^2\frac{f^*(r)}{r}e^{-\alpha z}$$

$$= -e^{-\alpha z}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\frac{\left(f^*(r)\right)}{r}\right] + \frac{x^2 f^*(r)}{r}\right\} - 16A,$$

Integrating (4.18), we obtain

(4.19) $$p(r,z) - p(0,0) = (1 - e^{-\alpha z})\left[\alpha\frac{f^*(r)}{r} + \alpha^3\int_0^r \frac{f(r)}{r}dr\right] - 16A_3z$$

$$f(r) = A_1\, rJ_1(\alpha r) + A_2r^2\, J_2(\alpha r)$$

Using

(4.20) $$\frac{d}{dr}[r^pJ_p(r)] = r^pJ_{p-1}(r)$$

we get

(4.21) $$\frac{f^*(r)}{r} = \alpha[A_1J_0(\alpha r) + A_2rJ_1(\alpha r)]$$

Again, since

(4.22) $$\int_0^r \frac{f(r)}{r} dr = A_1 \int_0^r J_1(\alpha r) dr + A_2 \int_0^r r J_2(\alpha r) dr,$$

using

(4.23) $$J_0^*(r) = -J_1(r),\ rJ_n^*(r) = nJ_n(r) - rJ_{n+1}(r),$$

we obtain

(4.24) $$\int_0^r \frac{f(r)}{r} dr = -A_1 \frac{J_0(\alpha r)}{\alpha} + A_2 \int_0^r \left[\frac{1}{\alpha} J_1(\alpha r) - rJ_1^*(\alpha r) \right] dr$$

$$= -\frac{A_1 J_0(\alpha r)}{\alpha} - \frac{A_2 J_0(\alpha r)}{\alpha^2} - A_2 \left[\frac{J_1(\alpha r)}{\alpha} r - \int_0^r \frac{J_1(\alpha r)}{\alpha} dr \right]$$

$$= -\frac{A_1 J_0(\alpha r)}{\alpha} - \frac{2A_2 J_0(\alpha r)}{\alpha^2} - \frac{A_2 J_1(\alpha r)}{\alpha}$$

Therefore,

(4.25) $$\alpha \frac{f^*(r)}{r} + \alpha^3 \int_0^r f(r) dr = \alpha^2 A_1 J_0(\alpha r) + \alpha^2 A_2 r J_1(\alpha r)$$

$$-\alpha^2 A_1(\alpha r) - \alpha^2 A_2 r J_1(\alpha r) - 2A_2 \alpha J_0(\alpha r) = -2A_2 \alpha J_0(\alpha r)$$

But from (4.10) and (4.11),

(4.26) $$A_2 = \frac{v_0}{\alpha R} \frac{J_0(\alpha R)}{J_0(\alpha R) J_2(\alpha R) - J_1^2(\alpha R)}$$

From (4.16), (4.19), (4.25) & (4.26)

(4.27) $$p(r,z) - p(0,0) = (1 - e^{-\alpha z}) \frac{2v_0}{R}$$

$$\frac{J_0(\alpha R)}{J_0(\alpha R) J_2(\alpha r) - J_1^2(\alpha R)} - \frac{16z}{R^2} \left(\frac{Q_0}{2\pi R^2} - \frac{v_0}{\alpha R} \right)$$

5. Remarks [7, 8]

Investigations in mathematical modelling for two dimensional flow in nephron or renal tubule is useful and relevant for studies in biomathematics and specially to understand the function of kidney.

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