



WINNING STRATEGIES FOR TWO-PERSON ZERO SUM GAME

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Abstract : A game of strategy is described by its set of rules which specify clearly what each person called a "player" is allowed to do under all possible circumstances. This paper provides a survey of some examples of game theory, defines game theory and the principle to always win the game, and discusses practical applications of game theory in both economic and social situations. This paper focuses on the simple overlook to the games which can be winning easily. By using simple methods of game theory, one can be able to solve what would be a confusing array of outcomes in a real-world situation. Game theory basically consists of Pure strategies and Mixed strategies, In this paper we have discussed examples which uses both the strategies.

IndexTerms - Players, Strategy, Pure strategies, Mixed strategies.

I. INTRODUCTION

Whenever two or more live entities come together, the fight for survival begins! And from which competition emerges. From competition strategy born, the strategy due to which one individual could win in every situation of life. But exactly which strategy one should adapt? - This is the most fundamental question, one may has. For such situation we can give mathematical form for the strategies from the field of OR (Operations Research) called game theory. Game theory is the branch of applied math used to create an optimum strategy in order to succeed in competitive situations of uncertainty. It's the mathematical study of decision making & modeling in situations of conflict that are found in everyday life. Game theory is a strategic interaction among two or more competing players and produce optimal decision-making. In other words, Game theory is the science of strategy, which is mathematical study of strategic decision making or at least the optimal decision-making of independent and competing players in a strategic setting contained to set rules and outcomes. This article will discuss some of the basic and fundamental concepts along with some amazing examples in game theory so far. Game theory is one of the most underrated cores in mathematics, which is widely applied in the real world. It can be used to analyze and predict different situations in daily life or decision of players in game. According to game theory, the choices of each individual player in a game affect the total outcome of the game. The assumption that all the players must be rational will follows in any game and the players should strive to maximize their every move in order to win the particular game.

2 Historical Background

Game theory, the study of strategic decision-making, brings together disparate disciplines such as mathematics, psychology, and philosophy. Game theory was invented by John von Neumann and Oskar Morgenstern in 1944 and has come a long way since then. The importance of game theory to modern analysis and decision-making can be gauged by the fact that since 1970, as many as 12 leading economists and scientists have been awarded the Nobel Prize in Economic Sciences for their contributions to game theory. Game theory is essentially a study of conflict situations between two or more opponents or players. Each player in the game situation must decide on a course of action, or strategy, and the strategy each player chooses affects the outcome for all players in the game. A two-player game is called a zero-sum game if the sum of the payoffs to each player is constant for all possible outcomes of the game. More specifically, the terms (or coordinates) in each payoff vector must add up to the same value for each payoff vector. Such games are sometimes called constant-sum games instead.

3 Basic Concepts in Game Theory

3.1 Definition of game

Game theory is a branch of applied [mathematics](#) that provides key for analyzing situations in which the entities make decisions that are in alliance with each other. That is, one's decision for a particular situation may affect others. For such situations we have game. The entities in a game are called players. The players can play a game individually or in groups. As discussed earlier players in any game are rational. According to game theory, One always lose, and another player always wins.

The word "play" will be employed to denote the number of times a particular game is played. The word "move" will mean a point in the game at which one of the players selects one of a set of alternatives.

Meaning of rational player is that, the one who makes the decisions based on what will lead him to have great benefit than others. A rational player always maximizes his strategies to win.

Here we mean **Strategy** as complete plan of action a player will take the decision for himself.

And **Payoff** is the payout value that a player receives from arriving at a particular outcome.

A Game is any set of circumstances that has a result dependent on the actions of two or more decision-makers (players).

A Player is a strategic decision-maker within the context of the game

Equilibrium: The point in a game where both players have made their decisions and an outcome is reached.

Outcomes or results: This includes the possible strategies and expected payoffs as per outcomes. It is assumed that all the participants are already aware of these payoffs.

There are two ways by which any game can be played, first is simultaneous games- in which the one player goes ahead with his/her actions simultaneously, irrespective of looking for information related to the actions chosen by other players. The other one is a sequential game, which includes the dependency of the player's actions on the previous action's results or another player's choice.

Pure strategy provides a complete definition of how a player will play a game. Pure strategy can be thought about as a singular concrete plan subject to the observations they make during the course of the game of play.

Mixed strategy (Randomized strategy) is used when player A playing his game pretty well and announces his moves in advance, while player B sees the result of the actual win of the game.

Key elements of Two-person zero sum game:

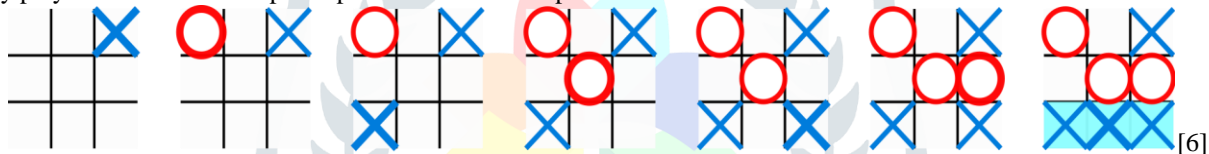
Each two person game consists of 2-players, strategies available to each Player, Payoff for each player that is amount of benefit or loss that a player derives when particular outcome happens.

3.2 Practical examples of Two-person zero sum game:

Let's check some examples of game theory used in everyday life. Consider the following real-world competitive situations: missile defense, sales price wars for new cars, energy regulation, auditing tax payers, the TV show "Survivor," terrorism, military conflicts, arbitration, advertising, elections and voting, conflict resolution, stock market, insurance, and telecommunications. There is something common in between these situations, to know what is common or how to use game theory in it? We'll see this by one-two simple examples.

A) Tic-tac-toe and rock paper scissors

Do you remember playing Tic-tac-toe? a game in which two players seek in alternate turns to complete a row, a column, or a diagonal with either three O's or three X's drawn in the spaces of a grid of nine squares, noughts and crosses before the opponent does. [5-wikipedia]. After learning how to play the game tic-tac-toe, we probably discovered a strategy of play that enables us to achieve at least a draw and even win if our opponent makes a mistake and we notice it. Sticking to that strategy ensures that we will not lose. A pure strategy is such as the one we found for tic-tac-toe, is an overall plan specifying moves to be taken in all eventualities that can arise in a play of the game. A game with throughout its play, all the rules, possible choices, and past history of play by any player are known to all participants is said to have perfect information.



Another example of a simple game is rock paper scissor. In many situations we can't decide who is right or wrong. Then the game of rock, paper, and scissors remain the only option and the one who wins; wins the argument. In this game, we know the consequences but are not aware of another player is going to do. Unlike tic-tac-toe, rock-paper-scissors is not a game with pure strategy, as one cannot predict the outcomes or moves of the other player. A Game without perfect information, such as matching pennies, rock-paper-scissors the players has a challenge because there is no pure strategy that ensures a win. For rock-paper-scissors you have three pure strategies: play rock or paper or scissors. For instances you cannot just continually play a pure strategy like stone because your opponent will soon catch on and play the associated winning strategy. What to do? We try to amaze our opponent by randomizing our choice of strategy for each play. For example, for rock-paper-scissors we can toss a six-sided die and decide to select rock half the time (the numbers 1, 2 or 3 are tossed), select paper one third of the time (the numbers 4 or 5 are tossed) or select scissors one sixth of the time (the number 6 is tossed). Doing so would tend to hide your choice from your opponent. But, by mixing strategies in this manner, should we expect to win or lose in the long run? What is the optimal mix of strategies we should play? How much would we expect to win? This is where the modern mathematical theory of games comes into play. This type of strategy is referred as Mixed Strategy.

B) Prisoner's Dilemma

One of the most popular and basic game theory strategies is the prisoner's dilemma. This concept explores the decision-making strategy taken by two individuals who, by acting in their own individual best interest, end up with worse outcomes than if they had cooperated with each other in the first place. In the prisoner's dilemma, two suspects apprehended for a crime are held in separate rooms and cannot communicate with each other. The prosecutor informs both Suspect 1 and Suspect 2 individually that if he confesses and testifies against the other, he can go free, but if he does not cooperate and the other suspect does, he will be sentenced to three years in prison. If both confess, they will get a two-year sentence, and if neither confesses, they will be sentenced to one year in prison. While cooperation is the best strategy for the two suspects, when confronted with such a dilemma, research shows most rational people prefer to confess and testify against the other person than stay silent and take the chance the other party confesses. A prisoner's dilemma describes a situation where, according to game theory, two players acting selfishly will ultimately result in a suboptimal choice for both. The prisoner's dilemma also shows us that mere cooperation is not always in one's best interests. For example, you and a colleague are in jail and suspected of committing a crime. You are isolated from each other and do not know how the other will respond to questioning. The police invite both of you to implicate the other in the crime (defect). What happens depends on what both of you do, but neither of you know how the other will respond.

If your colleague betrays you while you remain silent, then you receive the longest jail term while your colleague gets off free (and vice versa). If you both choose to cooperate with each other and not the cop by remaining silent, there is insufficient evidence to convict both of you, so you are both given a light sentence for a lesser crime. If you both decide to defect, then you have doomed each other.

The payoff in this game is a reduction in prison sentencing of very good, fairly good, fairly bad or very bad, which is translated into a point score system as follows:

		Your Colleague	
		Corporate	Defeat
You	Corporate	Great Decision Reward for mutual corporation <i>3 points</i>	Bad <i>0 points</i>
	Defeat	Good <i>5 points</i>	Totally bad decision Punishment for mutual defection <i>1 points</i>

It is implied that the prisoners will have no opportunity to reward or punish their partner other than the prison sentences they get and that their decision will not affect their reputation in the future. Because betraying a partner offers a greater reward than cooperating with them, all purely rational self-interested prisoners will betray the other, meaning the only possible outcome for two purely rational prisoners is for them to betray each other. In reality, humans display a [systemic bias](#) towards cooperative behavior in this and similar games despite what are predicted by simple models of "rational" self-interested action.

C) Collective Bargaining or Negotiation between Parties

Game theory plays an important role in different collective bargaining or negotiation activities among different parties or participants. For example, different negotiations take place between worker unions and the management during the situation of the strike of workers or lockout period to increase wages. Using game theory, both parties can arrive at the optimum solution of the issue, i.e., to increase wages by examining different options available for wages and benefits, which can maximize the welfare of both workers and management. Salary negotiation is also an example of the game theory application. The concept of game theory is used in other negotiations also like negotiations with suppliers while purchasing, compensation or incentive negotiations between management and suppliers or business partners, etc.

Conclusion:

A zero-sum game describes a relationship, competition, or business deal where one person's gain is the other person's loss. Zero sum games don't just apply to sports and games, in finance it highlights one investor earns the money that another investor loses. In this paper we have discussed the ways in which games like Tic-tac-toe, stone paper scissor and prisoner's dilemma can be considered mathematically while letting one player win the game.

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