



A new prospect on Bose- Einstein Condensation

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Abstract

This project is based on the discussion of the mean field theory for the dilute Bose gas then there is derivation of the Gross-pitaevskii equation for the order parameter and we have used it to calculate the ground state energy of the system, then we discuss Bogoliubov model, it is a theory beyond mean field and takes into account the fluctuation of order parameter. We introduce Bogoliubov effective Hamiltonian, discuss its diagonalization by means of Bogoliubov transformation and calculate the correction of the ground state energy arising from the quantum fluctuation then we discuss within the Bogoliubov model the effects of weak external random potential we have also studied the behavior of the one body density matrix.

Keywords : The non-interacting Bose gas, Condensed Phase, Normal Phase, The Gross-pitaevskii equation, The Thomas-Fermi approximation, Dilute Bose gas, Gross-pitaevskii equation, Ground state energy.

Introduction

The first experimental realization of Bose-Einstein condensation in dilute atomic gases happens in 1995.

1.1 A Bose condensate (BEC) is a state of matter in which separate atoms or subatomic particles, cooled to near absolute zero (0 K or -273.15°C or -459.67°F, K = kelvin), coalesce into a single quantum mechanical entity. BEC's most intriguing property is, it can

slow down light. In 1998 Lene Hau of Harvard University and group, slowed light travelling through a BEC from its speed in the vacuum of 3×10^8 m/sec to a mere 17 m/s, or up to 38 miles/hour. Lene Hau and group have completely stopped and halted a light pulse within a Bose-Einstein condensate, later releasing the light unchanged or sending it to the second BEC. These manipulations hold promise for newer light-based telecommunication types, quantum computing and optical storage of data through the low temperature requirements of BEC's offer practical difficulties.

Lene Hau does not only know how to stop light but also manipulate it and save its imprints for later use.

We can hold on to the light, move it around or even save it for later; we can actually manipulate it. Lecture of Hau at Southern Denmark

In those regard the speed of light takes on a new meaning . The light became compressed.

We are trying to unify Einsteins theory of relativity with quantum mechanics to find out how the univase came in to existence

According to photonics media in 1998, Hau slowed light which travelles in free space at speed of 1 86000 m/sec to just 38 mile/sec in a cloud of ultra cold atoms Einstein and others have theorized that the speed of light in free speed does not changed . Two years after hav and group stoped light completely in similar cloud , then restarted it without charging its characteristics.

The Bose Einstein condensates are very important to this work because within these clouds atoms become phase – locked , losing their individually and independence.

2.1 The non-interacting Bose gas

The Bose distribution function

$$F^0(Ev) = \frac{1}{e^{Ev-\mu} - 1} \quad \dots\dots\dots (1)$$

Where Ev is the energy of the single particle state for the particular trapping potential under consideration, μ is chemical potential

By Boltzmann distribution

$$F^0(Ev) = e^{-\frac{(Ev-\mu)}{kt}} \quad \dots\dots\dots (2)$$

At highttemperture $\mu \gg E_{min}$ $e^{-\frac{\mu-Ev}{kt}} \ll 1$ $\dots\dots\dots (3)$

The highest temperature at which the condensate exits is called Bose Eonstein temperature (T_c)

Were as $T_c \gg 1$ The number of the particle is excited states is given by

$$N_{ex} = \int_0^\infty d\epsilon g(\epsilon) f^0(\epsilon) \quad \dots\dots\dots (4) \quad \epsilon$$

This a achieves its greatest value $\mu = 0$

$$N = N_{ex}(T_c, \mu = 0) = \int_0^\infty d\epsilon g(\epsilon) \frac{1}{e^{kT_c} - 1} \quad \dots\dots\dots (5)$$

Let a dimensionless variable $x = \frac{\epsilon}{kT_c}$

$$N = C_\alpha (kT_c)^\alpha \int_0^\infty dx x^{\alpha-1} / e^{x-1} = c_\alpha \{ \tau(\alpha) \xi \} \propto (kT_c)^\alpha$$

$$\Rightarrow KT_c = \frac{N^{\frac{1}{\alpha}}}{c [\alpha \gamma(\alpha) \xi(\alpha)]^{\frac{1}{\alpha}}} \quad \dots\dots\dots (6)$$

For 3D harmonic – oscillator potential $\alpha = 3$

$$KT_c = \frac{\bar{h} \bar{\omega} N^{\frac{1}{3}}}{[\xi(3)]^{\frac{1}{3}}} \approx 0.99 \bar{h} \bar{\omega} N^{\frac{1}{3}} \quad \dots\dots\dots (7)$$

$$\omega \approx (\omega_x \omega_y \omega_z)^{1/3}$$

$$\Rightarrow T_c \approx 4.5 (f/100\text{Hz}) N^{1/3} n k \dots\dots\dots(8)$$

For uniform Bose gas in a 3- dimension box of volume V the index $\alpha = 3/2$

$$K T_c = \frac{2\pi}{\{\xi(3/2)\}^2} \frac{\hbar^2 h^3}{m} \approx \frac{\hbar^2 h^3}{m} \dots\dots\dots(9)$$

$$W = n \left(\frac{2\pi\hbar^2}{mkt} \right)^{3/2} \dots\dots\dots(10)$$

This majority of occupied states have energies of order KT or less , Hence the number of the states per unit volume that are occupied significantly is of the order of the total number of the state per unit volume with energy less then KT

After Solving the transition temperature Nex of particle in excited state

$$N_{ex} (T) = c_{\alpha} \int_0^{\infty} d \epsilon \epsilon^{\alpha-1} \frac{1}{e^{\frac{\epsilon}{RT}} - 1} \dots\dots\dots(11)$$

$$\text{We get } N_o = N [1-(T/T_c)^3] \dots\dots\dots(12)$$

For 3-D harmonic oscillator $\alpha = 3$

2.2 Density profile and velocity distribution

As BEC state is typically formed when a gas of bosons at low densities is cooled to temperature very close to absolute zero Also the distribution of the particle after a cloud is allowed to expand depends not only on the initial density distribution but also on the initial velocity distribution. In momentum space wave function become Fourier transform

$$\frac{-p^2x}{e_2 c^2 x} \frac{-p^2y}{e_2 c^2 y} \frac{-p^2z}{e_2 c^2 z} \dots\dots\dots(13)$$

$$\phi_0(p) = \frac{1}{\pi^{3/4} (c_x c_y c_z)^{1/2}} \dots\dots\dots(14)$$

$$\text{Which gives } n(p) = \frac{N}{\pi^4 (c_x c_y c_z)} \frac{-p^2x}{e_2 c^2 x} \frac{-p^2y}{e_2 c^2 y} \frac{-p^2z}{e_2 c^2 z} \dots\dots\dots(15)$$

2.3 In thermo dynamics quantities we have condensed phase

The energy of the micro scopically occupied state is taken to zero, there for only excited state contribute to the total energy of the system. Internal energy is $E = \int_0^{\infty} dE g(E) \frac{1}{e^{\frac{\epsilon}{RT}} - 1}$

$$\Rightarrow E = C_o \Gamma(\alpha + 1) \delta(\alpha + 1) (KT)^{\alpha+1} \dots\dots\dots(16)$$

2.4 we are familiar with Normal phase and theory of condensed state we get time – in dependent gross – pita evaskiz equation after solving

$$\phi(r_1, r_2, r_3, \dots, r_n) \prod^N \phi(r_i) e = 1 \dots\dots\dots(17)$$

As Hamiltonian of the system is given by

$$\hat{H} = \sum_{i=1}^N \left(\frac{p_i^2}{2m} + V_{ext}(r_i) \right) + \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N V(|r_i - r_j|) \dots\dots\dots(18)$$

We have

$$\frac{-\hbar^2}{2m} \Delta^2 \phi(r) + V_{ext}(r) \phi(r) + N \frac{4\pi\hbar^2}{m} \alpha |\phi(r)|^2 \phi(r) = \mu \phi(r) \dots\dots\dots(19)$$

2.5 the ground state for trapped bosons.

A variational calculation is

$$E(b_x, b_y, b_z) = N \sum \bar{h} w_i \left(\frac{a_i^2}{4b_j^2} + \frac{b_i^2}{4q_j^2} \right) + \frac{N^2 \mu_o}{2(2\pi)^2} b_x b_y b_z \dots\dots\dots(20)$$

Now we came on the Thomas Fermi approxi matron the ground state energy of the system,

$$\frac{-\hbar^2}{2m} \Delta^2 \phi(r) + V(r) \phi(r) + U_o |\phi(r)|^2 \phi(r) = \mu \phi(r) \dots\dots\dots(21)$$

At boundary of cloud $v(r) = \mu$ (22)

$$E = v(r) + n(r) U_0 \quad \text{.....(23)}$$

But $v(r) = \mu$

$$2\mu = mRi^2 \omega^2 \text{ and } Ri^2 = \frac{2\mu}{m\omega^2} \quad \text{.....(24)}$$

Number of particles N. For harmonic trap is

$$N = \frac{8\pi}{15} \left(\frac{2\pi}{m\omega}\right)^{3/2} \frac{\mu}{U_0} \quad \text{.....(25)}$$

$$\text{Length } \bar{\alpha} = \sqrt{\frac{h}{m\omega}} \quad \text{.....(26)}$$

$$\mu = \frac{15^{2/5}}{2} \left(\frac{Na}{a}\right)^{2/3} \bar{h}\omega \quad \text{.....(27)}$$

$$\bar{R} = (R_1, R_2, R_3)^{1/3} \quad \text{.....(28)}$$

$$R = 15^{1/5} \left(\frac{Na}{a}\right)^{1/5} \bar{a} \quad \text{.....(29)}$$

$$\text{Since } \mu = \frac{\delta E}{\delta N} \text{ and } \mu \propto N^{2/5} \quad \text{.....(30)}$$

$$\Rightarrow \frac{E}{N} = \frac{5}{7} \mu \quad \text{.....(31)}$$

This is the leading contribution to the energy at large N.

Which is our result.

Acknowledgement

The authors are thankful to Prof (Dr.) Basant Singh, Provice Chancellor, Dr. C. V. Raman, University Vaishali, Bihar, India and Prof (Dr.) Dharmendra Kumar Singh, Dean Academic, Dr. C.V.Raman University, Vaishali, Bihar, India. Thanks to library and its in charge, Dr. C.V. Raman University, Vaishali, Bihar, India. For extending all facilities in the completion of the present research work

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