



MATHEMATICAL MODEL IN CLIMATE CHANGE PREDICTION

Mayuri Devidas Borkar

Dr. Rajendra Gode Institute of Technology And Research Amravati (India).

Abstract : In this article, we look at how mathematical models can be used to predict climate change. Mathematical modelling is still the only reliable way to predict the climate. We talked about Energy Balance Models (EBMs), Earth Models of Intermediate Complexity (EMICs), and General Circulation Models (GCMs) to help explain this. (GCMs). We also talked about the things that affect climate.

I. INTRODUCTION

A shift in the typical weather patterns of an area is what scientists refer to as "climate change." The climate of the Earth has experienced an increase in temperature during the past couple of decades. This change is having an effect on the local climates in every region of the world. The climate is variable. Climate change is not a new phenomenon. It has changed in the past, is changing currently, and will undoubtedly continue to change in the future. Moreover, relevant climatic variations for human welfare need not be as dramatic as the previous ice age, when (20,000 years ago) kilometre-thick ice sheets covered areas of the northern hemisphere as far south as 40 degrees latitude. Since the retreat of the ice sheets 10,000 years ago, significant climatic changes have occurred. (for a recent examination of climatic variability see the paper by Lamb [1969]). Approximately one thousand years ago, a comparatively mild time allowed Vikings to explore the North Atlantic region. Several centuries later, at the beginning of the 'little ice age,' the Norse colonies in Greenland were wiped out, and the historical chronicles tell of long, severe winters that brought misery and destitution to Europe. In the high latitudes of the northern hemisphere, there has been a fairly rapid cooling since the 1940s, when the optimal (warmest) conditions occurred. The comparatively pleasant environment that we have grown accustomed to in the latter part of the twentieth century is not typical of all epochs since the end of the last ice age. The climate is variable.

Greenhouse gases, like carbon dioxide, methane, and nitrous oxide, are the main cause right now. These gases are made when things like burning fossil fuels, raising cattle, cutting down trees, and so on. Since the start of the Industrial Revolution in the 18th century, these kinds of activities have grown by leaps and bounds, and as a result, greenhouse gas emissions have reached dangerous levels. Due to the way their energies are quantized, these gases collect infrared radiation very well, which traps heat [1]. This causes the Earth's sunlight to be reflected back into space and stopped by the atmosphere, which forms a blanket of insulation that warms the Earth.

One of the most important things for science to find out in the 21st century is how to predict human-caused changes in the climate. Anthropogenic effects on the climate system include the burning of fossil fuels, which increases the amount of CO in the air, and changes in the amount of small gaseous species that control the amount of ozone in the air. Deforestation and desertification also change the albedo, which is the amount of light that reflects back into space.

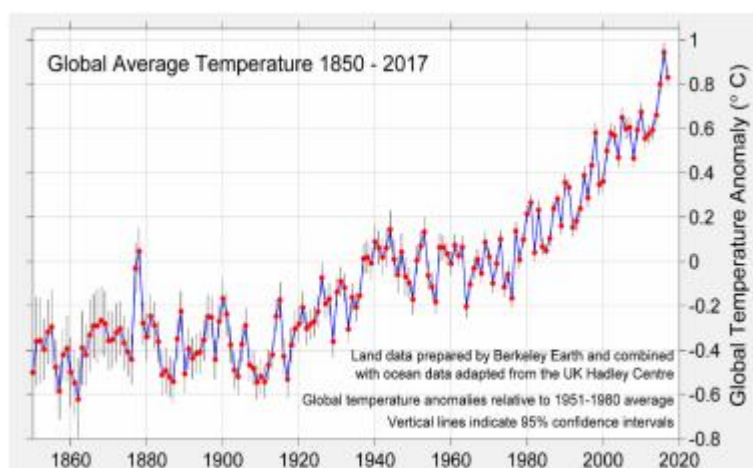
2. Mathematics and climate change

The biggest problem our world is facing right now is climate change. No part of our world is safe from the effects of melting glaciers, rising sea levels, and extreme weather. Galileo once wrote that "mathematics is the language of the universe" [3]. Math is a very useful and powerful tool for understanding the world and handling difficult problems in many fields of science, engineering, and technology [4–7]. In this way, you can't understate how important maths is to solving problems on the world, like the problem of how people and nature affect each other [8, 9]. People live and do all of their different and complex tasks in the world around them. On the one hand, people change the environment and how it works. On the other hand, the environment affects people, especially their health and even their ability to stay alive. At the turn of the century, it was clear that climate change had become a new and very important problem.

The climate change that is occurring now, in contrast to those that have occurred in the past, is caused by humans [10, 11] and is distinguished by an unprecedented pace of increase in the global mean surface temperature. (GMST). According to a report [12], the GMST has risen by an average of 0.07 degrees Celsius over the course of each decade from 1880. Meanwhile, the increase rate of the GMST in the first two decades of the XXI century reached a value of 0.17 C per decade, which is more than double the usual pace. This occurred during the period of time when the global mean surface temperature (GMST) was rising. An appreciable rise in the amount of carbon dioxide (CO₂) found in the air is one of the most telling indications that human activity is to blame for the current state of the environment. The production of this greenhouse gas (GHG), which is a by-product of the combustion of fossil fuels, is the primary contributor to the warming of the planet. The climate of practically every place on Earth has already been altered as a result of global warming [10, 11], but the degree to which this change has occurred varies greatly from one region to the next.

2.1 Global Warming

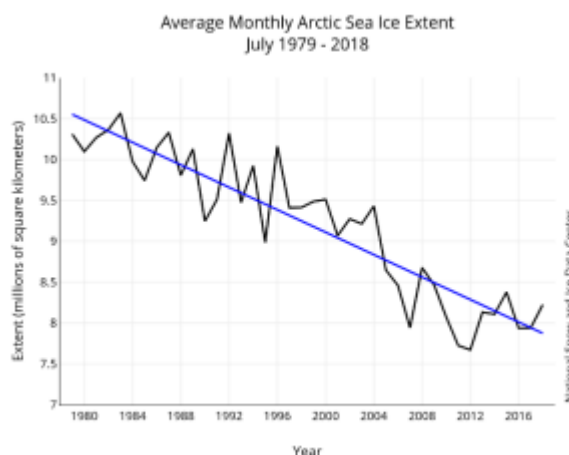
The Fifth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC) "There is no doubt that the climate system is warming, and many of the changes that have happened since the 1950s have never happened before in decades or millennia." [13] So, let's look at the real proof that this is true. Recent records of the Earth's temperature come from many different places, such as weather stations on the land of the Earth, satellites in orbit around the Earth, and buoys and ships in the ocean. Since the Met Office was founded in 1850, these records have been collected regularly at weather stations. The average temperature of the Earth for each year from 1851 to 1980 is shown in the graph below, which goes through 2017.



During this time period, the average temperature of the Earth has fluctuated from year to year due to a number of different factors. Some of these factors include the El Nino (a warming of the Southern Pacific Ocean as a result of the effect of ocean currents), other effects that are related to ocean currents, and also large volcanic eruptions such as the one that occurred in 1883 by Krakatoa. These fluctuations, however, are being placed on a trend that may easily be seen to be upward. In point of fact, the average temperature of the Earth in 2015 was 1 degree Celsius higher than it was when these records first began, and the temperature of the Earth over the past three decades has been the highest ever recorded.

2.2 Ice Loss and Sea Level Rise

The melting of Arctic sea ice is one of the most telling signs of the effects of global warming and a direct consequence of the warming of the planet as a whole. Since 1979, a satellite operated by NASA's National Snow and Ice Data Centre has been monitoring the extent of summer sea ice. This satellite has provided conclusive evidence that this is the case. The graph that was produced as a result, which may be shown below, reveals an unmistakable declining trend. (indicated by the blue line).



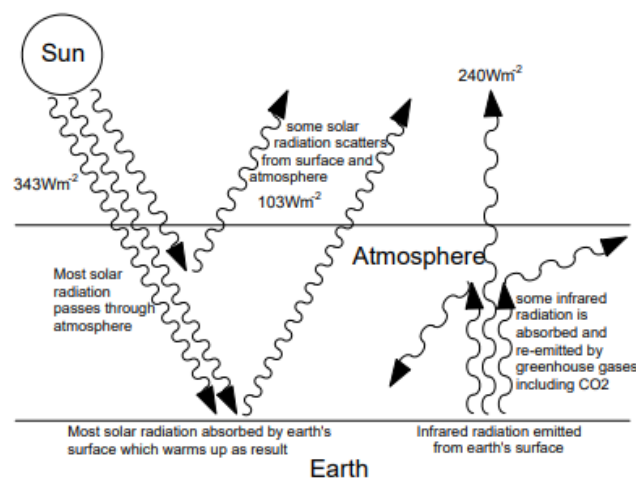
In the past 36 years, there has been a loss of around 2.5 million square kilometres worth of summer sea ice. This is equivalent to the total land area of Scotland being consumed each year. In one hundred years, all of the sea ice in the Arctic will have disappeared if the current rate of melting continues. At the same time, there has been a decrease in the amount of land ice found in both Antarctica and the ice sheets located on Greenland.

Several things will happen because of this. Loss of space for animals like polar bears is the one we hear about the most. The melting ice adds new water to the Atlantic Ocean, which changes the amount of salt in the water. This is the second effect of melting ice. Long-term, this could have a direct effect on how the oceans move, which could change the way of the North Atlantic Drift, which keeps the UK warm. (Ironically, global warming at the North Pole could possibly make the UK colder). A third long-term effect is that the Earth gets darker. The ice sheets help us stay cooler because they reflect a lot of the sun's heat. So, as the ice sheets melt, we will warm up. The biggest worry about sea level rise is what it will do to low-lying coastal areas, especially when combined with storm surges and other effects of events like hurricanes. If the sea level keeps going up, many of the world's cities and coastal areas will be in danger.

2.3 Increase in carbon dioxide in the air

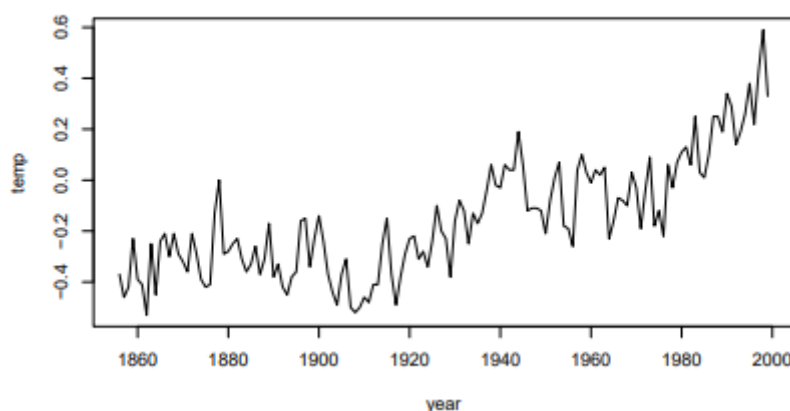
Climate change is one of the most important science, political, and environmental problems people face today. Scientists agree that people's actions, like cutting down trees and burning fossil fuels, have caused the amount of carbon dioxide in the air to rise. This carbon dioxide traps heat at the bottom of the earth's atmosphere, which changes the temperature of the earth. But some scientists and a lot of people who aren't scientists (like US President George W. Bush) aren't sure that rising amounts of carbon dioxide are changing the climate. In this case study, we will look at climate change using both linear and statistical models. Carbon dioxide is a good place to start. Life is based on carbon. That is, all of the complex organic molecules that make up living things are made up of a lot of carbon. So, when biological matter burns or breaks down, it gives off carbon, most of which is in the form of carbon dioxide. In the absence of human activities the amount of CO₂ in the atmosphere tends to stay roughly constant: the CO₂ given off by decaying/burning plants or animals that have died is eventually just re-absorbed by new living plants. What upsets this is when humans remove and burn large areas of forest, without replacing it, or burn fossil fuels. Fossil fuels, like oil, gas and coal are all made up from the bodies of organisms that lived and died many millions of years ago. The bodies of these animals formed layers of sediment at the bottom of ancient seas and over time were buried. Eventually this carbon rich organic material was buried deep under layers of sedimentary rock, and over the millenia ended up forming, oil, gas and coal deposits. When humans extract this fossil fuel and burn it, a large proportion of the carbon content of the fuel ends up as CO₂ in the atmosphere. There is no way around this problem since it is the carbon that is the energy source in the fuel. Carbon moves around in the earth's environment in a complicated way. Huge amounts of carbon are always moving through the system. In fact,

humans are only responsible for 3–4% of the CO₂ that goes into the air at any given time. The problem is that this is 2–3% more than what can be taken out by natural processes, so the amount of CO₂ in the air from people keeps going up. People hoped for a while that all of this extra carbon dioxide would just go away on its own. It's a good idea to look at some facts to see what's really going on. It's not easy to find long-term records of carbon dioxide. It wasn't until very recently that it was measured. As CO₂ levels can vary locally as a result of regional biological activities, even recent records can be a little challenging to understand. Because CO₂ absorbs infrared light with wavelengths in the region of 1319106 m, it is a cause for concern. Simply put, infrared radiation is light with a very long wavelength. It is what thermal imaging equipment of the kind used by police to look for people at night picks up. It continuously emanates from warm items. Infrared radiation released from the earth's surface would simply flow through the atmosphere and out into space if there were no greenhouse gases, such as CO₂. (thereby cooling the earth). However, greenhouse gases obstruct the process by absorbing infrared photons (light particles) and then re-emitting them. This indicates that the energy carried by these photons is confined deep into the atmosphere, warming it. Furthermore, this cools down because infrared radiation cannot carry heat to the higher atmosphere. The way it operates schematically is as follows:



When all aspects of the system are in balance, the numbers represent the typical amount of energy that is transferred from one type of radiation to another per square metre. As a result of human activity, the concentration of carbon dioxide (CO₂) in the atmosphere has grown, and there are sound scientific reasons to anticipate that this will result in an increase in temperature both in the lower atmosphere and on the surface of the globe.

Global mean temperature deviations from 1961–1990 mean



This plot does appear to demonstrate an increase in global mean temperature over the time period of the most rapid increase in CO₂ concentrations, but it is much less definitive than the plot for the increase in CO₂ concentrations. The remarkable degree to which the mean temperature appears to fluctuate is a unique feature that stands out. It is because of this that sceptics have reason to question whether or not the climate is actually changing.

3. Mathematical models of the climate

In order to properly understand climate change, a mathematical climate model is essential. Models are used by thousands of climate experts in order to gain a better understanding of the long-term implications of global events such as the rise in

greenhouse gases and the decline in the amount of ice covering the Arctic. We may make educated guesses about the likely future course of our planet's climate by using these models to do simulations covering time periods of hundreds of years.

Types of models

- Energy Balance Models, or EBMs
- EMICs (Earth Models of Intermediate Complexity)
- General Circulation Models (GCMs)

3.1 Energy balance models

As their name suggests, energy balance models predict changes in the climate system by looking at the Earth's energy budget. In their most basic form, they don't have any clear spatial dimensions and only give values for the computed variables that are the global average. So, they are called zero-dimensional EBMs. Both [14] and Sellers [15] explained what these EBMs are based on.

Changes in heat storage = absorbed solar radiation - emitted terrestrial radiation

$$C_E \frac{\partial T_s}{\partial t} = \left[(1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right] \quad (3.1)$$

where, C_E is the effective heat capacity of the media (measured in $\text{J}^{\text{m}^{-2}} \text{K}^{-1}$), T_s the surface temperature, t the time, α_p the planetary albedo, S_0 is the Total Solar Irradiance (TSI) and $A \uparrow$ the total amount of energy that is emitted by a 1 m^2 surface of the Earth. $A \uparrow$ could be represented on the basis of the Stefan-Boltzmann law, using a factor τ_a to represent the infrared transmissivity of the atmosphere (including the greenhouse gas effect), as.

$$A \uparrow = \varepsilon \sigma T_s^4 \tau_a \quad (3.2)$$

where ε is the emissivity of the surface. Using an albedo of 0.3, an emissivity of 0.97, and a value of τ_a of 0.64 leads to an equilibrium temperature $T_s = 287 \text{ K}$, very closely resembles the observed one. In some EBMs, Eq. 2 is linearized to make the model's explanation even easier. The other hand, τ_a and α_p are often parameterized as a function of the temperature, In particular, they need to take into account the fact that cold makes ice and snow cover more of the planet's surface, which raises the planet's albedo.

Zero-dimensional EBMs can be made to have one (usually latitude) or two horizontal dimensions in order to take into account how the temperature at the surface of the Earth varies from place to place. Then, in Eq. 1, an extra term called "transp" is added to show the net effect of heat input and output due to horizontal transport:

$$C_E \frac{\partial T_{s,j}}{\partial t} = \left[(1 - \alpha_p) \frac{S_0}{4} - A \uparrow \right] + \Delta transp \quad (3.3)$$

The surface temperature has been given an index of i to show that the variable is for the area i . Transport can be thought of as a linear function of temperature, which is the easiest way to describe it. However, more complex parameterizations are also used, such as a diffusion term.

3.2 General circulation models

General circulation models describe the climate system in the most accurate and detailed way. At the moment, their grid resolution is usually between 100 and 200 km. So, compared to EMICs, which have a grid resolution of between 300 and

thousands of kilometers, they give much more thorough information on a regional scale. A few years ago, GCMs only showed the weather, the land surface, and sometimes the flow of the oceans, and the sea ice was shown in a very simplified way. GCMs take into account more and more things these days. Models of the sea ice, the carbon cycle, the movement of ice sheets, and even the chemistry of the atmosphere are now used to describe the climate system.

GCM simulations take a lot of computer time because they include a lot of different processes and have a high level of detail. For example, even the fastest computers take a few weeks to run an experiment that lasts a century. As computer power goes up, it becomes cheaper to run longer simulations with a higher resolution, which gives more information about regions than the last generation of models.

4. GCMs and Climate Projections

Today, GCMs (which are sometimes used together with higher-resolution regional climate models to get results that are specific to a region) make predictions about the Earth's climate through the end of this century. In this case, the effect of model error is much bigger than the effect of uncertainty in the initial conditions. So, the model error is measured using both ensembles with different starting conditions and ensembles with different model parameters. Because many different GCMs are used, each with their own specific biases, a multi-model analysis also provides a measure of uncertainty in the projections.

Even though there are errors and biases in the models, it's important to note that GCMs still do a pretty good job of simulating how the climate works in general: storms form and move in a realistic way, temperatures change with the time of day and the day of the year, and rain generally falls where and when it should. But the details, like how strong the storms will get, where they will move or stop, and how much rain will fall, are not recorded well enough for the catastrophe modeller to be happy.

References

- [1] Brocker J., Calderhead B., Cheraghi D., Cotter C., Holm D., Kuna T., Pelloni B., Shepherd T., Weller H., Mathematics of Planet. Earth. A Premier; World Scientific Publishing: London, UK, 372p.(2017).
- [2] Budyko M. I. (1969). The effect of solar radiation variations on the climate of the Earth. *Tellus* 21: 611-619(1969).
Climate Report. December 2018. Available online:
- [3] Galileo G., The Assayer [Accessed January 2022] (1623).
- [4] Hannah R. and Max R., <https://ourworldindata.org/co2-and-other-greenhouse-gas-emissions>
[Accessed January 2022] (2020).
- [5] Lasse A. and Martin L. , GEO Ex Pro, Vol.17, No. 3 [Accessed February 2022] (2020).
- [6] Masson-Delmotte V., Zhai P., Pirani A., Connors S. L, Péan C., Berger S., Caud N., Chen Y., Goldfarb L., Gomis M. I., et al., IPCC. 2021: Summary for Policymakers. In *Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change;* Cambridge University Press: Cambridge, UK, in press (2021).
- [7] NOAA National Centers for Environmental Information, State of the Climate. Global
- [8] Reynolds J. J., Harshbarger R. J., Cengage: Boston, MA, USA, 1104p.(2012).
- [9] Riley K. F., Hobson M. P., M.P. Student Solution Manual for Mathematical Methods for Physics and Engineering, 3rd ed.; Cambridge University Press: Cambridge, UK, 2019; 542p. (2019).
- [10] Sellers W. D., A global climatic model based on the energy balance of the Earthatmosphere system. *J. Appl. Met.* 8: 392-400 (1969).
- [11] Stocker T. F., Qin D., Plattner G.-K, Tignor M., Allen S. K, Boschung J., Nauels A., Xia Y., Bex V., Midgley P. M. , IPCC 2013. Contribution of working group I to the Fifth assessment report of the intergovernmental panel on climate change. In *Climate Change 2013: The Physical Science Basis;* Eds.; Cambridge University Press: Cambridge, UK; New York, NY, USA, 1535p. (2013).
- [12] Stroud K., Industrial Press: South Norwalk, CT, USA, 728p.(2009).

[13] The 5th IPCC assessment report is given in <http://www.ipcc.ch/report/ar5/>.

www.ncdc.noaa.gov/sotc/global/201812 (accessed on 21 December 2020).

[14] Yang X. S., Introductory Mathematics for Earth Scientists; Dunedin Academic Press: Edinburgh, UK, 2009; 228p (2009).

[15] Yevick D., Yevik H., Fundamental Mathematics and Physics for Scientists and Engineers; John Wiley & Sons: New York, NY, USA, 2014; 464p. (2014).

