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DETERIORATING ITEMS SUPPLY CHAIN INVENTORY MODEL FOR SINGLE VENDOR SINGLE BUYER WHEN SHORTAGES ARE ALLOWED TO BUYERS WITH LEAD TIME CONSIDERATION

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ABSTRACT:

The optimal strategy for single vendor single buyer studied when components of integrated inventory models are based on deterioration and demand is exponential function of time under supply chain. The model introduced for single vendor single buyer system as profit maximized to determine the optimal cycle time without considering collaboration between vendor and buyer and also take joint decision under supply chain policy under the shortages and lead time. The model is illustrated with numerical examples and observed that both buyer and vendor earn significant profit in supply chain inventory system.

Key Words: Supply chain, optimal strategy, Deterioration, Exponential demand, shortages and lead time.

INTRODUCTION:

In supply chain inventory models mostly single buyer and single vendor manage collaborative inventory strategy to minimize total cost or maximize joint profit. This optimal strategy has limited global markets scenario. When a long-term relationship between two systems under supply chain has been developed, both systems can collaborate and information is shared to each other to accomplish improved benefits but during the shortage, period buyer has to face certain loss because of lacking of items and maintaining time to reorder the demand.

Lead-time has recently established enormous awareness in market for reducing shortages. Researcher has to focus on lead-time analysis to develop effective coordinated inventory model where integrated cost and profit of both vendor and buyer are minimum and maximum respectively under supply chain management.

Probability inventory model was given by Liao and Shyu(1991) when lead time is taken in consideration and found that customer service was improved and investment of inventory was reduced in safety stocks. To investigate the lead time at what time buyer should order his inventory, Daya and Raouf(1994) derived analytical inventory models by assuming that lead time could be decayed in to *n* equally autonomous mechanism, all components of mechanism is vary but has fixed breakdown or carrying cost which is independent of the ordered lot size.

Pan and Yang(2002) constructed the collaborative inventory model for single purchaser and single vendor by assuming finite production rate and demand. In addition, through-out the entire inventory, they considered lead-time following normal distribution, and the reorder point determined by cumulative expectation of demand through lead-time. Chang et al. (2006) constructed collaborative model when single vendor and single buyer considered lead time and ordering cost. They determined minimum joint cost under supply chain system. The collaborative inventory developed by Vijayashree and Uthayakumar (2015) for single manufactured goods when the system is considered single retailer and single consumer. They assumed that manufacturer ate is fixed/constant and lead time is following normal distribution. From the derived supply chain model they determined optimal order quantity, numbers of deliveries and lead time to minimize cost of supply chain system.

Pandian and Lakshmi (2018) derived supply chain inventory model when vendor and buyer have constant demand rate, production rate for vendor is infinite, shortages are absolutely backlogged, lead-time between buyer's order and order placed by vender follows uniform distribution. They determined optimal order quantity subject to minimized integrated costs.
In this article, we consider exponential demand function to derive inventory model where single vendor of system associated with single buyer take decision without collaboration and with collaboration strategy when carrying cost and deterioration cost depend on time, shortages are permitted to buyers and completely backlogged and considering lead time for buyer during supply chain policy.

NOTATIONS AND ASSUMPTIONS:

NOTATIONS

$D(t) = ae^{bt}$, where $a > 0$, $0 < b < 1$

$I_b(t)$ = Inventory level for buyer at any instant of time t

$I_v(t)$ =Inventory level for vendor at any instant of time t

A_b = Ordering cost per order for buyer

A_v = Ordering cost per order for vendor

C_b = Purchase cost per unit for buyer

C_{2b} = Shortage cost of buyer per unit

θ = Deterioration rate of items for buyer

x_b = Fixed holding cost for buyer

y_b = Varying holding cost for buyer

x_v = Fixed holding cost for vendor

y_v = varying holding cost for vendor

p = Selling price of buyer's per unit

n = Buyer's number of orders placed during cycle time.

TP_b= Total profit for buyer per unit time

TP_v = Total profit for vendor per unit time

TP = Total profit for both vendor and buyer per unit time

$t_1 = v_1*T/n$

t_2 = Time when inventory level reaches to zero (Decision variable)

L= Length of lead time (v_2*t_2)

T = Vendor's cycle time (a decision variable).

ASSUMPTIONS

Under the mentioned assumptions supply chain inventory models are developed:

1. Product's demand is decreasing function of exponential distribution depending on time.
2. Single vendor –single buyer are considered.
3. Shortages are permitted and completely backlogged.
4. Lead-time is taken in to consideration.
5. Deteriorated units can neither be repaired nor replaced during the cycle time and deterioration is dependent on time for buyer's inventory.
6. Time varying holding cost is considered for buyer and vendor.

MATHEMATICALMODELANDANALYSIS:

FollowingFigure-1shown the inventory level Ib(t) of buyer at time $t(0 \leq t \leq T/n)$

Buyer's Inventory

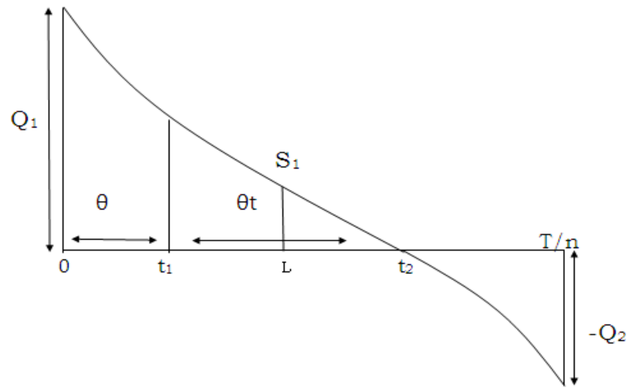

Figure-1

We discuses two situations, in the first situation the inventory model is developed without collaboration between vendor and buyer, while the second situation considers the vendor buyer collaboration.

The inventory level is depleted by exponential demand for both vendor and buyer. The differential equations are given for rate of change of inventory for the vendor and the buyer:

$$\frac{dI_b(t)}{dt} + \theta I_b(t) = -a e^{bt} \quad ; \qquad 0 < t < t_1 \tag{1}$$

$$\frac{dI_b(t)}{dt} + \theta t I_b(t) = -a e^{bt} \quad ; \qquad t_1 < t < L \tag{2}$$

$$\frac{dI_b(t)}{dt} + \theta t I_b(t) = -a e^{bt} \quad ; \qquad L < t < t_2 \tag{3}$$

$$\frac{dI_b(t)}{dt} = -a e^{bt} \quad ; \qquad t_2 < t < \frac{T}{n} \tag{4}$$

$$\frac{dI_v(t)}{dt} = -a e^{bt} \quad ; \qquad 0 < t < T \tag{5}$$

with the boundary conditions:

$$I_b(0) = Q_1 , \quad I_b(L) = S_1 , \quad I_b(t_2) = 0, \quad \text{and} \quad I_v(T) = 0$$

Their solutions are given by

$$I_b(t) = Q_1(1-\theta t) - a\left(t + \frac{bt^2}{2} + \frac{\theta t^2}{2} + \frac{b\theta t^3}{3} - \theta t^2 - \frac{b\theta t^3}{2}\right) \tag{6}$$

$$I_b(t) = a\begin{pmatrix} (L-t) + \frac{b}{2}\left(L^2 - t^2\right) + \frac{\theta}{6}\left(L^3 - t^3\right) + \frac{b\theta}{8}\left(L^4 - t^4\right) \\ -\frac{\theta t^2}{2}(L-t) - \frac{b\theta t^2}{4}\left(L^2 - t^2\right) \end{pmatrix}$$

$$S_1\left(1 + \frac{\theta}{2}\left(L^2 - t^2\right)\right) \tag{7}$$

$$I_b(t) = a\begin{pmatrix} \left(t_2 - t\right) + \frac{b}{2}\left(t_2^2 - t^2\right) + \frac{\theta}{6}\left(t_2^3 - t^3\right) + \frac{b\theta}{8}\left(t_2^4 - t^4\right) \\ -\frac{\theta t^2}{2}\left(t_2 - t\right) - \frac{b\theta t^2}{4}\left(t_2^2 - t^2\right) \end{pmatrix} \tag{8}$$

$$I_b(t) = a\left(\left(t_2 - t\right) + \frac{b}{2}\left(t_2^2 - t^2\right)\right) \tag{9}$$

$$I_v(t) = a\left\{(T-t) + \frac{b}{2}\left(T^2 - t^2\right)\right\} \tag{10}$$

Putting the value of t = t_1 in equations (6) and (7), we get the value of Q_1

$$\begin{aligned}Q_1 = &\frac{a}{(1-\theta t_1)}\begin{pmatrix}(L-t_1)+\frac{b}{2}\left(L^2-t_1^2\right)+\frac{\theta}{6}\left(L^3-t_1^3\right)+\frac{b\theta}{8}\left(L^4-t_1^4\right)\\ -\frac{\theta t^2}{2}(L-t_1)-\frac{b\theta t_1^2}{4}\left(L^2-t_1^2\right)\end{pmatrix}\\ &+\frac{a}{(1-\theta t_1)}\left(t_1+\frac{bt_1^2}{2}+\frac{\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}-\theta t_1^2-\frac{b\theta t_1^3}{2}\right)\\ &+\frac{S_1}{(1-\theta t_1)}\left(1+\frac{\theta}{2}\left(L^2-t_1^2\right)\right)\end{aligned} \tag{11}$$

Putting the value of t = Lin equations (5.7), we get the value of S_1

$$S_1 = a\begin{pmatrix}(t_2-L)+\frac{b}{2}\left(t_2^2-L^2\right)+\frac{\theta}{6}\left(t_2^3-L^3\right)+\frac{b\theta}{8}\left(t_2^4-L^4\right)\\ -\frac{\theta L^2}{2}(t_2-L)-\frac{b\theta L^2}{4}\left(t_2^2-L^2\right)\end{pmatrix} \tag{12}$$

Putting the values of Q1 and S1 in equations (6) and (7) respectively, we get

$$\begin{aligned}I_b(t) = &\frac{a(1-\theta t)}{(1-\theta t_1)}\begin{pmatrix}(L-t_1)+\frac{b}{2}\left(L^2-t_1^2\right)+\frac{\theta}{6}\left(L^3-t_1^3\right)+\frac{b\theta}{8}\left(L^4-t_1^4\right)\\ -\frac{\theta t_1^2}{2}(L-t_1)-\frac{b\theta t_1^2}{4}\left(L^2-t_1^2\right)\end{pmatrix}\\ &+\frac{a\left(1-\frac{\theta}{2}\left(L^2-t_1^2\right)\right)(1-\theta t)}{(1-\theta t_1)}\left(t_1+\frac{bt_1^2}{2}+\frac{\theta t_1^2}{2}+\frac{b\theta t_1^3}{3}-\theta t^2-\frac{b\theta t_1^3}{2}\right)\\ &-a\left(t+\frac{bt^2}{2}+\frac{\theta t^2}{2}+\frac{b\theta t^3}{3}-\theta t^2-\frac{b\theta t^3}{2}\right)\end{aligned} \tag{13}$$

$$\begin{aligned}I_b(t) = &a\begin{pmatrix}(L-t)+\frac{b}{2}\left(L^2-t^2\right)+\frac{\theta}{6}\left(L^3-t^3\right)+\frac{b\theta}{8}\left(L^4-t^4\right)\\ -\frac{\theta t^2}{2}(L-t)-\frac{b\theta t^2}{4}\left(L^2-t^2\right)\end{pmatrix}\\ &+a\left(1+\frac{\theta}{2}\left(L^2-t^2\right)\right)\begin{pmatrix}(t_2-L)+\frac{b}{2}\left(t_2^2-L^2\right)+\frac{\theta}{6}\left(t_2^3-L^3\right)+\frac{b\theta}{8}\left(t_2^4-L^4\right)\\ -\frac{\theta L}{2}(t_2-L)-\frac{b\theta L^2}{4}\left(t_2^2-L^2\right)\end{pmatrix}\end{aligned} \tag{14}$$

BUYER'S RELEVANT COSTS:

Holding Cost:

$$HC_b = n\begin{Bmatrix}x_b\left(\int_0^{t_1} I_b(t)dt+\int_{t_1}^{L} I_b(t)dt+\int_{t_1}^{t_2} I_b(t)dt\right)\\ +y_b\left(\int_0^{t_1} tI_b(t)dt+\int_{t_1}^{L} tI_b(t)dt+\int_{t_1}^{t_2} tI_b(t)dt\right)\end{Bmatrix} \tag{15}$$

Deterioration Cost:

$$DC_b = nC_b\theta\left\{\int_0^{t_1} I_b(t)dt + \int_{t_1}^{L} tI_b(t)dt + \int_{t_1}^{t_2} tI_b(t)dt\right\} \quad (16)$$

Shortage Cost:

$$SC_b = -nC_{2b}\int_{t_2}^{\frac{T}{n}} I_b(t)dt \quad (17)$$

Ordering Cost

$$OC_b = nA_b \quad (18)$$

Sales Revenue:

$$SR_b = np\int_0^{\frac{T}{n}} D(t)dt$$

$$= np\int_0^{\frac{T}{n}} ae^{bt}dt \quad (19)$$

(By neglecting higher power of b and θ)

Total Profit:

$$TP_b = \frac{1}{T}\left[SR_b - HC_b - DC_b - SC_b - OC_b\right] \quad (20)$$

VENDOR'S RELEVANT COSTS:

Holding Cost:

$$HC_v = x_v\left\{\int_0^{T} I_v(t)dt - n\left(\int_0^{t_1} I_b(t)dt + \int_{t_1}^{L} I_b(t)dt + \int_{t_1}^{t_2} I_b(t)dt + \int_{t_2}^{\frac{T}{n}} I_b(t)dt\right)\right\}$$

$$+ y_b\left\{\int_0^{T} tI_v(t)dt - n\left(\int_0^{t_1} tI_b(t)dt + \int_{t_1}^{L} tI_b(t)dt + \int_{t_1}^{t_2} tI_b(t)dt + \int_{t_2}^{\frac{T}{n}} tI_b(t)dt\right)\right\} \quad (21)$$

Ordering cost:

$$OC_v = A_v \quad (22)$$

Sales Revenue:

$$SR_v = C_b\int_0^{T} D(t)dt$$

$$= C_b\int_0^{T} ae^{bt}dt \quad (23)$$

(By neglecting higher power of b and θ)

Total Profit: (24

$$TP_V = \frac{1}{T}\left[SR_V - HC_V - OC_V\right]$$)

WITHOUTCOLLABORATIONANDWITHCOLLABORATIONDECISIONS:

In this section, we discuss two situations: when buyer and vendor take decisions independently and jointly under supply chain strategy.

Situation-I: Buyer and vendor take decision without collaboration

Here the buyer and vendor make decision without collaboration Buyer's maximum profit TP_b can be determined by following conditions

$$\frac{dTP_b}{dL}=0\ \frac{dTP_b}{dt_2}=0 \text{ and } \frac{dTP_b}{dT_b}=0 \text{ where } T_b=\frac{T}{n} \quad (25)$$

Provided it satisfies the condition

$$\begin{vmatrix} \frac{\partial^2 TP_b}{\partial t_2^2} & \frac{\partial^2 TP_b}{\partial t_2 \partial T} \\ \frac{\partial^2 TP_b}{\partial T \partial t_2} & \frac{d^2 TP_b}{\partial T^2} \end{vmatrix} > 0$$

This solution (n,t2,T) maximizes TPv.

Then the total profit without collaboration is given by; (26)

TP=max (TP_b+TP_v). (27)

Situation-II: Buyer and vendor take decision with collaboration

Here the buyer and vendor make decision with collaboration

Joint total profit and optimal values of t2 and T are obtained by the following conditions

$$\frac{dTP}{dL}=0\ \frac{dTP}{dt_2}=0 \text{ and } \frac{dTP}{dT}=0 \text{ for T and } t_2 \text{ provided} \quad (28)$$

Provided it satisfies the condition.

$$\begin{vmatrix} \frac{\partial^2 TP}{\partial t_2^2} & \frac{\partial^2 TP}{\partial t_2 \partial T} \\ \frac{\partial^2 TP}{\partial T \partial t_2} & \frac{d^2 TP}{\partial T^2} \end{vmatrix} > 0 \quad (29)$$

Where total profit (TP) with collaboration is given by;

TP=TP_b +TP_v. (30)

NUMERICAL EXAMPLE:

In order to illustrate our proposed model, we consider a=1000, b = 0.05, x_b = 10, y_b = 0.03, x_v = 8, y_v = 0.01, A_b = 140, A_v=1500, C_b=35, C_{2b}=15, p=45, v_1=0.4, v_2=0.7 in appropriate units. The optimal values of t2, T and profits for buyer and vendor are given in Table-1.The second order conditions given in equation (26) and equation (29) are also satisfied. The graphical representations of the concavity of the profits for independent and joint profits are also shown (Figures-2to5).

The optimal total profit TP = Rs. 74629.34 at n=3 for buyer's profit TPb*=Rs.43915.35, T*=0.7684, L=0.1026, t_2*=0.14652 and TPv=30713.98 when buyer and vendor take decision without collaboration. While when buyer and vendor take joint decision then the optimal total profit TP*= Rs. 75148.65 at n=1, T*= 0.6571, L=0.3995,t2* =0.5708,with buyer's profit TP_b =Rs.42556.90 andTP_v= Rs.32591.75.

Table-1

The optimal solutions

	Without	With
N	3	1
L	0.1026	0.3995
t2	0.1465	0.5708
T	0.7684	0.6571
Buyer's Profit	43915.35	42556.90
Vendor's Profit	30713.98	32591.75
Total Profit	74629.34	75148.65

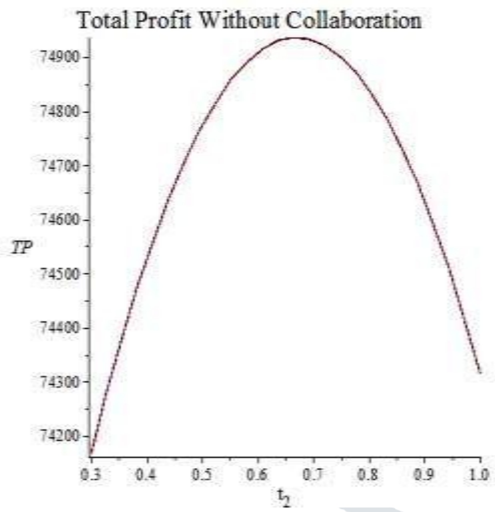

Figure-2

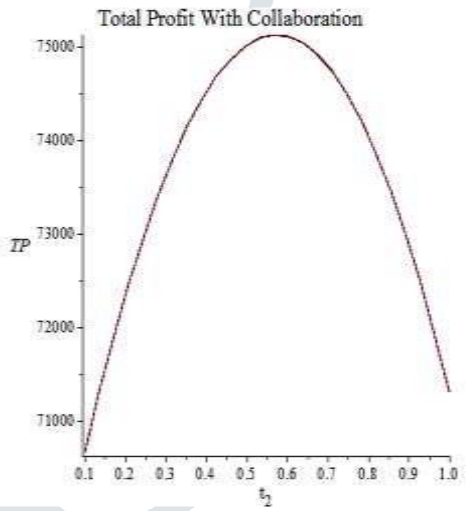

Figure-3

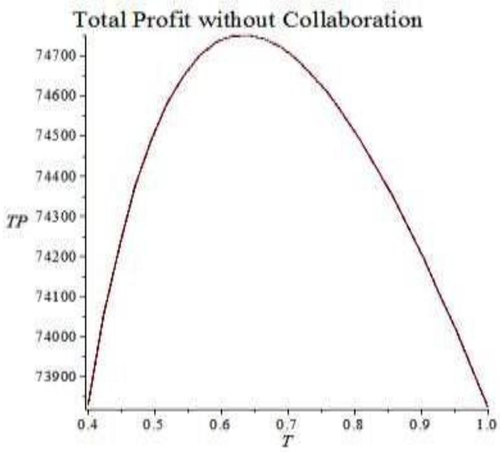

Figure -4

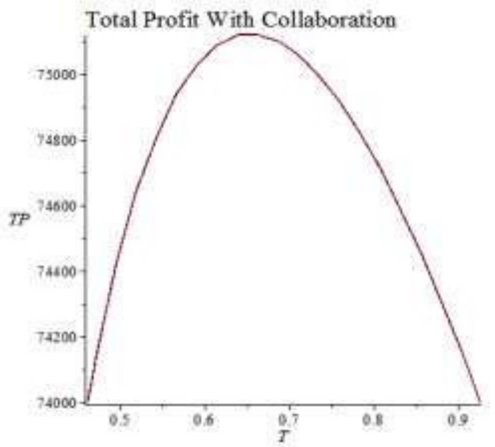

Figure -5

SENSITIVITYANALYSIS:

Table-2

Sensitive Analysis

	Parameters	Without Collaboration			With Collaboration		
		TPb	TPv	TP	TPb	TPv	TP
-20%	a	35028	24158	59187	33819	25827	59646
-10%		39471	27430	66901	38185	29206	67390
10%		48363	34009	82371	46935	35983	82919
20%		52813	37312	90125	51319	39380	90699
-20%	θ	43926	30801	74726	42543	32676	75219
-10%		43920	30595	74515	42550	32633	75183
10%		43910	30711	74621	42563	32551	75115
20%		43905	30707	74613	42570	32512	75081
-20%	Ab	44031	30738	74769	42617	32575	75191
-10%		43971	30732	74704	42587	32583	75170
10%		43862	30687	74549	42527	32600	75127
20%		43811	30690	74501	42498	32608	75106
-20%	xb	43915	30878	74793	42427	33266	75693
-10%		43960	30926	74886	42527	32893	75420
10%		43875	30686	74561	42577	32333	74909
20%		43840	30657	74496	42590	32106	74696
-20%	Av	43915	31168	75084	42745	32882	75627
-10%		43915	30909	74825	42649	32733	75382
10%		43915	30896	74812	42468	32457	74925
20%		43915	30047	73962	42383	32327	74710
-20%	xv	43915	31315	75230	42784	32541	75325
-10%		43915	31014	74930	42680	32547	75228
10%		43915	30414	74329	42414	32674	75087
20%		43915	30113	74028	42250	32793	75043

Sensitive analysis is carried out by taking the values of given parameters a, Ab, Av, xb, xv and θ respectively. This analysis is deliberating by changing one parameter at a time and the reaming parameters are kept constant. We observe that total profit increases when buyer and vendor take joint decision instead as compared to independent decision (Table-2). When a increases/decreases then total profit will increase/decrease, while if Ab, xb, xv, Av and θ increase/decrease then total profit will decrease/increase in independent and joint decision.

CONCLUSION:

The result shows that the optimal cycle time is significantly decreased and total profit significantly increased when buyer and vendor consider joint decision policy under supply chain as compared to independent decision taken by buyer and vendor. We also observe that the vendor's profit is increased and number of times order placed by buyer during cycle time is also decreased when buyer and vendor take joint decision.

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