



DAMPED VIBRATIONS OF AN ISOTROPIC RECTANGULAR PLATE OF PARABOLICALLY VARYING THICKNESS

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Abstract

Damped vibrations of an isotropic rectangular plate of parabolically varying thickness have been studied. The fourth order differential equation of motion is solved by the method of Frobenius. The Frequencies corresponding to the first two modes of vibration are computed for the rectangular plate with clamped-simply supported-clamped- simply supported (C-SS-C-SS), and clamped-simply supported-simply supported-simply supported (C-SS-SS-SS) edge conditions for different values of taper constant, and damping parameter. Effect of damping on natural frequencies of a rectangular plate of parabolically varying thickness has been observed.

Keywords— isotropic, transverse displacement, elastic foundation, deflection functions, damping, radial coordinates, Flexural rigidity, Poisson ratio

1 Introduction

The object of the work presented in this chapter is to study the effect on frequencies of an isotropic rectangular plate of parabolically varying thickness.

Leissa(3) has given the method for analyzing the vibration of rectangular plates. Several authors have studied the vibration problems of rectangular plate of uniform thickness using different boundary conditions and different method has finite difference method, Series method, Rayleigh Ritz method etc. the most accurate results were presented by Leissa on rectangular plate. Young(2) studied the vibration of rectangular plate Ritz methods. Jain and Soni(1) studied the free vibration on rectangular plate of parabolically varying thickness. In this chapter we have studied the damped vibration of an isotropic rectangular plate of parabolically varying thickness.

Keywords— isotropic, transverse displacement, elastic foundation, deflection functions, damping, radial coordinates, Flexural rigidity, Poisson ratio

2 Equation of Motion

The equation of motion of an element of a plates in transverse direction are

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (1)$$

$$\left. \begin{aligned} \text{with } Q_x &= \frac{\partial M_x}{\partial x} + \frac{\partial M_{yx}}{\partial y} \\ \text{and } Q_y &= \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} \end{aligned} \right\} \quad (2)$$

eliminating Q_x and Q_y from equation (1), one obtains

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (3)$$

Here $h = h(x, y)$

But for the plate $M_{xy} = -M_{yx}$, therefore,

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} + \frac{2\partial^2 M_{yx}}{\partial x \partial y} = \rho h \frac{\partial^2 w}{\partial t^2} \quad (4)$$

where

$$\left. \begin{aligned} M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right) \\ M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \\ M_{yx} &= -(1-\nu) D \frac{\partial^2 w}{\partial x \partial y} \end{aligned} \right\} \quad (5)$$

Q_x and Q_y are stress resultants, M_x , M_y and M_{xy} are moment resultants per unit length, ρ is the mass density per unit volume, ν is the Poisson's ratio, h is the thickness of the plate and E is the Young's modulus of the plate material.

In the case of non-homogeneous plates of variable thickness, E , ρ and h are functions of x and/or y . Now substituting the values of M_x , M_y and M_{xy} from (6.5) in to (6.4) one gets

$$\nabla^2 (D \nabla^2 W) - (1-\nu) \left(\frac{\partial^2 D}{\partial y^2} \cdot \frac{\partial^2 W}{\partial x^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \cdot \frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 D}{\partial x^2} \cdot \frac{\partial^2 W}{\partial y^2} \right) + \rho h \frac{\partial^2 W}{\partial t^2} = 0 \quad (6)$$

where

$$D = D(x, y) = \frac{E(x, y) h^3(x, y)}{12(1-\nu^2)}, \quad \rho = \rho(x, y) \text{ and } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

When the effect of damping is introduced, the differential equation (6.6) changes to

$$\nabla^2 (D \nabla^2 W) - (1-\nu) \left(\frac{\partial^2 D}{\partial y^2} \cdot \frac{\partial^2 W}{\partial x^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \cdot \frac{\partial^2 W}{\partial x \partial y} + \frac{\partial^2 D}{\partial x^2} \cdot \frac{\partial^2 W}{\partial y^2} \right) + \rho h \frac{\partial^2 W}{\partial t^2} + K \frac{\partial W}{\partial t} = 0 \quad (7)$$

where K = Damping constant, W = Transverse deflection, D = Flexural rigidity at any point of the plate

$$D = \frac{E h^3(x, y)}{12(1-\nu^2)}$$

Let the two opposite edges $y=0$ and $y=b$ of the plate be simply supported and thickness varies parabolically x -axis along the length i.e. in the direction x axis. And for simplicity one way assume that mass density ρ are also function of x -only. Thus, h and ρ are Independent of y , i.e.

$$h = h(x), \text{ and } \rho = \rho(x)$$

For a harmonic solution the deflection function W satisfying the condition at $y=0$ and $y=b$ is

$$w(x, y, t) = \bar{W}(x) \sin \frac{m\pi y}{b} e^{-\gamma t} \cos pt$$

where P = circular frequency of vibration, m = positive integer.

Substituting for w and D in the partial differential equation (6.7), one gets,

$$\left[h^3 \frac{\partial^4 w}{\partial x^4} + 6h^2 \frac{\partial h}{\partial x} \frac{\partial^3 w}{\partial x^3} + \left\{ 6h \left(\frac{\partial h}{\partial x} \right)^2 + 3h^2 \frac{\partial^2 h}{\partial x^2} - 2h^3 \frac{m^2 \pi^2}{b^2} \right\} \frac{\partial^2 w}{\partial x^2} - 6h^2 \frac{m^2 \pi^2}{b^2} \frac{\partial h}{\partial x} \frac{\partial w}{\partial x} \right] \cos pt + \left[\frac{m^4 \pi^4}{b^4} h^3 - 3\nu \frac{m^2 \pi^2}{b^2} \left\{ 2h \left(\frac{\partial h}{\partial x} \right)^2 + h^2 \frac{\partial^2 h}{\partial x^2} \right\} \right] w \cos pt + \left[\frac{12(1-\nu^2)\rho h}{E} \{ (\gamma^2 - p^2) \cos pt + 2\gamma \sin pt \} + \frac{12(1-\nu^2)}{E} K \{ -p \sin pt - \gamma \cos pt \} \right] w = 0 \quad (8)$$

Introducing the non-dimensional variables

$$H = \frac{h}{a}, X = \frac{x}{a}, \bar{W} = \frac{w}{a}, \bar{E} = \frac{E}{a} \text{ and } \bar{\rho} = \frac{\rho}{a}$$

The equation (6.8) reduces to

$$\begin{aligned} & \left[H^3 \frac{\partial^4 \bar{W}}{\partial X^4} + 6H^2 \frac{\partial^3 \bar{W}}{\partial X^3} \frac{\partial H}{\partial X} + \left\{ 6H \left(\frac{\partial H}{\partial X} \right)^2 + 3H^2 \frac{\partial^2 H}{\partial X^2} - 2H^3 \beta^2 \right\} \frac{\partial^2 \bar{W}}{\partial X^2} \right. \\ & \left. - 6H^2 \beta^2 \frac{\partial H}{\partial X} \frac{\partial \bar{W}}{\partial X} \right] \cos pt + \left[\beta^4 H^3 - 3\nu \beta^2 \left\{ 2H \left(\frac{\partial H}{\partial X} \right)^2 + H^2 \frac{\partial^2 H}{\partial X^2} \right\} \right] \\ & \bar{W} \cos pt + \left[\frac{12(1-\nu^2)}{\bar{E}} \bar{\rho} H \{ (\gamma^2 - p^2) \cos pt + 2\gamma p \sin pt \} \right. \\ & \left. + \frac{12(1-\nu^2)}{\bar{E}} K \{ -p \sin pt - \gamma \cos pt \} \right] \bar{W} = 0 \end{aligned} \quad (9)$$

where,

$$\beta^2 = m^2 \pi^2 \left(\frac{a}{b} \right)^2$$

Let the thickness of the plate varies Parabolically i.e.

$$H = H_0 (1 - \alpha X^2) \quad (10)$$

Where $H_0 = (H)_{x=0}$ and α is the taper constant, after substituting the equation(10) in the equation (9), equating the coefficient of $\sin pt$ and $\cos pt$ independently to zero, one obtains

$$\begin{aligned} & (1 - \alpha X^2)^4 \frac{\partial^4 \bar{W}}{\partial X^4} - 12\alpha X (1 - \alpha X^2)^3 \frac{\partial^3 \bar{W}}{\partial X^3} + \left\{ 24\alpha^2 X (1 - \alpha X^2)^2 - 2\beta^2 (1 - \alpha X^2)^4 \right\} \frac{\partial^2 \bar{W}}{\partial X^2} - \\ & 6\alpha (1 - \alpha X^2)^3 \frac{\partial^2 \bar{W}}{\partial X^2} + 12\beta^2 \alpha X (1 - \alpha X^2)^3 \frac{\partial \bar{W}}{\partial X} + \left\{ \beta^4 (1 - \alpha X^2)^4 - 24\beta^2 \nu \alpha^2 X^2 (1 - \alpha X^2)^2 \right\} \bar{W} + \\ & \{ 6\beta^2 \nu \alpha (1 - \alpha X^2)^3 \} \bar{W} - \{ D_k^2 (I^*)^2 + \Omega^2 I^* (1 - \alpha X^2)^2 \} \bar{W} = 0 \end{aligned} \quad (6.11)$$

Whereas

$$D_K = \frac{3(1-\nu^2)K^2}{\bar{\rho}\bar{E}}, \quad I^* = \frac{1}{H_0}, \quad \Omega^2 = \frac{12(1-\nu^2)a^2\bar{\rho}P^2}{\bar{E}}, \quad C^* = \frac{1}{H_0^3}$$

Where, p = circular frequency, Ω = frequency parameter, D_K = damping parameter

3 Solution

A series solution for \bar{W} is then assumed to be in the form

$$\bar{W}(x) = \sum_{\lambda=0}^{\infty} a_{\lambda} x^{C+\lambda}, \quad a_0 \neq 0, \quad (12)$$

On substituting the series expression (12) in the equation (11) one gets,

$$\begin{aligned} & \sum_{\lambda=0}^{\infty} a_{\lambda} F_1(\lambda) x^{C+\lambda-4} + \sum_{\lambda=0}^{\infty} a_{\lambda} F_2(\lambda) x^{C+\lambda-2} + \\ & \sum_{\lambda=0}^{\infty} a_{\lambda} F_3(\lambda) x^{C+\lambda} + \sum_{\lambda=0}^{\infty} a_{\lambda} F_4(\lambda) x^{C+\lambda+2} + \sum_{\lambda=0}^{\infty} a_{\lambda} F_5(\lambda) x^{C+\lambda+4} + \\ & \sum_{\lambda=0}^{\infty} a_{\lambda} F_6(\lambda) x^{C+\lambda+6} + a_{\lambda} F_7(\lambda) x^{C+\lambda+8} = 0 \end{aligned} \quad (13)$$

For series expression (6.12) to be the solution the coefficient of different powers of W in the equation (6.13) must be identically zero. Thus by equating the coefficient of the lowest power of X to zero, one get the identical equation

$$\begin{aligned} & a_0 F_1(0) = 0 \quad \text{Since } a_0 \neq 0 \quad \text{i.e. } F_1(0) = 0 \\ & F_1(0) = N_1(1) b_0(3) = 0 \end{aligned}$$

$$1. c(c-1)(c-2)(c-3) = 0$$

$c = 0, 1, 2, 3$

Now on equating the coefficients of the higher powers of X to be zero, it is found that the constants a_1, a_2 and a_3 are indeterminate for $c=0$, so these can be taken as arbitrary constants along with a_0 . The remaining constants a_λ ($\lambda = 4, 5, 6, \dots$) are all obtained in terms of a_0, a_1, a_2 and a_3 the remaining unknown constants are determined from recurrence relation.

$$a_\lambda = a_0 A_\lambda + a_1 B_\lambda + a_2 C_\lambda + a_3 D_\lambda \quad (\lambda = 4, 5, 6, \dots) \quad (14)$$

$$D_6 F_6(6) + D_4 F_7(4)]$$

and the remaining A_λ 's & B_λ 's ($\lambda = 17, 18, 19, \dots$) are determined from the recursion formula.

$$C_{(16+\lambda)} = \frac{-1}{F_1(16+\lambda)} [C_{(14+\lambda)} F_2(14+\lambda) + C_{(12+\lambda)} F_3(12+\lambda) + C_{(10+\lambda)} F_4(10+\lambda) + C_{(8+\lambda)} F_5(8+\lambda) + C_{(6+\lambda)} F_6(6+\lambda) + C_{(4+\lambda)} F_7(4+\lambda)]$$

$$D_{(16+\lambda)} = \frac{-1}{F_1(16+\lambda)} [D_{(14+\lambda)} F_2(14+\lambda) + D_{(12+\lambda)} F_3(12+\lambda) + D_{(10+\lambda)} F_4(10+\lambda) + D_{(8+\lambda)} F_5(8+\lambda) + D_{(6+\lambda)} F_6(6+\lambda) + D_{(4+\lambda)} F_7(4+\lambda)]$$

The solution for \bar{W} , corresponding to $c=0$ is

$$\bar{W} = a_0 \left[1 + \sum_{\lambda=4}^{\infty} A_\lambda X^\lambda \right] + a_1 \left[X + \sum_{\lambda=4}^{\infty} B_\lambda X^\lambda \right] + a_2 \left[X^2 + \sum_{\lambda=4}^{\infty} C_\lambda X^\lambda \right] + a_3 \left[X^3 + \sum_{\lambda=4}^{\infty} D_\lambda X^\lambda \right] \quad (15)$$

It is evident that no new solution will arise corresponding to other values of c i.e. for $c=1, 2, 3$. Solution corresponding to these values of c are already included in the solution corresponding to $c=0$.

4 Convergence of the Solution

The test the convergence of the solution (6.15) the technique used by Lamb has been applied. Writing the recurrence relation using (6.13) as

$$\frac{a_{\lambda+12}}{a_\lambda} + \frac{a_{\lambda+10}}{a_\lambda} \cdot \frac{F_2(\lambda+10)}{F_1(\lambda+12)} + \frac{a_{\lambda+8}}{a_\lambda} \cdot \frac{F_3(\lambda+8)}{F_1(\lambda+12)} + \frac{a_{\lambda+6}}{a_\lambda} \cdot \frac{F_4(\lambda+6)}{F_1(\lambda+12)} + \frac{a_{\lambda+4}}{a_\lambda} \cdot \frac{F_5(\lambda+4)}{F_1(\lambda+12)} + \frac{a_{\lambda+2}}{a_\lambda} \cdot \frac{F_6(\lambda+2)}{F_1(\lambda+12)} + \frac{F_7(\lambda)}{F_1(\lambda+12)} = 0 \quad (6.16)$$

$$\mu^{12} + \mu^{10} \frac{F_2(\lambda+10)}{F_1(\lambda+12)} + \mu^8 \frac{F_3(\lambda+8)}{F_1(\lambda+12)} + \mu^6 \frac{F_4(\lambda+6)}{F_1(\lambda+12)} + \mu^4 \frac{F_5(\lambda+4)}{F_1(\lambda+12)} + \mu^2 \frac{F_6(\lambda+2)}{F_1(\lambda+12)} + \frac{F_7(\lambda)}{F_1(\lambda+12)} = 0$$

Now taking the limit as $\lambda \rightarrow \infty$ one gets,

$$\mu^6 + N_1(1)\mu^5 + N_2(1)\mu^4 + N_3(1)\mu^3 + N_4(1)\mu^2 = 0$$

i.e. $\mu^2(\mu - \alpha)^4 = 0 \quad (17)$

$$\lim_{\lambda \rightarrow \infty} \frac{a_{\lambda+1}}{a_\lambda}$$

where $\mu =$

The roots of the equation (17) are :

$$\mu = 0, 0, \alpha, \alpha, \alpha, \alpha.$$

Thus the solution (15) is uniformly convergent in the interval $0 \leq X \leq 1$ when $|\mu| < 1$. Hence the solution is convergent for all $|\alpha| < 1$.

5 Boundary Conditions

The following combinations of boundary conditions at the edges $x=0$ and $x=1$ have been considered. While the other two edges $y=0$ and $y=1$ are simply supported in all cases.

(C-SS-C-SS) plate

Clamped at $X=0$ and simply supported at $X=1$.

(C-SS-SS-SS) Plate

Clamped at $X=0$ and simply supported at $X=1$. The boundary conditions for different edge conditions are as follows:

Clamped Edge Conditions (C-SS-C-SS)

At a clamped edge,

$$\frac{\partial W}{\partial X} = 0$$

$$W=0,$$

Simply-Supported Edge Conditions (C-SS-SS-SS)

At a simply supported edge,

$$W=0, \quad M_x = 0$$

(C-SS-C-SS) Plate

For a clamped plate the boundary conditions are :

At $X = 0$,

$$\bar{W} = \frac{d \bar{W}}{d X} = 0$$

At $X = 1$,

$$\bar{W} = \frac{d \bar{W}}{d X} = 0 \quad (18)$$

Applying the boundary conditions (18) to the solution (15), one gets

$$a_0 = a_1 = 0$$

$$a_2 \left[1 + \sum_{\lambda=4}^{\infty} C_{\lambda} \right] + a_3 \left[1 + \sum_{\lambda=4}^{\infty} D_{\lambda} \right] = 0$$

$$a_2 \left[2 + \sum_{\lambda=4}^{\infty} \lambda C_{\lambda} \right] + a_3 \left[3 + \sum_{\lambda=4}^{\infty} \lambda D_{\lambda} \right] = 0 \quad (19)$$

Eliminating the unknown constant a_2 and a_3 , one obtains the frequency equation for (C-SS-C-SS) plate as

$$\begin{vmatrix} V_1(\Omega) & V_2(\Omega) \\ V_3(\Omega) & V_4(\Omega) \end{vmatrix} = 0 \quad (20)$$

Whereas

$$V_1(\Omega) = 1 + \sum_{\lambda=4}^{\infty} C_{\lambda}, \quad V_2(\Omega) = 1 + \sum_{\lambda=4}^{\infty} D_{\lambda}, \quad V_3(\Omega) = 2 + \sum_{\lambda=4}^{\infty} \lambda C_{\lambda}, \quad V_4(\Omega) = 3 + \sum_{\lambda=4}^{\infty} \lambda D_{\lambda} \quad (21)$$

(C-SS-SS-SS) Plates

$$\bar{W} = \frac{d \bar{W}}{d X} = 0$$

For a (C-SS-SS-SS) plate the boundary conditions are, at $X=0$,

$$\bar{W} = \frac{d^2 \bar{W}}{d X^2} = 0$$

at $X=1$,

Applying the boundary condition (22) to the solution (6.15), one finds $a_0 = 0$, $a_1 = 0$

$$a_2 \left[1 + \sum_{\lambda=4}^{\infty} C_{\lambda} \right] + a_3 \left[1 + \sum_{\lambda=4}^{\infty} D_{\lambda} \right] = 0$$

$$a_2 \left[2 + \sum_{\lambda=4}^{\infty} \lambda(\lambda-1) C_{\lambda} \right] + a_3 \left[6 + \sum_{\lambda=4}^{\infty} \lambda(\lambda-1) D_{\lambda} \right] = 0 \quad (23)$$

Eliminating the unknown constants a_2 and a_3 one gets the frequency equation

$$\begin{vmatrix} V_1(\Omega) & V_2(\Omega) \\ V_5(\Omega) & V_6(\Omega) \end{vmatrix} = 0 \quad (24)$$

whereas

$$V_1(\Omega) = 1 + \sum_{\lambda=4}^{\infty} C_{\lambda}, \quad V_2(\Omega) = 1 + \sum_{\lambda=4}^{\infty} D_{\lambda},$$

$$V_5(\Omega) = 2 + \sum_{\lambda=4}^{\infty} \lambda(\lambda-1) C_{\lambda}, \quad V_6(\Omega) = 6 + \sum_{\lambda=4}^{\infty} \lambda(\lambda-1) D_{\lambda} \quad (25)$$

6 Result and Discussion

Numerical results for an isotropic rectangular plate of parabolically varying thickness resting on elastic foundation have been computed from the equation (20) and (24) using computer technology. In all the cases considered the Poisson's ratio has been assumed to remain constant and it has been taken to be 0.03. Terms of series up to an accuracy of 10^{-8} in their absolute values have been retained. Frequency parameter corresponding to first two modes of vibration of a clamped-simply supported-clamped-simply supported (C-SS-C-SS) and clamped-simply supported-simply supported-simply supported (C-SS-SS-SS) isotropic rectangular plate has been computed for different values of taper constant and damping parameter have been computed.

All the results are graphically shown in figures (6.1) to (6.8). The results up to accuracy of 10^{-4} have been given in the tables.

Verification of work is obtained if allowing the Damping parameter to be zero, the problem reduces to a well known problem of a rectangular plate of parabolically varying thickness. The results with damping parameter is equal to zero compared with very well known the results obtained by Jain and Soni [1]

Figure (1) shows the effect of variation of a taper constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $D_K = 0, h = .1$ and $D_K = .001, h = .1$) with clamped-simply supported -clamped-simply supported (C-SS-C-SS) edge conditions. From figure it is observed that the first mode of vibration increasing in frequency parameter with the increasing of taper constant for clamped-simply supported-clamped-simply supported plates.

Figure (2) shows the effect of variation of a taper constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $D_K = 0, h = .1$ and $D_K = .001, h = .1$) with clamped-simply supported -clamped-simply supported (C-SS-C-SS) edge conditions. From figure it is observed that the second mode of vibration will be increases in frequency parameter with the increasing of taper constant for clamped-simply supported-clamped-simply supported plates.

Figure (3) shows the effect of variation of a taper constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $D_K = 0, h = .1$ and $D_K = .001, h = .1$) with clamped-simply supported-simply supported (C-SS-SS-SS) edge conditions. From figure it is observed that the first mode of vibration increasing in frequency parameter with the increasing of taper constant for clamped- simply supported-simply supported-simply supported plates.

Figure (4) shows the effect of variation of a taper constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $D_K = 0, h = .1$ and $D_K = .001, h = .1$) with clamped- simply supported-simply supported-simply supported (C-SS-SS-SS) edge conditions. From figure it is observed that the second mode of vibration will be decreases in frequency parameter with the increasing of taper constant for clamped- simply supported-simply supported-simply supported plates.

Figure (5) shows the effect of variation of damping constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $\alpha = 0, h = .1$ and $\alpha = .001, h = .1$) with clamped-simply supported- clamped-simply supported (C-SS-C-SS) edge conditions. From figure it is observed that the first mode of vibration sharply decreases in the frequency parameter with the increasing of damping parameter for the clamped-simply supported- clamped-simply supported plates.

Figure (6) shows the effect of variation of damping constant on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $\alpha = 0, h = .1$ and $\alpha = .001, h = .1$) with clamped-simply supported- clamped-simply supported (C-SS-C-SS) edge conditions. From figure it is observed that the second mode of vibration decreases in the frequency parameter with the increasing of damping parameter for the clamped-simply supported- clamped-simply supported plates.

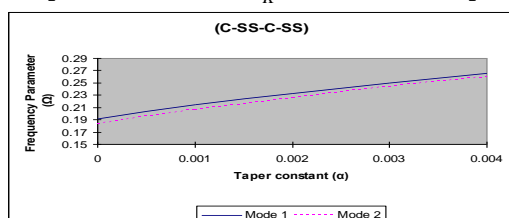
Figure (7) shows the effect of variation of damping parameter on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $\alpha = 0, h = .1$ and $\alpha = .001, h = .1$) with clamped- simply supported-simply supported-simply supported (C-SS-SS-SS) edge conditions. From figure it is observed that the first mode of vibration decreases in the frequency parameter with the increasing of damping parameter for the clamped- simply supported-simply supported-simply supported plates.

Figure (8) shows the effect of variation of damping parameter on the frequency parameter for a rectangular plate of parabolically varying thickness (i.e., for $\alpha = 0, h = .1$ and $\alpha = .001, h = .1$) with clamped- simply supported-simply supported-simply supported (C-SS-SS-SS) edge conditions. From figure it is observed that the second mode of vibration decreasing in the frequency parameter with the increasing of damping parameter for the clamped- simply supported-simply supported-simply supported plates.

FIGURE = 1

($H=0.03, \nu=0.3, \frac{a}{b}=0.25$)

Graph _____ = ($D_K=0.0$) and Graph ----- = ($D_K=0.001$)

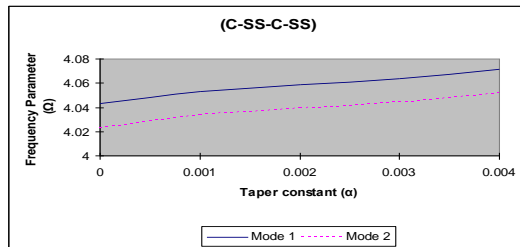


Variation of Ω for the first mode of vibration of a damped (C-SS-C-SS) rectangular plate of parabolically varying thickness for different values of taper constant.

FIGURE = 2

$$(H=0.03, \nu=0.3, \frac{a}{b}=0.25)$$

Graph _____ = ($D^K=0.0$) and Graph ----- = ($D^K=0.001$)

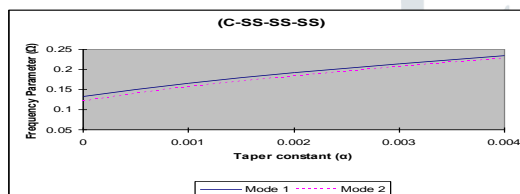


Variation of Ω for the second mode of vibration of a damped (C-SS-C-SS) rectangular plate of parabolically varying thickness for different values of taper constant.

FIGURE = 3

$$(H=0.03, \nu=0.3, \frac{a}{b}=0.25)$$

Graph _____ = ($D^K=0.0$) and Graph ----- = ($D^K=0.001$)

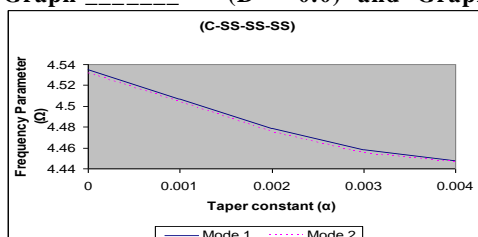


Variation of Ω for the first mode of vibration of a damped (C-SS-SS-SS) rectangular plate of parabolically varying thickness for different values of taper constant.

FIGURE = 4

$$(H=0.03, \nu=0.3, \frac{a}{b}=0.25)$$

Graph _____ = ($D^K=0.0$) and Graph ----- = ($D^K=0.001$)

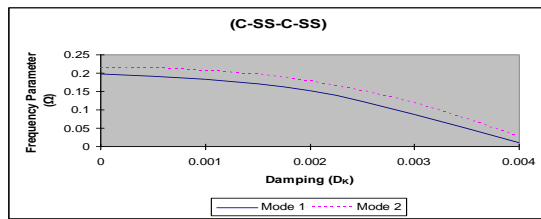


Variation of Ω for the second mode of vibration of a damped (C-SS-SS-SS) rectangular plate of parabolically varying thickness for different values of taper constant.

FIGURE 5

$$(H=0.03, \nu=0.3, \frac{a}{b}=0.25)$$

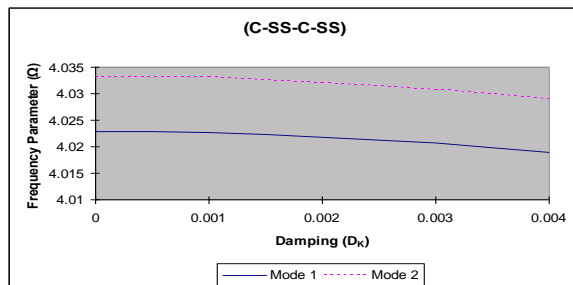
Graph _____ = ($\alpha=0.0$) and Graph ----- = ($\alpha=0.001$)



Variation of Ω for the first mode of vibration of a damped (C-SS-C-SS) rectangular plate of parabolically varying thickness for different values of damping parameter.

FIGURE = 6

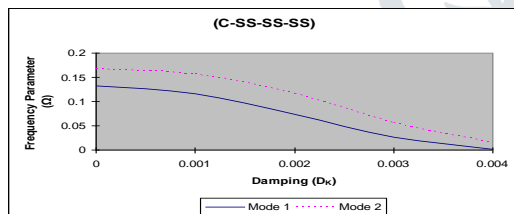
$\frac{a}{b} = 0.25$
 $(H=0.03, \nu=0.3)$
 Graph _____ = $(\alpha=0.0)$ and Graph ----- = $(\alpha=0.001)$



Variation of Ω for the second mode of vibration of a damped (C-SS-C-SS) rectangular plate of parabolically varying thickness for different values of damping parameter.

FIGURE = 7

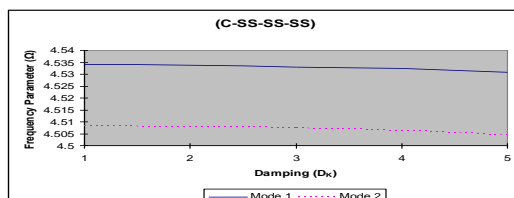
$\frac{a}{b} = 0.25$
 $(H=0.03, \nu=0.3)$
 Graph _____ = $(\alpha=0.0)$ and Graph ----- = $(\alpha=0.001)$



Variation of Ω for the first mode of vibration of a damped (C-SS-SS-SS) rectangular plate of parabolically varying thickness for different values of damping parameter.

FIGURE = 8

$\frac{a}{b} = 0.25$
 $(H=0.03, \nu=0.3)$
 Graph _____ = $(\alpha=0.0)$ and Graph ----- = $(\alpha=0.001)$



Variation of Ω for the second mode of vibration of a damped (C-SS-SS-SS) rectangular plate of parabolically varying thickness for different values of damping parameter.

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