JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

A STUDY ON PROPERTIES OF $Nrc(s)$-CLOSED SETS WITH $Nrc(s)$-CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACES.

J. Mugaibullah,
Assistant Professor, Department of Mathematics, Pollachi College of Arts and Science, Poosaripatti (Pollachi), Tamilnadu, India

N. Rajalakshmi,
Assistant Professor, Department of Mathematics, Pollachi College of Arts and Science, Poosaripatti (Pollachi), Tamilnadu, India

ABSTRACT: The main purpose of this paper is to defined the nano sets called nano $rc(s)$-closed set and in nano topological spaces. Also we discuss its properties in nano topological spaces. Then we have continued research work and introduced a nano continuous function called nano $rc(s)$ continuous function in nano topological spaces. In future, we will study the concept of connectedness and kernel etc.

KEYWORDS: $Nrc(s)$- closed sets, $Nrc(s)$-continuous functions.

1.INTRODUCTION:

In 2014, K. Bhuvaneswari and K. Mythili Gnanapriya had introduced nano generalized closed sets in nano topological spaces. Nano c(s)-sets in nano topological spaces was introduced S. Visalakshi and A. Pushpalatha in 2020. In this dissertation we introduced to study about properties of $Nrc(s)$- closed sets with $Nrc(s)$-continuous functions in Nano Topological spaces.

Nano semi generalized closed (briefly Nsg - closed set [ ] if $Nscl(A) \subseteq U$ whenever $A \subseteq U$.
U is a nano semi open set in $(U, \tau_R(X))$

Nano regular generalized closed (briefly Nrg – closed) sets [ ] if $Nrcl \subseteq U$ whenever $A \subseteq U$ .and U is a nano regular open set in $(U, \tau_R(X))$

Nano semi c(s) generalized closed (briefly $Nsc(s)g$ – closed set [ ] if $Nscl \subseteq U$. whenever $A \subseteq U$ and U is a nano c(s) set.

Nano semi c* generalized closed (briefly Nsc^*g – closed set[ ] if $Nscl(A) \subseteq U$ .whenever $A \subseteq U$ and U is a nano c* set.

2. PRELIMINARIES

Definition 2.1

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernibility with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$. Then

(i) The lower approximation of X with respect to R is the set of all object, which can be certain classified as X with respect to R and its is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all object, which can be classified neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$ That is $B_R(X) = U_R(X)$ - $L_R(X)$

Definition 2.2

(i) nano continuous if $f^{-1}(A)$ is nano closed set in $(U, \tau_R(X))$ for every nano closed set (A) in $(V, \tau_R(Y))$.

(ii) nano α-continuous (briefly $N\alpha$-continuous) [ ] if for every nano closed set A in $(V, \tau_R(Y))$ its inverse image $f^{-1}(A)$ is nano α-closed in $(U, \tau_R(X))$.

(iii) nano regular continuous (briefly Nr-continuous) [ ] if for every nano closed set A in $(V, \tau_R(Y))$ its inverse image $f^{-1}(A)$ is nano regular closed in $(U, \tau_R(X))$.

(iv) nano semi generalized continuous (briefly Nsg-continuous) [ ] if for every nano closed set A in $(V, \tau_R(Y))$ its inverse image $f^{-1}(A)$ is nano semi generalised closed set in $(U, \tau_R(X))$.

3. PROPERTIES OF $Nrc(s)$-CLOSED SETS IN NANO TOPOLOGICAL SPACES.

Theorem 3.1. The intersection of two $Nrc(s)$-closed sets are also $Nrc(s)$-closed sets in nano topological spaces.

Proof : Let both the sets F and G be $Nrc(s)$-closed sets in $(U, \tau_R(X))$. Let V be $Nrc(s)$-closed set with $F \subseteq V$ and $G \subseteq V$. Then $F \cap G \subseteq V$ since F and G are $Nrc(s)$-closedsets, $Nrcl(F) \subseteq V$ and $Nrcl(G) \subseteq V$. Hence $Nrcl(F \cap G) \subseteq Nrcl(F) \cap Nrcl(G) \subseteq V$, therefore $F \cap G$ is $Nrc(s)$-closed set in $(U, \tau_R(X))$.

Remark 3.2. The union of two $Nrc(s)-$ closed sets need not be a $Nrc(s)-$ closed set as seen from the following example.

Example 3.3. Let $U = \{\alpha,\beta,\gamma,\delta\}$ with $U/R = \{\{\alpha\},\{\gamma\},\{\beta,\delta\}\}$ and $X = \{\alpha,\beta\}$. Then $\tau_R(X) = \{\phi,U,\{\alpha\},\{\alpha,\beta,\delta\},\{\beta,\delta\}\}$ is a nano topology on U. From Example 3.2, we have $\{\beta\}$ and $\{\delta\}$ are $Nrc(s)$-closed sets but their union $\{\beta,\delta\}$ is not a $Nrc(s)$-closed set in $(U,\tau_R(X))$.

Definition 3.4. The nano $rc(s)$-closure of a set A denoted by $Nrc(s)cl(A)$, is the intersection of nano $Nrc(s)$-closed sets containing a set A.

Remark 3.5. It is clear that $Nrc(s)cl(A)$ is a $Nrc(s)$-closed set.

Theorem 3.6. $A \subseteq Nrc(s)cl(A)$ and $A = Nrc(s)cl(A)$ if and only if A is a nano $Nrc(s)$-closed set in nano topological spaces.

Proof: It is obvious by above remark 4.5.

Theorem 3.7. If A is a $Nrc(s)$-closed set and $A \subseteq B \subseteq Nrc(s)cl(A)$, then B is a $Nrc(s)$-closed set in $(U,\tau_R(X))$.

Proof: Let $B \subseteq V$ where V is $Nc(s)$ set in $\tau_R(X)$ and by given $A \subseteq B$ implies $A \subseteq V$. Since A is a $Nrc(s)$-closed set, $Nrc(s)cl(A) = A$. So $Nrc(s)cl(A) \subseteq V$. Also by given, $B \subseteq Nrc(s)cl(A)$ implies $Nrc(s)cl(B) \subseteq Nrc(s)cl(A)$. Thus $Nrc(s)cl(B) \subseteq V$ and so B is a $Nrc(s)$- closed set in $(U,\tau_R(X))$.

4. NANO $rc(s)$ - CONTINUOUS FUNCTION IN NANO TOPOLOGICAL SPACES

Definition 4.1. Let $(U,\tau_R(X))$ and $(V,\tau_{R^1}(X))$ be to nano topological spaces, then a mapping $f:(U,\tau_R(X)) \to (V,\tau_{R^1}(X))$ is called as nano $rc(s)$-continuous (briefly $Nrc(s)$-continuous) function on $(U,\tau_R(X))$. If the inverse image of every nano open set in $(V,\tau_{R^1}(X))$ is nano $rc(s)$ open set in $(U,\tau_R(X))$.

Example 4.2. Let $U = \{\alpha,\beta,\gamma,\delta\}$ with $U/R = \{\{\alpha\},\{\gamma\},\{\beta,\delta\}\}$ and $X = \{\alpha,\beta\}$. Then $\tau_R(X) = \{\phi,U,\{\alpha\},\{\alpha,\beta,\delta\},\{\beta,\delta\}\}$ and $\tau_R^c(X) = \{\phi,U,\{\gamma\},\{\beta,\gamma,\delta\},\{\alpha,\gamma\}\}$. The $Nrc(s)-$closed sets are $\{\phi,U,\{\beta\},\{\gamma\},\{\delta\},\{\alpha,\gamma\},\{\beta,\gamma\},\{\gamma,\delta\},\{\alpha,\beta,\gamma\},\{\alpha,\gamma,\delta\},\{\beta,\gamma,\delta\}\}$. Let $V = \{p,q,s,r\}$ with $V/R^1 = \{\{q\},\{r\},\{p,s\}\}$ and $Y = \{p,r\}$ then its nano topology is $\tau_{R^1}(Y) = \{\phi,V,\{r\},\{p,s,r\},\{p,s\}\}$ and its complement is $\tau_{R^1}^c(Y) = \{\phi,V,\{q\},\{q,r\},\{p,q,s\}\}$. The $Nrc(s)$-closed sets are $\{\phi,V,\{p\},\{q\},\{s\},\{p,q\},\{q,s\},\{q,r\},\{p,q,s\},\{p,q,r\},\{q,s,r\}\}$. Define $f:(U,\tau_R(X)) \to (V,\tau_{R^1}(X))$ by $f(\alpha) = r.\ f(\beta) = p,\ f(\gamma) = q,\ f(\delta) = s.$ Then the inverse images of nano open sets in $(V,\tau_{R^1}(X))$ are $f^{-1}(\{r\}) = \{\alpha\}$, $f^{-1}(\{p,q,s\}) = \{\alpha,\beta,\delta\}$, $f^{-1}(\{p,s\}) = \{\beta,\delta\}$ and $f^{-1}(V) = U.$ That is the inverse image of every nano open set in $(V,\tau_{R^1}(X))$ is $Nrc(s)-$open set in $(U,\tau_R(X))$. Therefore f is a continuous function in $(U,\tau_R(X))$.

Theorem 4.3. A function $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ is said to be $Nrc(s)-$continuous if and only if the inverse image of every nano closed set in $(V,\tau_{R^1}(Y))$ is $Nrc(s)-$closed set in $(U,\tau_R(X))$.

Proof: Let f be a $Nrc(s)-$continuous function and F be a nano closed set in $(V,\tau_{R^1}(Y))$. That is $(V-F)$ is a nano open set in $(V,\tau_{R^1}(Y))$. Since f is a $Nrc(s)-$continuous function, $f^{-1}(V-F)$ is a $Nrc(s)-$open set in $(U,\tau_R(X))$. So $f^{-1}(F)$ is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. Conversely, let us assume that the inverse image of every nano closed set in $(V,\tau_{R^1}(Y))$ is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. Let H be a nano open set in $(V,\tau_{R^1}(Y))$. Then $(V-H)$ is a nano closed set in $(V,\tau_{R^1}(Y))$. By our assumption $f^{-1}(V-H)$ is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. Hence $f^{-1}(V)-f^{-1}(H)=U-f^{-1}(H)$ is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. So $f^{-1}(H)$ is a $Nrc(s)-$open set in $(U,\tau_R(X))$. That is the inverse image of every nano open set in $(V,\tau_{R^1}(Y))$ is $Nrc(s)-$open set in $(U,\tau_R(X))$. That is f is a $Nrc(s)-$continuous function on $(U,\tau_R(X))$.

Theorem 4.4. Every nano continuous function is $Nrc(s)-$continuous function.

Proof: Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ be a nano continuous function. Let A be any nano closed set in $(V,\tau_{R^1}(X))$. Then $f^{-1}(A)$ is nano closed set in $(U,\tau_R(X))$. Since every nano closed set in $Nrc(s)-$closed, $f^{-1}(A)$ is $Nrc(s)-$closed set in $(U,\tau_R(X))$. Therefore f is $Nrc(s)-$closed set.

Converse of the above theorem is not true. It is Proved by the following example.

Example 4.5. In Example 5.2, f is $Nrc(s)-$continuous function. But, f is not a nano continuous function. Since inverse image of nano closed sets $\{p\},\{s\},\{p,q\},\{q,s\},\{p,q,s\}$ and $\{q,s,r\}$ are not nano closed sets in $(U,\tau_R(X))$.Hence every $Nrc(s)-$continuous function is not a nano continuous function.

Theorem 4.6. Every nano regular continuous function is $Nrc(s)-$continuous function in nano topological spaces.

Proof: Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ be a nano regular continuous function. Let A be any nano closed set in $(V,\tau_{R^1}(Y))$. Then $f^{-1}(A)$ is nano regular closed set in $(U,\tau_R(X))$. Since every nano regular closed set is $Nrc(s)$-closed set, $f^{-1}(A)$ is $Nrc(s)$ closed set in $(U,\tau_R(X))$. Hence f is $Nrc(s)-$continuous function in $(U,\tau_R(X))$.

Converse of the above theorem is not true. It is Proved by the following example.

Example 4.7. Let $U=\{\alpha,\beta,\gamma,\delta\}$ with $U/R=\{\{\alpha\},\{\gamma\},\{\beta,\delta\}\}$ and $X=\{\alpha,\beta\}$. Then the nano topology is $\tau_R(X)=\{\phi,U,\{\alpha\},\{\alpha,\beta,\delta\},\{\beta,\delta\}\}$. Let $V=\{p,q,s,r\}$ with $V/R^1=\{\{q\},\{r\},\{p,s\}\}$ and $\boldsymbol{Y=\{p,r\}}$ then

$\tau_{R^1}(Y)=\{\phi,V,\{r\},\{p,s,r\},\{p,s\}\}$. The complement of $\tau_R(X)$ and $\tau_{R^1}(Y)$ are $\tau_R^{\,c}(X)$ = $\{\phi,U,\{\gamma\},\{\beta,\gamma,\delta\},\{\alpha,\gamma\}\}$ is $\tau_R^{\,c}(Y)=\{\phi,V,\{q\},\{q,r\},\{p,q,s\}\}$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by $f(\alpha)=s,\ f(\beta)=p,\ f(\gamma)=q,\ f(d)=r$,then f is a $Nrc(s)-$continuous function. But f is not a $Nr-$continuous function. Since $f^{-1}(\{q,r\})=\{\gamma,\delta\}$ is not $Nr-$closed set in $(U,\tau_R(X))$.

Theorem 4.8. Every $N\alpha-$continuous function is $Nrc(s)-$continuous function in the nano topological spaces in $(U,\tau_R(X))$.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a $N\alpha-$continuous function and A be a nano closed set in $(V,\tau_{R^1}(X))$. Then the inverse image of A under the map f is a $N\alpha-$closed set in $(U,\tau_R(X))$. Since every $N\alpha-$closed set is $Nrc(s)-$closed set, $f^{-1}(A)$ is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. Hence f is a $Nrc(s)-$continuous function.

Converse of the above theorem is not true. It is Proved by the following example.

Example 4.9. Let $U=\{\alpha,\beta,\gamma,\delta\}$ with $U/R=\{\{\alpha\},\{\gamma\},\{\beta,\delta\}\}$ and $X=\{\alpha,\beta\}$. Then the nano topology is $\tau_R(X)=\{\phi,U,\{\alpha\},\{\alpha,\beta,\delta\},\{\beta,\delta\}\}$. Let $V=\{p,q,s,r\}$ with $V/R^1=\{\{p,s\},\{q\},\{r\}\}$ and $Y=\{p,r\}$. Then $\tau_{R^1}(Y)=\{\phi,V,\{r\},\{p,s,r\},\{p,s\}\}$. The complement of $\tau_R(X)$ and $\tau_{R^1}(Y)$ ares(X) = $\{\phi,U,\{\gamma\},\{\beta,\gamma,\delta\},\{\alpha,\gamma\}\}$ and $\tau_R^{\,c}(Y)=\{\phi,V,\{q\},\{q,r\},\{p,q,s\}\}$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(X))$ by $f(\alpha)=p,\ f(\beta)=s,\ f(\gamma)=q,\ f(d)=r$, then f is a $Nrc(s)-$continuous function. But f isnot a $N\alpha-$continuous function. Since $f^{-1}(\{q,r\})=\{\gamma,\delta\}$ and $f^{-1}(\{p,q,s\})=\{\alpha,\beta,\gamma\}$ are not $Nr-$closed setsin $(U,\tau_R(X))$.

Theorem 4.10. Every $Nrc(s)-$continuous function is $Nsg-$continuous function in the nano topological space $(U,\tau_R(X))$.

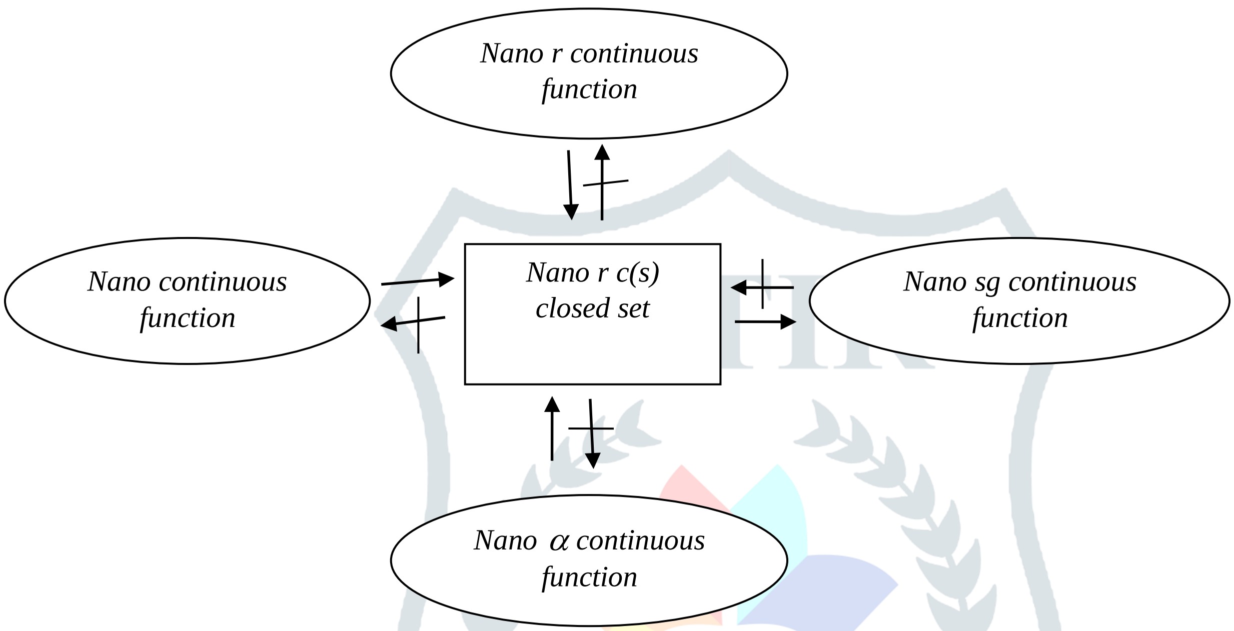
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(X))$ be a $Nrc(s)-$continuous function and A be a nano closed set in $(V,\tau_{R^1}(X))$. Then the inverse image of A under the map f is a $Nrc(s)-$closed set in $(U,\tau_R(X))$. Since every $Nrc(s)-$closed set in $Nsg-$closed set, $f^{-1}(A)$ is a $Nsg-$closed set in $(U,\tau_R(X))$. Hence f is a $Nsg-$continuous function.

Converse of the above theorem is not true. It is Proved by the following example.

Example 4.11. Let $U=\{\alpha,\beta,\gamma,\delta\}$ with $U/R=\{\{\alpha\},\{\gamma\},\{\beta,\delta\}\}$ and $X=\{\alpha,\beta\}$. Then the nano topology is $\tau_R(X)=\{\phi,U,\{\alpha\},\{\alpha,\beta,\delta\},\{\beta,\delta\}\}$. Let $V=\{p,q,s,r\}$ with $V/R^1=\{\{p\},\{s\},\{q,r\}\}$ and

$Y = \{p, s\}$. Then the nano topology is $\tau_{R^1}(Y) = \{\phi, V, \{p\}, \{p, q, s\}, \{q, s\}\}$. The complement of $\tau_R(X)$ and $\tau_{R^1}(Y)$ are $\tau_R^{\ c}(X) = \{\phi, U, \{\gamma\}, \{\beta, \gamma, \delta\}, \{\alpha, \gamma\}\}$ is $\tau_{R^1}^{\ c}(Y) = \{\phi, V, \{r\}, \{p, s, r\}, \{p, r\}\}$ respectively. Define $f : (U, \tau_R(X)) \to (V, \tau_{R^1}(X))$ by $f(\alpha) = q$, $f(\beta) = p$, $f(\gamma) = s$, $f(d) = r$, then f is a $Nsg-$continuous function. But f is not a $Nrc(s)-$continuous function, since $f^{-1}(\{p, r\}) = \{\beta, \delta\}$ is not a $Nr-$closed set in $(U, \tau_R(X))$.

Remark 4.12. From the above discussion we get the following diagram which represents the relationship between $Nrc(s)-$continuous function and existing nano continuous functions in nano topological spaces.



REFERENCES

[1] Bhuvaneswari. K and Ezhilarasi. K, On Nano semi generalised and Nano generalized semi closed sets in Nano Topological spaces, International Journal of Mathematics and Computer Application Research, 4(3) (June 2014), 117-124.

[2] Bhuvaneswari. K and Mythili Gnanapriya. K , Nano Generalized Closed Sets in Nano Topological Spaces, International Journal of scientific and Research Publication, 4(5) (May 2014), 1-3.

[3] Jayalakshmi. A and Janaki. C, A New Form of Nano Locally Closed Sets in Nano Topological Spaces, Global Journal of Pure and Applied Mathematics, 13 (9) (2017), 5997-6006.

[4] Lellis Thivagar. M, Carmel Richard, Note on nano topological spaces, Communicated.

[5] Lellis Thivagar. M, Carmel Richard, On Nano Continuity, Mathematical Theory Modeling, 3(7) (Jan 2013), 32-37.

[6] Pushpalatha. A and Sridevi. S, Nano semi c^* generalized Closed Sets in nano topological spaces, Malaya Journal of Mathematics, S(1) (Jan 2019), 361-366.

[7] Pushpalatha. A and Vishalakshi. S, Nano Semi $c(s)$ Generalized Closed Sets in Nano Topological Spaces, International Journal of Future Generation Communication and Networjing, 13 (4) (2020), 1586-1601.

[8] Sulochana Devi. P and Bhuvaneswari. K, On Nano Regular Generalized and Nano Generalized Regular Closed sets, IJETT, 8 (13) (2014), 386-390.