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TOTAL DOMINATING SETS AND TOTAL DOMINATION POLYNOMIALS OF STAR GRAPH

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Abstract - Let $G = (V, E)$ be a simple graph. A set $S \subseteq V$ is a total dominating set of G if every vertex is adjacent to an element of S. Let $D_t(S_n, i)$ be the family of all total dominating set of the graph S_n ,$n \geq 2$ with cardinality i, and let $d_t(S_n, i) = |D_t(S_n, i)|$.In this paper we construct $d_t(S_n, i)$, and obtain the polynomial $D_t(S_n, x) = \sum_{i=\Upsilon_t(S_n)}^{n} d_t(S_n, i)x^i$,which we call total domination polynomial of S_n ,$n \geq 2$ and obtain some properties of this polynomial.

Keywords- Star graph, Total dominating set, Total domination polynomial

1.INTRODUCTION

Let $G = (V, E)$ be a simple graph of order $|V| = n$. A set $S \subseteq V$ is a dominating set of G if every vertex in $V - S$ is adjacent to atleast one vertex in S . A set $S \subseteq V$ is a total dominating set of G if every vertex of G is adjacent to an element of S. The total domination number of a graph G is the minimum cardinality of a total dominating set in G, and is denoted by $\Upsilon_t(G)$. A star graph denoted by S_n is the complete bipartite graph $K_{1,n}$. let G_n^i be the family of dominating sets of a graph G_n with cardinality i and let $d(G_n, i) = |G_n^i|$. We call the polynomial $D(G_n, x) = \sum_{i=\Upsilon(G_n)}^{n} d(G_n, i)x^i$, the domination polynomial of graph G_n [2]. In this paper, we study the concept of total dominating set and total domination polynomial of star S_n ,$n \geq 2$. Let $D_t(S_n, i)$ be the family of total dominating sets of a star S_n,$n \geq 2$ with cardinality i, and let $d_t(S_n, i) = |D_t(S_n, i)|$. We call the polynomial $D_t(S_n, x) = \sum_{i=2}^{n} d_t(S_n, i)x^i$,the total domination polynomial of star. As usual we use n_{c_i} for the combination n to i and we denote the set $\{1,2, . . . , n\}$ simply by $[n]$.

2. TOTAL DOMINATING SETS OF STAR (S_n)

Let $n \geq 2$,be the star with n vertices $V(S_n) = [n]$ and $E(S_n) = \{(1,2), (1,3), . . . , (1, n)\}$. Let $D_t(S_n, i)$ be the family of total dominating sets of S_n with cardinality i. We shall investigate total dominating sets of stars.

Theorem:1

Let S_n be a star with order ≥ 3 , then $d_t(S_n, i) = n_{C_i} - (n-1)_{C_i}\ \forall\ i \leq n-1$.

Proof:

Let S_n be a star and $v \in V(S_n)$ such that v is center of S_n, that is ($\rho(v) = n-1$) and let $H = S_n - v$, then $D_t(H_n, i) = \varphi \; \forall \; i < n$ that is ($d_t(H_n, i) = 0 \; \forall \; i < n$) .

Since $|V(H)| = n-1$ then $(n-1)_{C_i}$ number of subsets of H with cardinality i, and n_{C_i} number of subsets of S_n with cardinality i and since total dominating set of S_n is every subset of S_n has vertex v and since H is a total dominating set of S_n with cardinality $n-1$, therefore $d_t(S_n, i) = n_{C_i} - (n-1)_{C_i} \; \forall \; i \leq n-1$.

Theorem:2

Let S_n be a star with order ≥ 3 , then

(i) $d_t(S_n, i) = (n-1)_{C_{i-1}} \; \forall \; i \leq n-1$

(ii) $d_t(S_n, i) = d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1)$ if $i \neq 2$

(iii) $d_t(S_n, i) = d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1) + 1$ if $i = 2$

Proof:

(i) By theorem:1,

$$d_t(S_n, i) = n_{C_i} - (n-1)_{C_i}$$

$$= \frac{n(n-1)\ldots(n-i+1)}{i!} - \frac{(n-1)(n-2)\ldots(n-1-i+1)}{i!}$$

$$= \frac{n(n-1)\ldots(n-i+1)}{i(i-1)!} - \frac{(n-1)(n-2)\ldots(n-i+1)(n-i)}{i(i-1)!}$$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)}{(i-1)!}\left[\frac{n}{i} - \frac{n-i}{i}\right]$$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)}{(i-1)!}\left[\frac{n-n+i}{i}\right]$$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)}{(i-1)!}$$

$$d_t(S_n, i) = (n-1)_{C_{i-1}}$$

(ii) Since $n_{C_i} = \frac{n(n-1)\ldots(n-i+1)}{i!}$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)}{i!} n$$

$$\therefore \quad (n-1)_{C_i} = \frac{(n-1-1)(n-1-2)\ldots(n-1-i+1)}{i!}(n-1)$$

$$= \frac{(n-2)(n-3)\ldots(n-i)}{i!}(n-1)$$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)(n-i)}{i!}$$

and

$$(n-2)_{C_i} = \frac{(n-2-1)(n-2-2)\ldots(n-2-i+1)}{i!}(n-2)$$

$$= \frac{(n-3)(n-4)\ldots(n-i-1)}{i!}(n-2)$$

$$= \frac{(n-2)(n-3)\ldots(n-i)(n-i-1)}{i!}$$

$$= \frac{(n-1)(n-2)\ldots(n-i)(n-i-1)}{i!\,(n-1)}$$

$$\therefore \quad (n-1)_{C_{i-1}} = \frac{(n-1)(n-2)\ldots(n-i+1+1)(n-i+1)}{(i-1)!}$$

$$= \frac{(n-1)(n-2)\ldots(n-i)(n-i+1)i}{i!}$$

and

$$(n-2)_{C_{i-1}} = \frac{(n-1)(n-2)\ldots(n-i+1)(n-i+1-1)}{(i-1)!\,(n-1)}$$

$$= \frac{(n-1)(n-2)\ldots(n-i+1)(n-i)i}{i!\,(n-1)}$$

Let $m = \frac{(n-1)(n-2)\ldots(n-i+1)}{i!}$

Now, LHS $= d_t(S_n, i)$

$= n_{C_i} - (n-1)_{C_i}$ by theorem:1

$= mn - m(n-i)$

$= mi$ -----------------------------(1)

RHS $= d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1)$

$= (n-1)_{C_i} - (n-2)_{C_i} + (n-1)_{C_{i-1}} - (n-2)_{C_{i-1}}$ by theorem:1

$= m(n-i) - \frac{m(n-i)(n-i-1)}{n-1} + mi - \frac{m(n-i)i}{n-1}$

$= \frac{m(n-i)(n-1)-m(n-i)(n-i-1)+mi(n-1)-mi(n-i)}{n-1}$

$= \frac{m}{n-1} \left[(n-i)(n-1) - (n-i)(n-i-1) + i(n-1) - i(n-i) \right]$

$= \frac{m}{n-1} \left[(n-1)(n-i+i) - (n-i)(n-i-1+i) \right]$

$= \frac{m}{n-1} \left[n(n-1) - (n-i)(n-1) \right]$

$= \frac{m(n-1)}{n-1} \left[n - (n-i) \right]$

$= \frac{m(n-1)}{n-1} \left[n - (n-i) \right]$

$= mi$ ------------------------------ (2)

From (1) and (2) , LHS = RHS

That is $d_t(S_n, i) = d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1)$

(iii) If $i = 2$, then $d_t(S_n, i) = d_t(S_n, 2)$

$= n_{C_2} - (n-1)_{C_2}$ by theorem:1

$= \frac{n(n-1)}{2} - \frac{(n-1)(n-2)}{2}$

$= \frac{n^2 - n - n^2 + 3n - 2}{2}$

$= \frac{2n-2}{2}$

$= n - 1$

By theorem :2(i), $d_t(S_{n-1}, i) = d_t(S_{n-1}, 2) = (n-1)_{C_1} = n-2$

$d_t(S_{n-1}, i-1) = d_t(S_{n-1}, 1) = 0$

Now $d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1) = n - 2 + 0$

$= n - 2$

$= (n-1) - 1$

$= d_t(S_n, i) - 1$

$\therefore d_t(S_n, i) = d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1) + 1$

Using Theorem:1 and Theorem:2, we obtain the coefficients of $D_t(S_n, x)$ for $2 \le n \le 15$ in Table:1. Let $d_t(S_n, i) = \left| D_t(S_n, i) \right|$. There are interesting relationships between the numbers $d_t(S_n, i)$; $2 \le n \le 15$ in the table:1. In the following theorem, we obtain some properties of $d_t(S_n, i)$.

Table:1

i	2	3	4	5	6	7	8	9	10	11	12	13	14	15
n														
2	1													
3	2	1												
4	3	3	1											
5	4	6	4	1										
6	5	10	10	5	1									
7	6	15	20	15	6	1								
	7	2	3	3	2	7	1							

8		1	5	5	1									
9	8	28	56	70	56	28	8	1						
10	9	36	84	126	126	84	36	9	1					
11	10	45	120	210	252	210	120	45	10	1				
12	11	55	165	330	462	462	330	165	55	11	1			
13	12	66	220	495	792	924	792	495	220	66	12	1		
14	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1	
15	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1

Theorem:3

The following properties hold for coefficients of $D_t(S_n, x)$. For every $\in Z^+$,

(i) $d_t(S_n, n) = 1 \; \forall \; n > 1$

(ii) $d_t(S_n, 2) = n - 1 \; \forall \; n \geq 3$

(iii) $d_t(S_n, i) = d_t(S_n, n - i + 1)\,; 2 \leq i \leq n - 1$

(iv) $\Upsilon_t(S_n) = 2$

Proof:

(i) Since for any graph G with n vertices, $d_t(G, n) = 1$.
Then $d_t(S_n, n) = 1$.

(ii) By theorem: 1,
$d_t(S_n, 2) = n_{C_2} - (n-1)_{C_2}$

$$= \frac{n!}{(n-2)!2!} - \frac{(n-1)!}{(n-3)!2!}$$

$$= \frac{n(n-1)!}{(n-2)(n-3)!2!} - \frac{(n-1)!}{(n-3)!2!}$$

$$= \frac{n(n-1)(n-2)(n-3)!}{(n-2)(n-3)!2!} - \frac{(n-1)(n-2)(n-3)!}{(n-3)!2!}$$

$$= \frac{n(n-1) - (n-1)(n-2)}{2}$$

$$= \frac{n^2-n-n^2+3n-2}{2}$$

$$= \frac{2n-2}{2}$$

$$d_t(S_n, 2) = n - 1$$

(iii) $d_t(S_n, i) = n_{C_i} - (n-1)_{C_i}$

$$= \frac{n!}{(n-i)!i!} - \frac{(n-1)!}{(n-1-i)!i!}$$

$$= \frac{n(n-1)!}{(n-i)(n-i-1)!i(i-1)!} - \frac{(n-1)!}{(n-i-1)!i(i-1)!}$$

$$= \frac{(n-1)!}{(n-i-1)!(i-1)!i}\left[\frac{n}{n-i} - 1\right]$$

$$= \frac{(n-1)!}{(n-i-1)!(i-1)!i}\left[\frac{n-n+i}{n-i}\right]$$

$$= \frac{(n-1)!}{(n-i)!(i-1)!} \quad \text{---------------------------(1)}$$

$$d_t(S_n, n-i+1) = n_{C_{n-i+1}} - (n-1)_{C_{n-i+1}}$$

$$= \frac{n!}{(i-1)!(n-i+1)!} - \frac{(n-1)!}{(i-2)!(n-i+1)!}$$

$$= \frac{n(n-1)!}{(i-1)(i-2)!(n-i+1)!} - \frac{(n-1)!}{(i-2)!(n-i+1)!}$$

$$= \frac{(n-1)!}{(i-2)!(n-i+1)!}\left[\frac{n}{i-1} - 1\right]$$

$$= \frac{(n-1)!}{(i-2)!(n-i+1)!}\left[\frac{n-i+1}{i-1}\right]$$

$$= \frac{(n-1)!(n-i+1)}{(i-1)!(n-i+1)(n-i)!}$$

$$= \frac{(n-1)!}{(n-i)!(i-1)!} \text{ ---------------------------(1)}$$

From (1) and (2) we get ,$d_t(S_n, i) = d_t(S_n, n - i + 1)$

(iv) Let S_n be a star and $v \in V(S)$ such that v is center of S_n. Then $\{v, v_i\}; i = 1, 2, \ . \ . \ . \ , n$ is a total dominating set of S_n Therefore $\Upsilon_t(S_n) = 2$.

3.Total domination polynomial of a star

In this section we introduce and investigate the domination polynomial of stars.

Definition:1

Let $D_t(S_n, i)$ be the family of total dominating sets of a star S_n with cardinality i, and let $d_t(S_n, i) = |D_t(S_n, i)|$ and since $\Upsilon_t(S_n) = 2$.Then the total domination polynomial $D_t(S_n, x) = \sum_{i=2}^{n} d_t(S_n, i)x^i$.

Theorem :3

The following properties hold for all $D_t(S_n, x)$ $\forall\, n \geq 3$

(i) $D_t(S_n, x) = D_t(S_{n-1}, x) + x\, D_t(S_{n-1}, x) + x^2$

(ii) $D_t(S_n, x) = \sum_{i=2}^{n} n_{C_i} x^i - \sum_{i=2}^{n-1} (n-1)_{C_i} x^i$

(iii) $D_t(S_n, x) = \sum_{i=2}^{n} (n-1)_{C_{i-1}} x^i$

Proof:

(i) $D_t(S_n, x) = \sum_{i=2}^{n} d_t(S_n, i)x^i$

$= \sum_{i=2}^{n} [d_t(S_{n-1}, i) + d_t(S_{n-1}, i-1)]x^i$

$= \sum_{i=2}^{n} d_t(S_{n-1}, i)\ x^i + \sum_{i=2}^{n} d_t(S_{n-1}, i-1)\, x^i$

Now, $\sum_{i=2}^{n} d_t(S_{n-1}, i)\ x^i = \sum_{i=2}^{n-1} d_t(S_{n-1}, i)\ x^i = D_t(S_{n-1}, x)$

$\sum_{i=2}^{n} d_t(S_{n-1}, i-1)\, x^i = x \sum_{i=2}^{n} d_t(S_{n-1}, i-1)\, x^{i-1} = xD_t(S_{n-1}, x)$

Therefore $D_t(S_n, x) = D_t(S_{n-1}, x) + x\, D_t(S_{n-1}, x)$ -------------------------(1)

But $\sum_{i=2}^{n} d_t(S_n, 2)x^2 = \sum_{i=2}^{n} [d_t(S_{n-1}, 2) + d_t(S_{n-1}, 1) + 1]x^2$

$= \sum_{i=2}^{n} d_t(S_{n-1}, 2)\ x^2 + \sum_{i=2}^{n} d_t(S_{n-1}, 1)\, x^2 + x^2$---------------------(2)

From (1) and (2),we get $D_t(S_n, x) = D_t(S_{n-1}, x) + x\, D_t(S_{n-1}, x) + x^2$

(ii) $D_t(S_n, x) = \sum_{i=2}^{n} d_t(S_n, i)x^i$

$= \sum_{i=2}^{n} [n_{C_i} - (n-1)_{C_i}]x^i$

by theorem :1

$D_t(S_n, i) = \sum_{i=2}^{n} n_{C_i} x^i - \sum_{i=2}^{n-1} (n-1)_{C_i} x^i$

(iii) $D_t(S_n, x) = \sum_{i=2}^{n} d_t(S_n, i)x^i$

$= \sum_{i=2}^{n}(n-1)_{C_{i-1}}x^i$ by theorem:2

Example:1

Let S_7 be a star with order 7, then (by theorem:4) we have

(i) $D_t(S_7,x) = D_t(S_6,x)+x\, D_t(S_6,x) + x^2$

(ii) $D_t(S_7,x) = \sum_{i=2}^{7} 7_{C_i}x^i - \sum_{i=2}^{6} 6_{C_i}x^i$

(iii) $D_t(S_7,x) = \sum_{i=2}^{7} 6_{C_{i-1}}x^i$

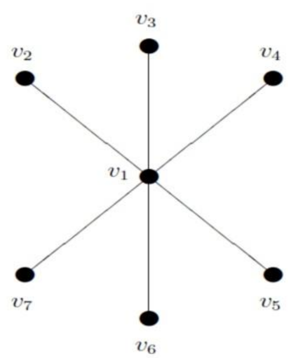

Figure .1:S_7

(i) From table:1 ,we get $D_t(S_6,x) = 5x^2 + 10x^3 + 10x^4 + 5x^5 + x^6$ and

$x\, D_t(S_6,x) = 5x^3 + 10x^4 + 10x^5 + 5x^6 + x^7$

Now, $D_t(S_6,x)+x\, D_t(S_6,x) + x^2 = 5x^2 + 10x^3 + 10x^4 + 5x^5 + x^6 + 5x^3 + 10x^4 + 10x^5 + 5x^6 + x^7 + x^2$

$= 6x^2 + 15x^3 + 20x^4 + 15x^5 + 6x^6 + x^7$

$= D_t(S_7,x)$

(ii) $\sum_{i=2}^{7} 7_{C_i}x^i - \sum_{i=2}^{6} 6_{C_i}x^i = (7_{C_2}x^2 + 7_{C_3}x^3 + 7_{C_4}x^4 + 7_{C_5}x^5 + 7_{C_6}x^6 + 7_{C_7}x^7) - (6_{C_2}x^2 + 6_{C_3}x^3 + 6_{C_4}x^4 + 6_{C_5}x^5 + 6_{C_6}x^6)$

$= (21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7) - (15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6)$

$= 6x^2 + 15x^3 + 20x^4 + 15x^5 + 6x^6 + x^7$

$= D_t(S_7,x)$

(iii) $\sum_{i=2}^{7} 6_{C_{i-1}}x^i = 6_{C_1}x^2 + 6_{C_2}x^3 + 6_{C_3}x^4 + 6_{C_4}x^5 + 6_{C_5}x^6 + 6_{C_6}x^7$

$= 6x^2 + 15x^3 + 20x^4 + 15x^5 + 6x^6 + x^7$

$= D_t(S_7,x)$

Conclusion

Domination theory in graphs has been consolidated as an important area of study within Discrete mathematics, due to its multiple theoretical and applied applications. This research opens a door to analyze the properties associated to the Domination Theory in the studied unitary operators $\Upsilon_t(S_n)$, $d_t(S_n)$, $D_t(S_n)$. This research continues with the study of an important parameter of the Domination Theory(total domination number) when unitary operators $\Upsilon_t(S_n)$, $d_t(S_n)$, $D_t(S_n)$ act on graphs. In particular, we find optimal bounds, and, in some cases, we give closed formulas for the number of total domination when certain operators act on the graph.

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