



A STUDY ON CERTAIN FLOW SHOP SCHEDULING PROBLEM

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Abstract : Flow shop scheduling is used to determine the optimal sequence of n jobs to be processed on m machines in the same order permutation Flow shop Scheduling Problems (PFSP) require same job sequence on all the machines with the constraint that machines can only process on job at a time and jobs can be processed by only one machine at a time.

Keywords: Palmer's Heuristic algorithm, Gupta's Heuristic algorithm, Flow shop scheduling.

I. INTRODUCTION

Flexible Manufacturing System (FMS) is an automated manufacturing system which consists of group of automated machine tools, interconnected with an automated material handling and storage system and controlled by computer to produce products according to the right schedule.

FMS Scheduling system is one of the most important information-processing subsystems of CIM system. The productivity of CIM is highly depending upon the quality of FMS scheduling. The basic work of scheduler is to design an optimal FMS schedule according to a certain measure of performance, or scheduling criterion. This work focuses on productivity oriented-make span criteria. Make span is the time length from the starting of the first operation of the first demand to the finishing of the last operation of the last demand.

II. LITERATURE REVIEW:

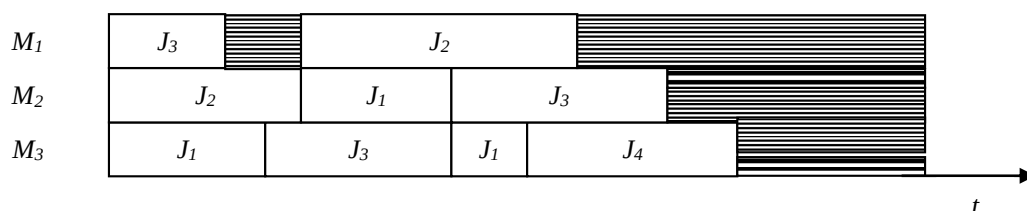
Chen and Askin [8] developed a model for project selection, scheduling and resource allocation with time dependent returns. They formulate and analyse the joint problem of project selection and task scheduling. They study the situation where a manager has many alternative projects to pursue such as developing new product platforms or technologies, incremental product upgrades, or continuing education of human resources. Project return is assumed to be a known function of project completion time.

Biskup and Hermann [4] developed a model for Single-machine scheduling against due dates with past-sequence-dependent setup times. Their objective is to minimize the due date.

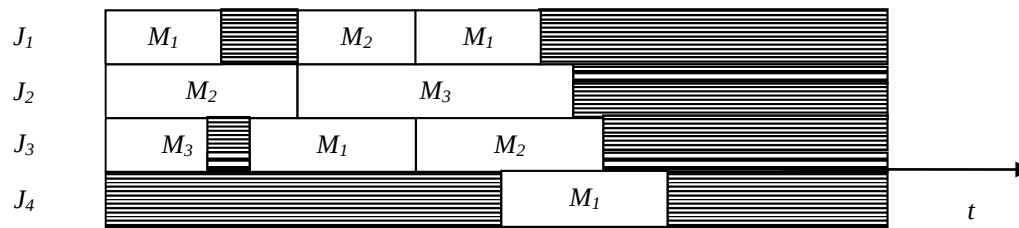
Chen and Lee [5] developed a model for Logistics scheduling with batching [LSB] and transportation. Their objective is to minimize the sum of weighted job delivery time and total transportation cost. Since their problem involves not only the traditional performance measurement, such as weighted completion time, but also transportation arrangement and cost, key factors in logistics management.

III. SCHEDULING PROBLEMS

Suppose that m machines M_j ($j = 1, \dots, m$) have to process n jobs J_i ($i = 1, \dots, n$). A schedule is for each job an allocation of one or more time intervals to one or more machines. Schedules may be represented by Gantt charts as shown in Figure 1.1. Gantt charts may be machine-oriented (Figure 1.1(a)) or job-oriented (Figure 1.1(b)). The corresponding scheduling problem is to find a schedule satisfying certain restrictions.



(a)



(b)

Figure 1.1: Machine- and job-oriented Gantt charts.

The general shop problem may be defined as follows. We have n jobs $i = 1 \dots n$ and m machines $M_1, \dots, M_m$. Each job i consists of a set of operations O_{ij} ($j = 1 \dots n_i$) with processing times p_{ij} . Each operation O_{ij} must be processed on a machine $\mu_{ij} \in \{M_1, \dots, M_m\}$. There may be precedence relations between the operations of all jobs. Each job can only be processed only by one machine at a time and each machine can only process one job at a time. The objective is to find a feasible schedule that minimizes some objective function of the finishing times C_i of the jobs $i = 1 \dots n$. The objective functions are assumed to be regular.

1.1. CLASSIFICATION OF SCHEDULING PROBLEMS:

As per the environment, the scheduling problems are basically classified into four types. They are as follows.

1. Flow shop scheduling problem
2. Job shop scheduling problem
3. Open shop scheduling problem
4. Mixed shop scheduling problem

1.2. FLOW SHOP SCHEDULING PROBLEM

It is a typical combinatorial optimization problem, where each job has to go through the processing in each and every machine on the shop floor. Each machine has same sequence of jobs. The jobs have different processing time for different machines. So in this case we arrange the jobs in a particular order and get many combinations and we choose that combination where we get the minimum make span

Now we classify flow shop problems as:

- Flow shop (there is one machine at each stage)
- No-wait flow shop (a succeeding operation starts immediately after the preceding operation completes).
- Flexible (hybrid) flow shop (more than one machine exist in at least one stage)
- Assembly flow shop (each job consists of specific operations, each of which has to be performed on a pre-determined machine of the first stage, and an assembly operation to be performed on the second stage machine.)

1.3. FLOW SHOP SCHEDULING METHODS

For the two- Machine Flow- shop problems, there are two methods. They are,

- Johnson's Rule.
- Kusiak's Rule.

For the general m -Machine Problems, there several Heuristics available, they are

- Palmer's Heuristic Algorithm.
- Gupta's Heuristic Algorithm.
- CDS Heuristic Algorithm.
- RA Heuristic Algorithm.

1.4. GENERAL ASSUMPTIONS IN FLOW SHOP PROBLEMS

Generally the following assumptions are made in Flow shop scheduling problems:

They are,

- There are m machines and n jobs.
- Each job consists of m operations and each operation requires a different machine
- n jobs have to be processed in the same sequence on m machines.
- Every job has to be processed on all machines in the order ($j=1,2,\dots,m$)
- Every machine processed only one job at a time.
- Every job is processed on one machine at a time.
- Operations are not pre-emptive.
- Set-up time for the operations are sequence- independent and are included in the processing times.
- Operating sequence of the jobs are the same on every machine, and the common sequence has to determine.

1.5. THREE CATEGORIES OF FSP:

There are three categories of Flow shop scheduling problem. They are as follows,

1. Deterministic flow-shop scheduling problem. Assume that fixed processing times of jobs are known.
2. Stochastic flow-shop scheduling problem. Assume that processing times vary according to chosen probability distribution
3. Fuzzy flow- shop scheduling problem. Assume that a fuzzy due date is assigned to each job to represent the grade of satisfaction of decision makers for the completion time of the job.

4. PROBLEM STATEMENT

There is a flow shop scheduling problem in which all the parameters like processing machines in a flow shop based on batch- processing machines in a flow shop based on comparisons of Gupta's, Palmer's heuristics, are proposed. Analytic solutions in all the heuristics are investigated. Gantt chart is generated to verify the effectiveness of the proposed approaches. Here the heuristics approach for planning problems are proposed which provides a way to optimize the make span which is our objective function.

4.1. PALMER'S Heuristic algorithm

Procedure: Palmer's Heuristic

Input: job list I, machine m;

Output: Schedule "s";

Step 1

```
begin
fori=1 to n
for j=1 to m
Calculate  $S_i = (2j - m - 1)t_{i,j}$ ;
```

Step 2

Permutation schedule is constructed by sequencing the jobs in Non-increasing order of S_i such as:

$$S_{i1} \geq S_{i2} \geq \dots \geq S_{in};$$

Step 3

```
end
Output optimal sequence is obtained as schedule "s";
End
```

4.2. GUPTA HEURISTIC ALGORITHM

Procedure: Gupta's Heuristic

Input: job list i, machine m;

Output: Schedule "s";

Step 1

```
begin
fori=1 to n
for k=1 to m-1
ifti1 < tim then
ei = -1;
else
ei = -1;
Calculate  $s_i = t_i / \min\{t_{i,k} + t_{i,k+1}\}$ 
end
```

Step 2

Permutation schedule constructed by sequencing the jobs in non-increasing order of S_i such as:

$$S_{i1} \geq S_{i2} \geq \dots \geq S_{in};$$

Step 3

```
end
```

Output optimal sequence is obtained as schedule "s";

```
end
```

5. Examples for Palmer's Algorithm:

Consider a 10 job 8 machine flow shop scheduling problem.

JOB M/c	1	2	3	4	5	6	7	8	9	10
1	6	3	8	4	9	3	5	2	1	6
2	5	9	1	6	8	2	9	8	4	3
3	1	5	6	3	2	4	4	9	6	5
4	7	7	4	1	9	3	2	1	2	5
5	9	2	3	5	2	7	4	6	5	2
6	3	5	9	6	5	2	8	3	4	7
7	4	6	5	7	9	3	6	4	3	1
8	2	1	9	7	6	5	6	8	9	9

5.1. Solution by using Palmer's Algorithm

The solution constructed as follows:

Step 1

Set the slope index s_i for job i as:

$$s_i = (m-1)t_{i,8} + (m-3)t_{i,7} + (m-5)t_{i,6} + (m-7)t_{i,5} + (m-9)t_{i,4} + (m-11)t_{i,3} + (m-13)t_{i,2} + (m-15)t_{i,1}$$

For 8 machines ($m=8$) and $i = 1$

$$\begin{aligned} s_1 &= (m-1)t_{1,8} + (m-3)t_{1,7} + (m-5)t_{1,6} + (m-7)t_{1,5} + (m-9)t_{1,4} + (m-11)t_{1,3} + (m-13)t_{1,2} + (m-15)t_{1,1} \\ &= (8-1) \times 2 + (8-3) \times 4 + (8-5) \times 3 + (8-7) \times 9 + (8-9) \times 7 + (8-11) \times 1 + (8-13) \times 5 + (8-15) \times 6 \\ &= 14 + 20 + 9 - 7 - 3 - 25 - 42 \\ s_1 &= -25 \end{aligned}$$

Similarly we get for other machines using the formula

For 8 machines ($m=8$) and $i = 2$

$$\begin{aligned} s_2 &= (m-1)t_{2,8} + (m-3)t_{2,7} + (m-5)t_{2,6} + (m-7)t_{2,5} + (m-9)t_{2,4} + (m-11)t_{2,3} + (m-13)t_{2,2} + (m-15)t_{2,1} \\ s_2 &= -34 \end{aligned}$$

For 8 machines ($m=8$) and $i = 3$

$$\begin{aligned} s_3 &= (m-1)t_{3,8} + (m-3)t_{3,7} + (m-5)t_{3,6} + (m-7)t_{3,5} + (m-9)t_{3,4} + (m-11)t_{3,3} + (m-13)t_{3,2} + (m-15)t_{3,1} \\ s_3 &= 35 \end{aligned}$$

For 8 machines ($m=8$) and $i = 4$

$$\begin{aligned} s_4 &= (m-1)t_{4,8} + (m-3)t_{4,7} + (m-5)t_{4,6} + (m-7)t_{4,5} + (m-9)t_{4,4} + (m-11)t_{4,3} + (m-13)t_{4,2} + (m-15)t_{4,1} \\ s_4 &= 39 \end{aligned}$$

For 8 machines ($m=8$) and $i = 5$

$$\begin{aligned} s_5 &= (m-1)t_{5,8} + (m-3)t_{5,7} + (m-5)t_{5,6} + (m-7)t_{5,5} + (m-9)t_{5,4} + (m-11)t_{5,3} + (m-13)t_{5,2} + (m-15)t_{5,1} \\ s_5 &= -14 \end{aligned}$$

For 8 machines ($m=8$) and $i = 6$

$$\begin{aligned} s_6 &= (m-1)t_{6,8} + (m-3)t_{6,7} + (m-5)t_{6,6} + (m-7)t_{6,5} + (m-9)t_{6,4} + (m-11)t_{6,3} + (m-13)t_{6,2} + (m-15)t_{6,1} \\ s_6 &= 17 \end{aligned}$$

For 8 machines ($m=8$) and $i = 7$

$$\begin{aligned} s_7 &= (m-1)t_{7,8} + (m-3)t_{7,7} + (m-5)t_{7,6} + (m-7)t_{7,5} + (m-9)t_{7,4} + (m-11)t_{7,3} + (m-13)t_{7,2} + (m-15)t_{7,1} \\ s_7 &= 6 \end{aligned}$$

For 8 machines ($m=8$) and $i = 8$

$$\begin{aligned} s_8 &= (m-1)t_{8,8} + (m-3)t_{8,7} + (m-5)t_{8,6} + (m-7)t_{8,5} + (m-9)t_{8,4} + (m-11)t_{8,3} + (m-13)t_{8,2} + (m-15)t_{8,1} \\ s_8 &= 9 \end{aligned}$$

For 8 machines ($m=8$) and $i = 9$

$$\begin{aligned} s_9 &= (m-1)t_{9,8} + (m-3)t_{9,7} + (m-5)t_{9,6} + (m-7)t_{9,5} + (m-9)t_{9,4} + (m-11)t_{9,3} + (m-13)t_{9,2} + (m-15)t_{9,1} \\ s_9 &= 48 \end{aligned}$$

For 8 machines ($m=8$) and $i = 10$

$$\begin{aligned} s_{10} &= (m-1)t_{10,8} + (m-3)t_{10,7} + (m-5)t_{10,6} + (m-7)t_{10,5} + (m-9)t_{10,4} + (m-11)t_{10,3} + (m-13)t_{10,2} + (m-15)t_{10,1} \\ s_{10} &= 14 \end{aligned}$$

Step 2

Jobs are sequenced according to decreasing order of slope index numbers.

$$48 \geq 39 \geq 35 \geq 17 \geq 14 \geq 9 \geq 6 \geq -14 \geq -25 \geq -34$$

$$s_9 \geq s_4 \geq s_3 \geq s_6 \geq s_{10} \geq s_8 \geq s_7 \geq s_5 \geq s_1 \geq s_2$$

Step 3

Output optimal sequence is {9, 4, 3, 6, 10, 8, 7, 5, 1, 2}

The total processing time can be calculated as:

Job i	M/c 1		M/c 2		M/c 3		M/c 4		M/c 5		M/c 6		M/c 7		M/c 8	
	Time		Time		Time		Time		Time		Time		Time		Time	
	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out
9	0	1	1	5	5	11	11	13	13	18	18	22	22	25	25	34
4	1	5	5	11	11	14	14	15	18	23	23	29	29	36	36	43
3	5	13	13	14	14	20	20	24	24	27	29	38	38	43	43	52
6	13	16	16	18	20	24	24	27	27	34	38	40	43	46	52	57
10	16	22	22	25	25	30	30	35	35	37	40	47	47	48	57	66
8	22	24	25	33	33	42	42	43	43	49	49	52	52	56	66	74
7	24	29	33	42	42	46	46	48	49	53	53	61	61	67	74	80
5	29	38	42	50	50	52	52	61	61	63	63	68	68	77	80	86
1	38	44	50	55	55	56	61	68	68	77	77	80	80	84	86	88
2	44	47	55	64	64	69	69	76	77	79	80	85	85	91	91	92

Thus total processing time can be calculated as:

Total idle time for M/c 1=92-47=45 (Units)

Total idle time for M/c 2=1+2+2+4+ (92-64) =37 (Units)

Total idle time for M/c 3= 5+1+3+4+3+8 (92-69) =47 (Units)

Total idle time for M/c 4=11+1+5+3+7+3+4+1+ (92-76) =51 (Units)

Total idle time for M/c 5=13+1+1+6+8+5+ (92-79) =47 (Units)

Total idle time for M/c 6=18+1+2+1+2+9+ (92-85) =40 (Units)

Total idle time for M/c 7=22+4+2+1+4+5+1+3+1+ (92-91) =44 (Units)

Total idle time for M/c 8=25+2+3=30 (Units)

5.2. Solution by using Gupta's Heuristic Algorithm

The solution constructed as follows:

Step 1

Set the slope index s_i for jobs i as:

$$s_1 = 1/\min\{t_{11} + t_{12} + t_{13} + t_{14} + t_{15} + t_{16} + t_{17}, t_{12} + t_{13} + t_{14} + t_{15} + t_{16} + t_{17} + t_{18}\}$$

If $t_{1,1} < t_{1,8}$ then $e_1 = 1$ otherwise $e_1 = -1$

Here $t_{1,8} < t_{1,1}$ (i.e.2<6)

Therefore $e_1 = -1$

$$s_1 = e_1/\min\{6 + 5 + 1 + 7 + 9 + 3 + 4, 5 + 1 + 7 + 9 + 3 + 4 + 2\}$$

$$s_1 = -1/\min\{35, 31\}$$

$$s_1 = -1/31$$

$$s_1 = -0.0323$$

If $t_{2,1} < t_{2,8}$ then $e_2 = 1$ otherwise $e_2 = -1$

Here $t_{2,1} > t_{2,8}$ (i.e.3>5) the condition is not satisfied.

$$\therefore e_2 = -1$$

$$s_2 = -0.0286$$

If $t_{3,1} < t_{3,8}$ then $e_3 = 1$ otherwise $e_3 = -1$

Here $t_{3,1} < t_{3,8}$ (i.e.8<9) the condition is satisfied. $\therefore e_3 = 1$

$$s_3 = 0.0278$$

If $t_{4,1} < t_{4,8}$ then $e_4 = 1$ otherwise $e_4 = -1$

Here $t_{4,1} < t_{4,8}$ (i.e.4<7) the condition is satisfied.

Therefore $e_4 = 1$

$$s_4 = 0.0313$$

If $t_{5,1} < t_{5,8}$ then $e_5 = 1$ otherwise $e_5 = -1$

Here $t_{5,1} > t_{5,8}$ (i.e. $9 > 6$) the condition is not satisfied.

Therefore $e_5 = -1$

$$s_5 = -0.0244$$

If $t_{6,1} < t_{6,8}$ then $e_6 = 1$ otherwise $e_6 = -1$

Here $t_{6,1} < t_{6,8}$ (i.e. $3 < 5$) the condition is satisfied.

Therefore $e_6 = 1$

$$s_6 = 0.0417$$

If $t_{7,1} < t_{7,8}$ then $e_7 = 1$ otherwise $e_7 = -1$

Here $t_{7,1} < t_{7,8}$ (i.e. $5 < 6$) the condition is satisfied.

Therefore $e_7 = 1$

$$s_7 = 0.0263$$

If $t_{8,1} < t_{8,8}$ then $e_8 = 1$ otherwise $e_8 = -1$

Here $t_{8,1} < t_{8,8}$ (i.e. $2 < 8$) the condition is satisfied.

Therefore $e_8 = 1$

$$s_8 = 0.0303$$

If $t_{9,1} < t_{9,8}$ then $e_9 = 1$ otherwise $e_9 = -1$

Here $t_{9,1} < t_{9,8}$ (i.e. $1 < 9$) the condition is satisfied.

Therefore $e_9 = 1$

$$s_9 = 0.0400$$

If $t_{10,1} < t_{10,8}$ then $e_{10} = 1$ otherwise $e_{10} = -1$

Here $t_{10,1} < t_{10,8}$ (i.e. $6 < 9$) the condition is satisfied.

Therefore $e_{10} = 1$

$$s_{10} = 0.0345$$

Step 2

Jobs are sequenced according to decreasing order of slope index values

$$0.0417 \geq 0.0400 \geq 0.0345 \geq 0.0313 \geq 0.0303 \geq 0.0278 \geq 0.0263 \geq -0.0244 \geq -0.0286 \geq -0.0323$$

$$s_6 \geq s_9 \geq s_{10} \geq s_4 \geq s_8 \geq s_3 \geq s_7 \geq s_5 \geq s_2 \geq s_1$$

Step 3

Output optimal sequence is {6,9,10,4,8,3,7,5,2,1}

Thus total processing time can be calculated as:

Job i	M/c 1		M/c 2		M/c 3		M/c 4		M/c 5		M/c 6		M/c 7		M/c 8	
	Time		Time		Time		Time		Time		Time		Time		Time	
	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out
6	0	3	3	5	5	9	9	12	12	19	19	21	21	24	24	29
9	3	4	5	9	9	15	15	17	19	24	24	28	28	31	31	40
10	4	10	10	13	15	20	20	25	25	28	28	35	35	36	40	49
4	10	14	14	20	20	23	25	26	27	35	35	41	41	48	49	56
8	14	16	20	28	28	37	37	38	38	44	44	47	48	52	56	64
3	16	24	28	29	29	43	43	47	47	50	50	59	59	64	64	73
7	24	29	29	38	38	47	47	50	50	54	59	67	67	73	73	79
5	29	38	38	46	46	49	49	58	58	60	67	72	73	82	82	88
2	38	41	46	55	55	60	60	67	67	69	72	77	82	88	88	89
1	41	47	55	60	60	61	61	74	74	74	83	86	88	92	92	94

Therefore, total processing time = 94 (Units)

Total idle time for M/c 1 = $94 - 47 = 47$ (Units)

Total idle time for M/c 2 = $3 + 1 + 1 + (94 - 60) = 39$ (Units)

Total idle time for M/c 3 = $5 + 5 + 6 + (94 - 61) = 49$ (Units)

Total idle time for M/c 4 = $9 + 3 + 3 + 11 + 5 + 2 + (94 - 74) = 53$ (Units)

Total idle time for M/c 5 = $12 + 1 + 6 + 3 + 4 + 7 + 5 + (94 - 83) = 49$ (Units)

Total idle time for M/c 6 = $19 + 3 + 3 + 3 + 6 + (94 - 86) = 42$ (Units)

Total idle time for M/c 7 =21+4+4+5+7+3+ (94-92) =46 (Units)

Total idle time for M/c 8 =24+2+3+3=32 (Units)

5.3. CONCLUSION

Rule
Make span

Gupta's
94 Units

Palmer's
92 Units

From the above table we conclude that a result of the work found that that out of the Palmer's Heuristic Model and Gupta's Heuristic Model, for the flow shop scheduling problem using Make span Criterion, the Palmer's Heuristic Model is the best one because of Make span is minimum than that of Gupta's Heuristic Model.

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