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Optimizing profit in an Inventory Model using Pentagonal dense fuzzy and dense fuzzy lock set

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Absract: In this research paper, pentagonal dense fuzzy and Pentagonal dense fuzzy lock set with single key and double keys are used to optimize the profit obtained in inventory problems with penalty cost. Pentagonal dense fuzzy lock sets help the decision makers to predict the demand of an item and quantity of an item to be purchased under the critical situation by changing the parameter limit at any time during particular critical time. In this paper the parameter demand is considered as pentagonal dense fuzzy and pentagonal dense fuzzy lock. Numerical examples are provided to compare the profit in general fuzzy number, dense fuzzy number and dense fuzzy lock number.

Keywords: Dense set, Fuzzy inventory, Dense fuzzy set, Dense fuzzy lock set, Pentagonal dense fuzzy set and Pentagonal dense fuzzy lock set.

1. Introduction:

Researchers developed the inventory problems under fuzzy environment to face the uncertainty in the inventory problems. At first Zadeh [1] introduced the concept of fuzzy set. Later Bellmann and Zadeh [2] used fuzzy set in decision making problems. Bobylev [3] developed fuzzy control theory with the help of Cauchy problem. The generalization of fuzzy sets for practical applications was discussed by Buckley [4].Diamond[5,6] developed k-type fuzzy numbers and star type fuzzy numbers. Chutia et al. [7] found the membership of general fuzzy number. In olden days demand was assumed as a constant value in inventory management problems. But nowadays the demand varies uncertainly due to some unpredictable
circumstances. This uncertainity can be calculated and predicted by means of learning experiences. The recent study of dense fuzzy set has been able to solve this problem to some extent. The complexities of such situation can be solved further by dense fuzzy lock sets. The upper and lower bound of the parameters involved in inventory can be restricted using fuzzy dense lock set. De and Beg[8] described the triangular dense fuzzy set along with new defuzzification methods. In their observations, the Cauchy sequence was employed, which naturally converged to zero. Using this property, they studied the triangular dense fuzzy set, in which the fuzziness decreases with time. Sujit Kumar [9] introduced triangular dense fuzzy lock sets. De and Mahata [10] developed the cloudy fuzzy set, which is an extension of the triangular dense fuzzy set using a continuous time variable. Recently, Karmakar et al. [11] studied a pollution control scheme for a sponge iron production plant model utilizing the triangular dense fuzzy rule. To get better optimization in decision making problems, Maity et al. [12] developed a two-decision-makers single-decision inventory model utilizing the concept of the triangular dense fuzzy lock set. To get defuzzified index value of the fuzzy lock set, they proved that whenever the Cauchy sequence converges everywhere without zero, then a lock has come which contradicts the fuzzy lock set to reach into the crisp singleton. Applying proper key, it is possible to reach the crisp set alone. Suman Maity, Sujit Kumar

De and Sankar Prasad Mondal[13] developed an EOQ model under fuzzy lock environment. In that research work, they used triangular dense fuzzy lock set. If the parameter in the inventory problem had five values and the decision maker had to make decision to maximize the profit in inventory decision making then pentagonal dense fuzzy lock set will be helpful. In this research work the inventory model considered for optimization involves linear penalty cost[14].Penalty cost is the amount loss to the owners due to some unpredictable circumstances like, the current pandemic situation. In this paper the parameter demand is considered as pentagonal dense fuzzy and pentagonal dense fuzzy lock to find the optimum solution. Also this work reveals the importance of lock keys and also exhibits that the decision can be changed even after the set operation is over.

This paper is structured as follows: section 2 describes the preliminaries needed for the work, the section 3 defines pentagonal dense fuzzy and pentagonal dense fuzzy lock set. In section 4, crisp EOQ model is considered. In section 5, derives fuzzy inventory model with demand rate as pentagonal dense fuzzy set and fuzzy lock set. Section 6 illustrates an example. Section 7 analyzes the effect of changes of lock keys in PDFLS with double keys. The final section concludes the research work.

2. Preliminaries

2.1.Dense fuzzy set [8]

Let $\tilde{A}$ be the fuzzy number whose components are the elements of R × N, R being the set of real numbers and N being the set of natural numbers with the membership grade satisfying the functional relation μ : R × N → [0, 1]. Now as n → ∞ if μ(x, n) → 1 for some x ∈ R and n ∈ N then the set $\tilde{A}$ is called dense fuzzy set. If $\tilde{A}$ is triangular then it is called TDFS. Now, if for some n in N, μ(x, n) attains the highest membership degree 1 then the set itself is called "Normalized

2.2.Triangular Dense Fuzzy Set" or NTDFS.[8]

Let us assume the TDFS as follows

If $\tilde{A}=\left\langle a_2\left(1-\frac{\rho}{1+n}\right), a_2, a_2\left(1+\frac{\sigma}{1+n}\right)\right\rangle$ for $0<\rho,\sigma<1$, n ≥ 0 and the corresponding membership function along with the graphical illustration are given in equation (1) and figure1.

$$\mu(x,n)=\begin{cases}\dfrac{x-a_2\left(1-\frac{\rho}{1+n}\right)}{\frac{\rho a_2}{1+n}}, & a_2\left(1-\frac{\rho}{1+n}\right)\le x\le a_2\\[2ex] \dfrac{a_2\left(1+\frac{\sigma}{1+n}\right)-x}{\frac{\sigma a_2}{1+n}}, & a_2\le x\le a_2\left(1+\frac{\sigma}{1+n}\right)\end{cases} \qquad \text{------------------(1)}$$

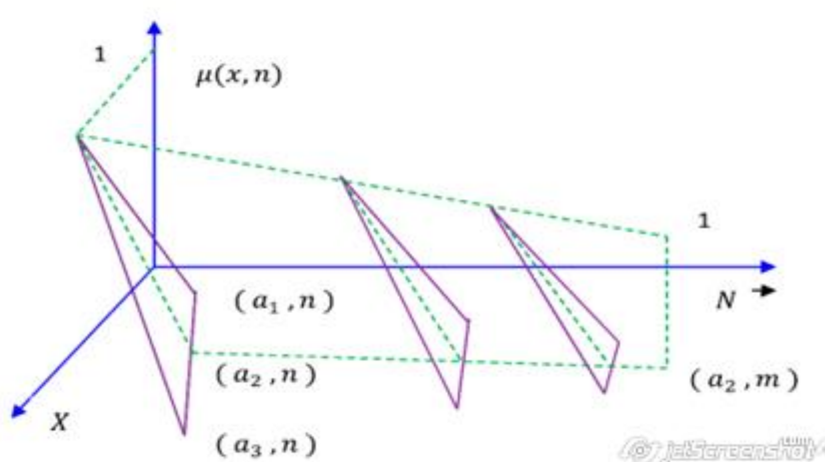


Figure 1: Membership function of NDTFS

Figure 1 shows initially the span of triangular fuzzy components are higher but as the progress of learning experiences the components become more closer and the membership degree gets near 1**. [11]**

2.3.Triangular dense fuzzy lock sets [9]

Let $\tilde{A}=a_1,a_2,a_3$ be the TDF S whose components are the elements of R × N, then if its membership function μ : R × N → [0, 1] satisfies for n → ∞ if μ(x, n)=1 with some x ∈ R and n∈ N and in this case the index value $I(\tilde{A}) \nrightarrow a_2$, then the TDFS A is called triangular dense fuzzy lock set. It is also normal by its initial assumptions over NTDFS.

Let the TDFS A $\tilde{A}=\langle a\{1-\rho f_n, a, a\{1+\sigma g_n\}\rangle$ for 0 < $\rho,\sigma \in$ R and f_n, g_n are two Cauchy sequences of functions having converging points $\frac{1}{k_1},\frac{1}{k_2}$,$0\neq k_1,k_2 \in R$, respectively, then the fuzzy set A is called triangular dense fuzzy lock set with double keys k_1 and k_2, and they depend upon ρ and σ, respectively. The membership function of $\tilde{A}$ is stated as follows

$$\mu(x,n)=\begin{cases}\dfrac{x-a(1-\rho f_n)}{a\rho f_n}, & a(1-\rho f_n)\le x\le a\\ \dfrac{a(1+\sigma g_n)-x}{a\sigma g_n}, & a\le x\le a(1+\sigma g_n)\end{cases}$$

The graphical representation of TDFLS is given in Figure 2.

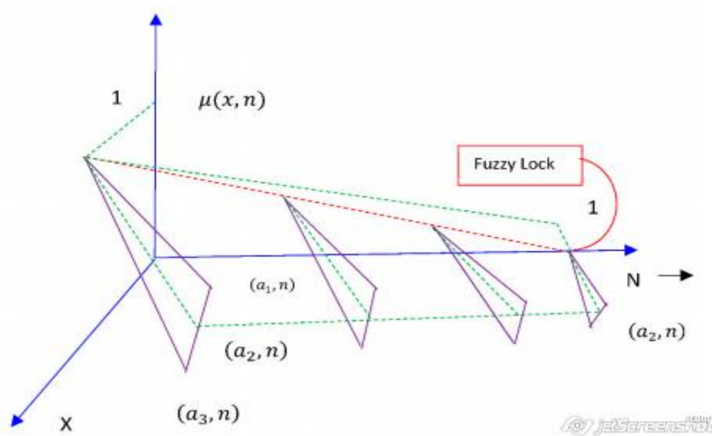


Figure 2 : Membership function of TDFLS

3.Pentagonal dense fuzzy set and Pentagonal dense fuzzy lock set

3.1. Pentagonal dense fuzzy set[15]

A fuzzy number $\tilde{A}$=(a_1 , a_2 , a_3 , a_4 ,a_5) with $a_1 = a_3 f_n$, $a_2 = a_3 g_n$, $a_4 = a_3 h_n$, $a_5 = a_3 t_n$ where f_n, g_n, h_n, t_n are all converges to 1 as $n\to\infty$ then the fuzzy set $\tilde{A}$=(a_1 , a_2 , a_3 , a_4 ,a_5) converges to t crisp singleton set {a_3}. This fuzzy set is called as Pentagonal dense fuzzy set(PDFS). The graphical representation of PDFS are given in figure 2.

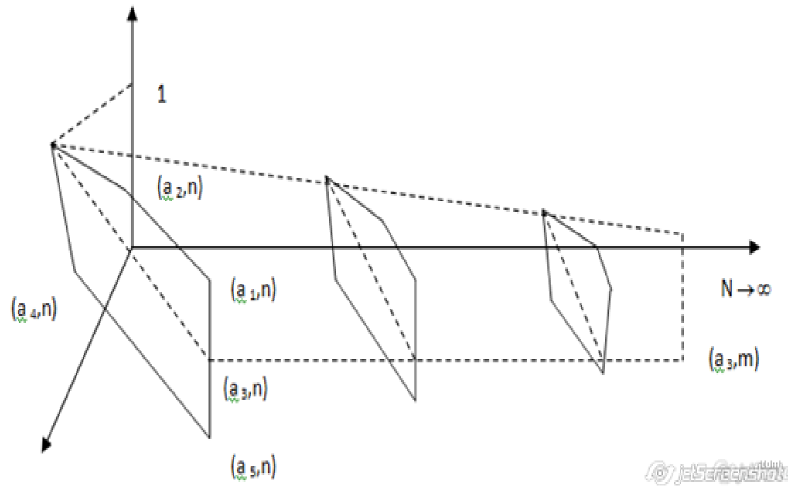


Figure 3: Membership function of PDFS

In particular by using the definition of PDFS we assume the membership function of PDFS as

$$\mu(x,n)=\begin{cases} r\dfrac{x-a_3\left(1-\frac{\tau}{1+n}\right)}{\frac{a_3(\tau-\beta)}{1+n}}, & a_3\left(1-\frac{\tau}{1+n}\right)\le x\le a_3\left(1-\frac{\beta}{1+n}\right) \\ 1-(1-r)\dfrac{x-a_3\left(1-\frac{\beta}{1+n}\right)}{a_3\left(\frac{\beta}{1+n}\right)}, & a_3\left(1-\frac{\beta}{1+n}\right)\le x\le a_3 \\ 1 & x=a_3 \\ 1-(1-r)\dfrac{a_3\left(1+\frac{\gamma}{1+n}\right)-x}{a_3\left(\frac{\gamma}{1+n}\right)}, & a_3\le x\le a_3\left(1+\frac{\gamma}{1+n}\right) \\ r\dfrac{a_3\left(1+\frac{\sigma}{1+n}\right)-x}{\frac{a_3(\sigma-\gamma)}{1+n}}, & a_3\left(1+\frac{\gamma}{1+n}\right)\le x\le a_3\left(1+\frac{\sigma}{1+n}\right) \\ 0 & otherwise \end{cases}$$

$0< r < 1, 0<\tau,\beta,\gamma,\sigma<1$

--------(2)

3.2.Pentagonal dense fuzzy lock set

Let $\tilde{A}$=(a_1 , a_2 , a_3 , a_4 ,a_5) be the PDFS whose components are the elements of R×N then its membership function μ : R × N ⟶ [0, 1] satisfies for n⟶∞ if μ(x.n)↛1 with some xϵR , nϵN and the index value I($\tilde{A}$)↛ a_3 then the PDFS is called pentagonal dense fuzzy lock set.

A fuzzy number $\tilde{A}$=(a_1 , a_2 , a_3 , a_4 ,a_5) with a_1 = $a_3 f_n$, a_2 = $a_3 g_n$, a_4 = $a_3 h_n$, a_5= a_3 t_n where f_n, g_n, h_n, t_n are all converged to 1 as $n\to\infty$ then the fuzzy set $\tilde{A}$= (a_1,a_2,a_3,a_4,a_5) does not converge to a singleton set { a_3 }. This fuzzy set is called as Pentagonal dense fuzzy lock set(PDFLS). The graphical representation of PDFLS are given in figure 4.

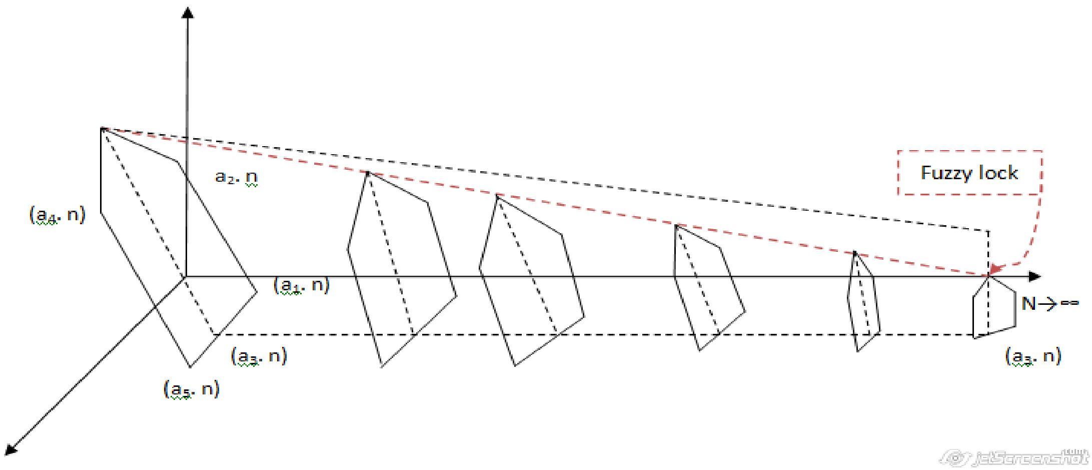


Figure 4 : Graphical Representation of PDFLS

The membership function of PDFLS is

$$\mu(D)=\begin{cases} r\dfrac{x-a_3\left(1-\tau\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}{a_3(\tau-\beta)\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)} & a_3\left(1-\tau\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)\le x\le a_3\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)\\[2ex] 1-(1-r)\left[\dfrac{x-a_3\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}{a_3\left(\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}\right] & a_3\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)\le x\le a_3\\[2ex] 1-(1-r)\dfrac{\left(D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)\right)-D}{D_0\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)} & D_0\le D\le D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)\\[2ex] r\dfrac{D_0\left(1+\sigma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)-D}{D_0(\sigma-\gamma)\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)} & D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)\le D\le D_0\left(1+\sigma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right) \end{cases}$$

Now consider the inventory model derived in [14]

4.Crisp Inventory Model

4.1 Assumptions

1. A single product is considered over the period T unit of time.
2. Lead time is zero.
3. Demand is price dependent.
4. Replenishment is instantaneous.
5. Shortages are allowed.

4.2 Notations

A - set up cost per cycle.

B – shortage cost

H - Holding cost per cycle
Θ – Time at which the product starts to deteriorate.
C - Average total cost per unit time.
D_0 - Demand .
$\widetilde{D}$ - fuzzy demand.
Q - Number of items to be purchased.
T - cycle length.
T_1- Time horizon.

For this model,
Shortage cost over (t_1, T) is –BD(T-t_1)[14]
penalty cost due to deterioration of the product over (Θ,t) is $\pi D\left[\frac{T^2}{2} - \theta t + \frac{\theta^2}{2}\right]$.[14]

The inventory holding cost over the time period (0,T) is $\frac{HDT}{2}$ [10]

Setup cost= $\frac{A}{T}$ [10]

The total cost per unit time is $C = \frac{A}{T} + \frac{\pi D t_1^2}{2T} - \frac{\pi t_1 \theta D}{T} + \frac{\pi \theta^2 D}{2T} + \frac{HDT}{2} - BD + \frac{BDt_1}{T}$ ------(3) [14]

Differentiate with respect to T and equate to zero we get the optimal cycle time T^* is

$$T^* = \sqrt{\frac{2A + \pi D\theta^2 + \pi t_1^2 D - 2\pi t_1 \theta D + 2BDt_1}{HD}} \text{ --------- (4) [14]}$$

and the optimal order quantity is $Q^* = D\sqrt{\frac{2A + \pi D\theta^2 + \pi t_1^2 D - 2\pi t_1 \theta D + 2BDt_1}{HD}}$ -----------(5).[14]

The profit for this inventory model can be calculated using the formula
Z= [Revenue-[holding cost+ setup cost +shortage cost + penalty cost]]×Time horizon
Now the Objective function of the inventory model reduces to maximize the profit using

$$Z = \left[pDt_1 - \frac{A}{T} - \frac{\pi D t_1^2}{2T} + \frac{\pi t_1 \theta D}{T} - \frac{\pi \theta^2 D}{2T} - \frac{HDT}{2} + BD - \frac{BDt_1}{T}\right] \times T_1 \text{ ---------------(6)}$$

(6) can be written as Z=gd-h -------------------(7)

Where $g = \left[pt_1 - \frac{\pi t_1^2}{2T} + \frac{\pi t_1 \theta}{T} - \frac{\pi \theta^2}{2T} - \frac{HT}{2} + B - \frac{Bt_1}{T}\right]T_1$ and $h = \frac{A}{T}$ -----------------------(8)

From (7) we have $D = \frac{z+h}{g}$

5. Fuzzy Mathematical Model

This section defines membership function, graphical representation and its defuzzification method of pentagonal dense fuzzy set(PDFS) and pentagonal dense fuzzy lock set(PDFLS)

5.1. Demand as Pentagonal dense fuzzy number

For the model defined in section 4, consider the demand as pentagonal dense fuzzy number since the demand plays vital role in inventory and it cannot be predicted. Using dense fuzzy number it is able to predict by learning frequencies.

The membership function of the demand rate having n learning frequency of linear pentagonal dense fuzzy set with symmetry is

$$\mu(D,n)=\begin{cases} r\dfrac{D-D_0\left(1-\frac{\tau}{1+n}\right)}{\frac{D_0(\tau-\beta)}{1+n}}, & D_0\left(1-\frac{\tau}{1+n}\right)\le d\le D_0\left(1-\frac{\beta}{1+n}\right) \\ 1-(1-r)\dfrac{D-D_0\left(1-\frac{\beta}{1+n}\right)}{D_0\left(\frac{\beta}{1+n}\right)}, & D_0\left(1-\frac{\beta}{1+n}\right)\le d\le D_0 \\ 1-(1-r)\dfrac{D_0\left(1+\frac{\gamma}{1+n}\right)-D}{D_0\left(\frac{\gamma}{1+n}\right)}, & D_0\le D\le D_0\left(1+\frac{\gamma}{1+n}\right) \\ r\dfrac{D_0\left(1+\frac{\sigma}{1+n}\right)-D}{\frac{D_0(\sigma-\gamma)}{1+n}}, & D_0\left(1+\frac{\gamma}{1+n}\right)\le D\le D_0\left(1+\frac{\sigma}{1+n}\right) \\ 0 & otherwise \end{cases} \quad 0<r<1 \text{-----------------(9)}$$

Where the α cuts are given by

$$A_{1L}=D_0\left(1-\frac{\tau}{1+n}+\frac{\alpha(\tau-\beta)}{r(1+n)}\right),\ A_{2L}=D_0\left(1-\frac{\beta}{1+n}+\frac{1-\alpha}{1-r}\frac{\beta}{1+n}\right),\ D_0\ A_{2R}=D_0\left(1+\frac{\gamma}{1+n}-\frac{(1-\alpha)\gamma}{(1-r)(1+n)}\right),$$

$$A_{1R}=D_0\left(1+\frac{\sigma}{1+n}-\frac{\alpha(\sigma-\gamma)}{1+n}\right)$$

The defuzzification formula is

$$I(\tilde{D})=\frac{1}{4}\sum_{n=1}^{N}\int_0^1(A_{1L}+A_{2L}+A_{2R}+A_{1R})d\alpha$$

On integration and simplification we get the following equation

$$I(\tilde{D})=D_0+\frac{D_0}{8N}\left(\begin{array}{l}\frac{-2r\tau+\tau-\beta}{r}+(\beta-\gamma)\frac{2r-1}{1-r}+\\ \frac{2r\sigma-\sigma+\gamma}{r}\end{array}\right)\left[\frac{1}{2}+\frac{1}{3}+......\frac{1}{1+N}\right] \text{------------------(10)}$$

The membership function of the fuzzy objective function is

$$\mu(Z,n)=\begin{cases} r\dfrac{\frac{z+h}{g}-D_0\left(1-\frac{\tau}{1+n}\right)}{\frac{D_0(\tau-\beta)}{1+n}}, & D_0g\left(1-\frac{\tau}{1+n}\right)-h\le z\le D_0g\left(1-\frac{\beta}{1+n}\right)-h \\ 1-(1-r)\dfrac{\frac{z+h}{g}-D_0\left(1-\frac{\beta}{1+n}\right)}{D_0\left(\frac{\beta}{1+n}\right)}, & D_0g\left(1-\frac{\beta}{1+n}\right)-h\le z\le D_0g-h \\ 1-(1-r)\dfrac{D_0\left(1+\frac{\gamma}{1+n}\right)-\frac{z+h}{g}}{D_0\left(\frac{\gamma}{1+n}\right)}, & D_0g-h\le z\le D_0\left(1+\frac{\gamma}{1+n}\right)-h \\ r\dfrac{D_0\left(1+\frac{\sigma}{1+n}\right)-\frac{z+h}{g}}{\frac{D_0(\sigma-\gamma)}{1+n}}, & D_0\left(1+\frac{\gamma}{1+n}\right)-h\le z\le D_0\left(1+\frac{\sigma}{1+n}\right) \\ 0 & otherwise \end{cases} \quad 0<r<1 \text{-----------------(11)}$$

The left and right α cuts are given by

$$Z_{1L} = D_0 g - h - \frac{D_0 g \tau}{1+n} + \frac{\alpha D_0 g(\tau - \beta)}{r(1+n)}$$

$$Z_{2L} = D_0 g - h - \frac{D_0 g \beta}{1+n} + \frac{(1-\alpha) D_0 \beta}{(1+n)(1-r)}$$

$$Z_{2R} = D_0 g + \frac{D_0 g \gamma}{1+n} - h - \frac{(1-\alpha) D_0 \gamma g}{(1-r)(1+n)}, \quad Z_{1R} = D_0 g - h + \frac{D_0 g \sigma}{1+n} - \frac{\alpha D_0 g(\sigma - \gamma)}{r(1+n)}$$

Then the defuzzification is

$$I(\tilde{z}) = \frac{1}{4N} \sum_{n=1}^{N} \int_0^1 (Z_{1L} + Z_{2L} + Z_{1R} + Z_{2R})\, d\alpha$$

After integrating and simplifying we get the equation as

$$I(\tilde{Z}) = D_0 g - h + \frac{D_0 g}{8N} \left(\frac{-2r\tau + \tau - \beta}{r} + (\beta - \gamma)\frac{2r-1}{1-r} + \frac{2r\sigma - \sigma + \gamma}{r} \right) \left[\frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{1+N} \right] \text{---------------(12)}$$

where N is the total number of learning frequency to achieve the maturity in learning experiences.
Thus the problem of dense fuzzy model becomes

Max Z = $D_0 g - h + \frac{D_0 g}{8N} \left(\frac{-2r\tau + \tau - \beta}{r} + (\beta - \gamma)\frac{2r-1}{1-r} + \frac{2r\sigma - \sigma + \gamma}{r} \right) \left[\frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{1+N} \right]$ subject to the constraints

$$Q = Tt_1 \left[D_0 + \frac{D_0}{8N} \left(\frac{-2r\tau + \tau - \beta}{r} + (\beta - \gamma)\frac{2r-1}{1-r} + \frac{2r\sigma - \sigma + \gamma}{r} \right) \left[\frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{1+N} \right] \right] \text{-------(13)}$$

5.2.Demand as Pentagonal dense fuzzy lock

Now if the demand rate is pentagonal fuzzy dense lock number then its membership function is defined as

$$\mu(D,n) = \begin{cases} r\dfrac{D - D_0\left(1 - \tau\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right)}{D_0(\tau - \beta)\left(\frac{1}{k_1} - \frac{1}{n+1}\right)} & D_0\left(1 - \tau\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right) \le D \le D_0\left(1 - \beta\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right) \\ 1 - (1-r)\left[\dfrac{D - D_0\left(1 - \beta\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right)}{D_0\left(\beta\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right)}\right] & D_0\left(1 - \beta\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\right) \le D \le D_0 \\ 1 - (1-r)\dfrac{D_0\left(1 + \gamma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right) - D}{D_0\gamma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)} & D_0 \le D \le D_0\left(1 + \gamma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right) \\ r\dfrac{D_0\left(1 + \sigma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right) - D}{D_0(\sigma - \gamma)\left(\frac{1}{k_2} - \frac{1}{n+1}\right)} & D_0\left(1 + \gamma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right) \le D \le D_0\left(1 + \sigma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\right) \end{cases} \quad ----- (14)$$

$0<r<1$

The left and right α cuts are defined as

$$\mu_{1L} = D_0 - \frac{D_0\tau}{k_1}\left(1-\frac{\alpha}{r}\right)+\frac{D_0\tau}{1+n}\left(1-\frac{\alpha}{r}\right)-\frac{\alpha D_0\beta}{rk_1}+\frac{\alpha D_0\beta}{r(1+n)},\ \mu_{2L} = D_0 - \frac{D_0\beta}{k_1}\left(1+\frac{1}{1-r}\right)+\frac{D_0\beta}{1+n}\left(1-\frac{1}{1-r}\right)-\frac{\alpha D_0\beta}{(1-r)k_1}+\frac{\alpha D_0\beta}{(1-r)(1+n)}$$

$$\mu_{2R} = D_0 + \frac{D_0\gamma}{k_2}\left(1-\frac{1}{1-r}\right)-\frac{D_0\gamma}{1+n}\left(1+\frac{1}{1-r}\right)-\frac{\alpha D_0\beta}{(1-r)k_1}+\frac{\alpha D_0\beta}{(1-r)(1+n)}$$

$$\mu_{1R} = D_0 - \frac{D_0\sigma}{k_2}\left(1-\frac{\alpha}{r}\right)-\frac{D_0\sigma}{1+n}\left(1-\frac{\alpha}{r}\right)+\frac{\alpha D_0\gamma}{rk_2}-\frac{\alpha D_0\gamma}{r(1+n)}$$

Then the defuzzification is

$$I(\tilde{D}) = \frac{1}{4}\sum_{n=1}^{N}\int_0^1 \left(\mu_{1L}+\mu_{2L}+\mu_{2R}+\mu_{1R}\right)d\alpha$$

On integration and simplification we get the equation as

$$I(\tilde{D}) = D_0 + \frac{D_0}{8}\left[\frac{\tau-\beta-2r\tau}{rk_1}+(1-r)\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)+\frac{2r\sigma-\sigma+\gamma}{k_2 r}-\frac{r^2}{1-r}\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)\right]$$

$$+\frac{D_0}{8N}\left[\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{1+N}\right]\left[\frac{-\tau+\beta+2r\tau}{r}+\frac{-2r\sigma+\sigma-\gamma}{r}+(1-r)(\beta-\gamma)-\frac{r^2}{1-r}(\beta-\gamma)\right] \quad \text{---(15)}$$

The membership function of the fuzzy objective function is defined as

$$\mu(Z,n) = \begin{cases} r\dfrac{\dfrac{z+h}{g}-D_0\left(1-\tau\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}{D_0(\tau-\beta)\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)} & D_0\left(1-\tau\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)-h \le Z \le D_0\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)-h \\ 1-(1-r)\left[\dfrac{\dfrac{z+h}{g}-D_0\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}{D_0\left(\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)}\right] & D_0\left(1-\beta\left(\dfrac{1}{k_1}-\dfrac{1}{n+1}\right)\right)-h \le Z \le D_0-h \\ 1-(1-r)\dfrac{\left(D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)\right)-\dfrac{z+h}{g}}{D_0\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)} & D_0-h \le Z \le D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)-h \\ r\dfrac{D_0\left(1+\sigma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)-\dfrac{z+h}{g}}{D_0(\sigma-\gamma)\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)} & D_0\left(1+\gamma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)-h \le Z \le D_0\left(1+\sigma\left(\dfrac{1}{k_2}-\dfrac{1}{n+1}\right)\right)-h \end{cases} \quad \text{-----(16)}$$

The right and left α cuts are defined as

$$B_{1L} = D_0 g - h + \frac{D_0 g\left(\frac{1}{k_1}-\frac{1}{n+1}\right)}{r}\left(\alpha(\tau-\beta)-r\tau\right),\ B_{2L} = D_0 g - h - \frac{D_0 g\beta\left(\frac{1}{k_1}-\frac{1}{n+1}\right)}{1-r}(\alpha-r)$$

$$B_{2R} = D_0 g - h + \frac{D_0 g\gamma\left(\frac{1}{k_2}-\frac{1}{n+1}\right)}{1-r}(\alpha-r),\ B_{1R} = D_0 g - h + \frac{D_0 g\left(\frac{1}{k_2}-\frac{1}{n+1}\right)}{r}\left(-\alpha(\sigma-\tau)+r\sigma\right)$$

Then the membership function is calculated by using the formula

$$I(\tilde{z}) = \frac{1}{4N}\sum_{n=1}^{N}\int_0^1 (B_{1L}+B_{2L}+B_{3L}+B_{4L})\ d\alpha$$

After integrating and simplifying we get

$$I(\tilde{Z}) = D_0 g - h + \frac{D_0 g}{8}\left[\frac{\tau-\beta-2r\tau}{rk_1}+(1-r)\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)+\frac{2r\sigma-\sigma+\gamma}{k_2 r}-\frac{r^2}{1-r}\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)\right]+$$

$$\frac{gD_0}{8N}\left[\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{1+N}\right]\left[\frac{-\tau+\beta+2r\tau}{r}+\frac{-2r\sigma+\sigma-\gamma}{r}+(1-r)(\beta-\gamma)-\frac{r^2}{1-r}(\beta-\gamma)\right] \quad \text{------------(17)}$$

Thus the problem of dense fuzzy lock model becomes

$$\text{Max } Z = \begin{matrix} D_0 g - h + \frac{D_0 g}{8}\left[\frac{\tau-\beta-2r\tau}{rk_1} + (1-r)\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right) + \frac{2r\sigma-\sigma+\gamma}{k_2 r} - \frac{r^2}{1-r}\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)\right] + \\ \frac{gD_0}{8N}\left[\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{1+N}\right]\left[\frac{-\tau+\beta+2r\tau}{r} + \frac{-2r\sigma+\sigma-\gamma}{r} + (1-r)(\beta-\gamma) - \frac{r^2}{1-r}(\beta-\gamma)\right] \end{matrix}$$

subject to the constraints

$$Q = Tt_1 \begin{bmatrix} D_0 + \frac{D_0}{8}\left[\frac{\tau-\beta-2r\tau}{rk_1} + (1-r)\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right) + \frac{2r\sigma-\sigma+\gamma}{k_2 r} - \frac{r^2}{1-r}\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)\right] \\ + \frac{D_0}{8N}\left[\frac{1}{2}+\frac{1}{3}+\ldots\ldots\frac{1}{1+N}\right]\left[\frac{-\tau+\beta+2r\tau}{r} + \frac{-2r\sigma+\sigma-\gamma}{r} + (1-r)(\beta-\gamma) - \frac{r^2}{1-r}(\beta-\gamma)\right] \end{bmatrix} \quad (18)$$

If N→∞ then (16) becomes $I(\tilde{Z}) = D_0 g - h + \frac{D_0 g}{8}\left[\frac{\tau-\beta-2r\tau}{rk_1} + (1-r)\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right) + \frac{2r\sigma-\sigma+\gamma}{k_2 r} - \frac{r^2}{1-r}\left(\frac{\gamma}{k_2}-\frac{\beta}{k_1}\right)\right]$ ------(17)

If $k_1=k_2=k$ then (17) becomes $I(\tilde{z}) = D_0 g - h + \frac{D_0 g}{8}\left[\frac{\tau-\beta-2r\tau}{rk} + \frac{2r-1}{(1-r)k}(\beta-\gamma) + \frac{2r\sigma-\sigma+\gamma}{rk}\right]$ -----(19) , lock fuzzy set with single key.

If k=1 in (18) then we get general fuzzy number $I(\tilde{z}) = D_0 g - h + \frac{D_0 g}{8}\left[\frac{\tau-\beta-2r\tau}{r} + \frac{2r-1}{(1-r)}(\beta-\gamma) + \frac{2r\sigma-\sigma+\gamma}{r}\right]$ ------(20)

5.3.Rules for finding Pentagonal lock keys

For every fuzzy parameter there are upper limits and lower limits. To find the limits of double lock keys, the index value of A is defined as

$$\mu(\tilde{A}) = \frac{1}{4N}\sum_{n=1}^{N}\int_0^1 (K_{1L} + K_{2L} + K_{1R} + K_{2R})\, d\alpha$$

Where $K_{1L} = a + a\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\left(\frac{\alpha(\tau-\beta)-\tau r}{r}\right)$, $K_{2L} = a - a\beta\left(\frac{1}{k_1} - \frac{1}{n+1}\right)\left(\frac{\alpha - r}{1-r}\right)$

$K_{2R} = a + a\gamma\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\left(\frac{\alpha - r}{1-r}\right)$, $K_{1L} = a + a\left(\frac{1}{k_2} - \frac{1}{n+1}\right)\left(\frac{-\alpha(\sigma-\gamma)+\sigma r}{r}\right)$

After substituting and simplifying we get

$$\mu(\tilde{A}) = \frac{a}{2} + \frac{a}{8r}\left[\frac{\tau-\beta-2r\tau-\beta r+r^2\beta}{k_1}\right] + \frac{a}{2} + \frac{a}{8r}\left[\frac{\gamma-\sigma+2r\sigma+\gamma r-r^2\gamma}{k_2}\right]$$

Therefore $\frac{a}{2} + \frac{a}{8r}\left[\frac{\tau-\beta-2r\tau-\beta r+r^2\beta}{k_1}\right] \geq a_L$ where a_L is the lower limit of the fuzzy parameter which implies

$k_1 \leq \frac{a}{4r(2a_L - a)}\left(\tau-\beta-2r\tau-\beta r+r^2\beta\right)$ ----(21)and $\frac{a}{2} + \frac{a}{8r}\left[\frac{\gamma-\sigma+2r\sigma+\gamma r-r^2\gamma}{k_2}\right] \leq a_U$ where a_U is the upper limit of the fuzzy parameter which implies $\frac{a}{4r(2a_U - a)}\left(\gamma r-\gamma r^2+2r\sigma-\sigma+\gamma\right) \leq k_2$ ------(22)

6. Numerical Example

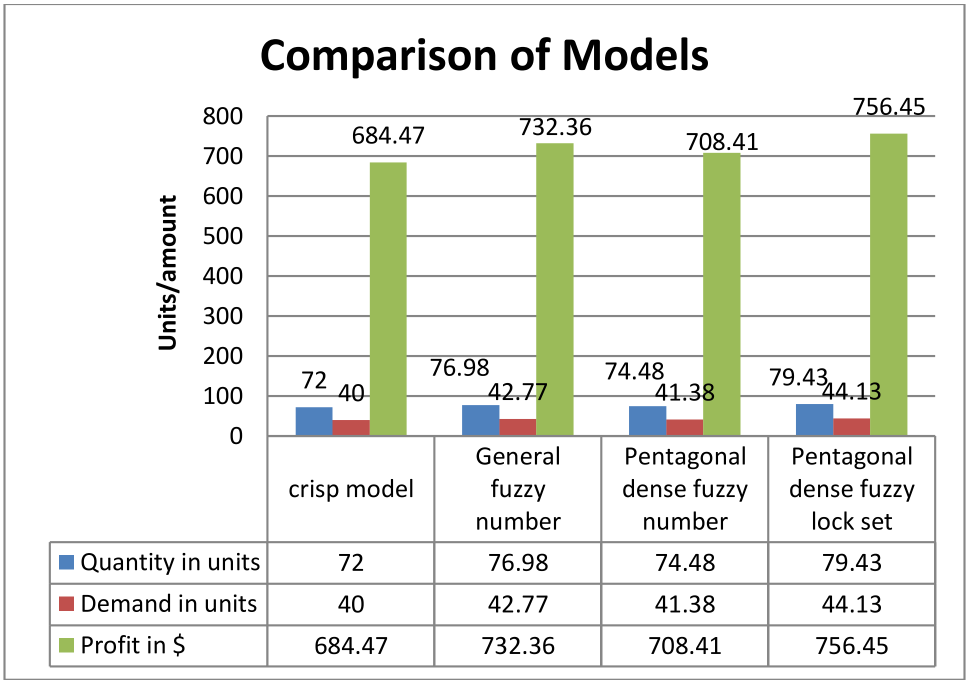
Consider the data set used in [14] $D_0=40$, $t_1=0.6$, θ=0.02, H=3, B=5 p =30 T_1 = 40 days and A =25, r=0.6, β=0.2, τ=0.1, γ=0.5, σ =0.7. Substitute the values in (21) and (22) we get $k_1 \geq -0.233$ and $k_2 \geq 0.211$. hence choose k_1 = 0.5 and k_2 = 0.475.

Substituting the above values in (6), (7),(8),(10),(12),(13),(15) and (18) and tabulated in table 1.

Table 1: Optimum solution of Inventory Model

Model	Learning frequencies N	Cycle time T	Order quantity Q	predicted demand $I(\tilde{D})$	Profit $I(\tilde{Z})$
Crisp Model	-	3	72	40	$684.47
General fuzzy Model	-	3	76.98	42.77	$732.36
Pentagonal Dense fuzzy Model	1 2 3 4	3	74.48 74.07 73.79 73.59	41.38 41.15 40.99 40.89	$708.41 $704.42 $701.76 $699.83
Pentagonal dense fuzzy lock with single key Model k=0.5	1 2 3 4	3	79.43 79.86 80.14 80.33	44.13 44.37 44.52 44.63	$756.45 $760.37 $762.97 $764.87
Pentagonal dense fuzzy lock with double keys Model k_1=0.5 k_2= 0.475	1 2 3 4	3	80.28 80.68 84.47 81.14	44.59 44.82 46.93 45.08	$764.09 $768.02 $770.62 $772.52

From table 1, it is observed that at fourth learning frequencies the order quantity decreased but the profit is maximum in both pentagonal dense fuzzy number and pentagonal dense fuzzy lock set with double keys. In pentagonal dense fuzzy lock with single keys after 4 learning experiences the order quantity decreases and the profit also decreases. The optimality exists after four learning frequencies in PDFS, PDFLS with single key and PDFLS with double key.



Graph 1 compares the values of total cost, quantity and demand obtained in crisp model, general fuzzy, pentagonal dense fuzzy and pentagonal dense fuzzy lock.

Now, for the inventory model [14], consider the demand as triangular dense fuzzy lock with double keys then the following formula is obtained for demand and total cost using the membership function defined in [13].

$$I(\tilde{D}) = D_0 + \frac{D_0}{4}\left(\frac{\sigma}{k_2} - \frac{\rho}{k_1}\right) + \frac{D_0(\rho-\sigma)}{4N}\left(\frac{1}{2} + \frac{1}{3} + \dots\dots + \frac{1}{N+1}\right) \text{------------(23) and}$$

$$I(\tilde{Z}) = D_0 f_1 - f_2 + \frac{D_0}{4}\left(\frac{\sigma}{k_2} - \frac{\rho}{k_1}\right) + \frac{D_0(\rho-\sigma)}{4N}\left(\frac{1}{2} + \frac{1}{3} + \dots\dots + \frac{1}{N+1}\right) \text{--------(24)}$$

Substitute the above values in (23) and (24) , the following value is obtained.

7.Sensitivity analysis

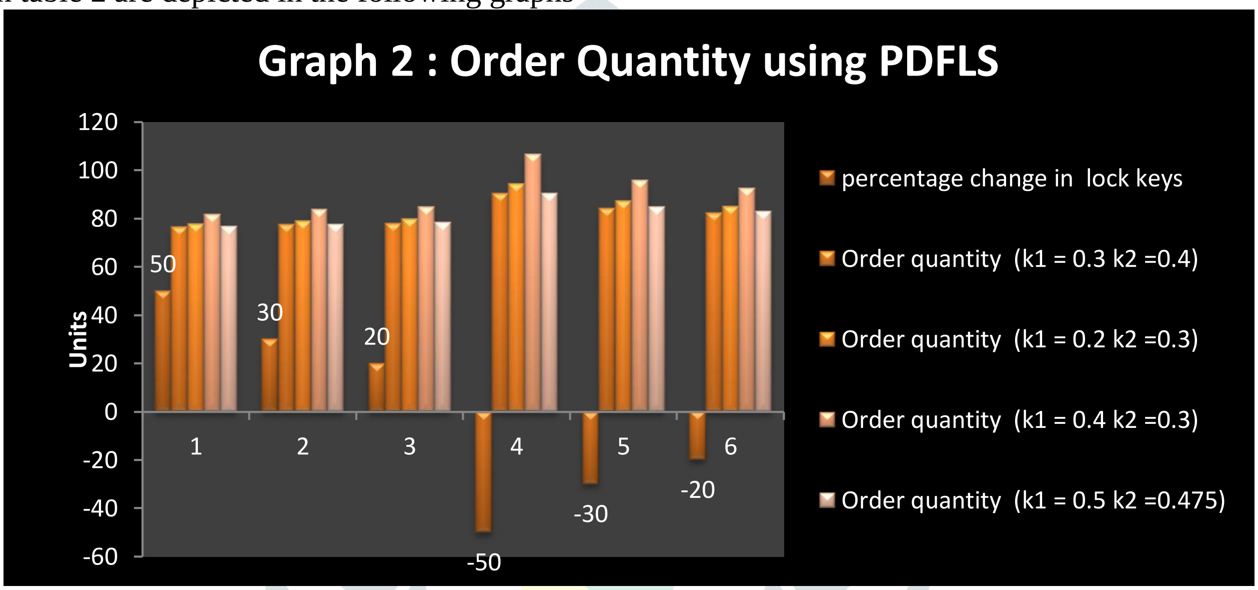
The effect of lock keys k_1 and k_2 in pentagonal dense fuzzy lock set with double keys affect the profit and quantity order that are shown in table 2.

Table 2: Effect of lock keys

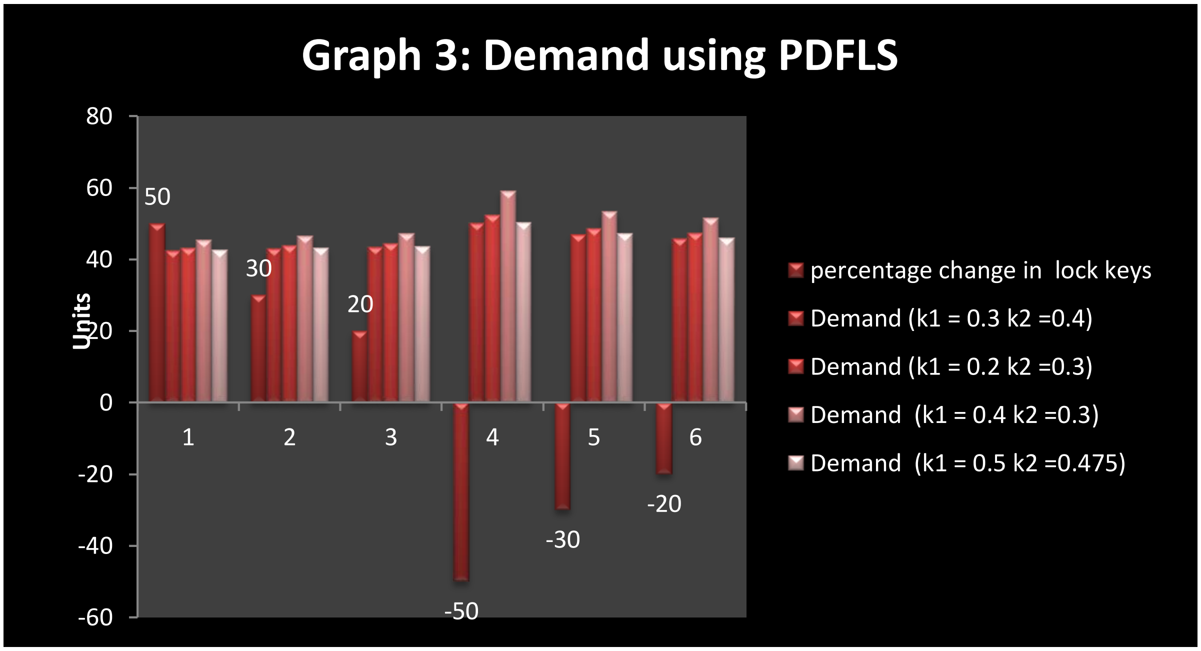
Associated Keys	% change	quantity order units	predicted demand units	profit in $	profit increase compares to crisp in %
$k_1 = 0.3$ $k_2 = 0.4$	+50	76.45	42.47	727..32	+6.26
	+30	77.50	43.06	737.39	+7.73
	+20	78.13	43.41	743.54	+8.63
	-50	90.37	50.21	861.45	+25.86
	-30	84.31	46.84	803.26	+17.36
	-20	82.45	45.80	785.25	+14.72
$k_1 = 0.2$ $k_2 = 0.3$	+50	77.84	43.25	740.68	+8.21
	+30	79.18	43.99	753.38	+10.07
	+20	79.88	44.35	760.29	+11.07
	-50	94.48	52.49	900.85	+31.61
	-30	87.38	48.54	832.47	+21.62
	-20	85.12	47.29	810.74	+18.45

k_1 = 0.4 k_2 =0.3	+50 +30 +20 -50 -30 -20	81.86 83.72 84.91 106.53 95.95 92.61	45.48 46.51 47.17 59.18 53.30 51.45	779.29 797.14 808.60 1016.58 914.91 882.69	+13.85 +16.46 +18.14 +48.52 +33.67 +28.96
k_1=0.5 k_2= 0.475	+50 +30 +20 -50 -30 -20	76.76 77.69 78.43 90.47 84.92 82.87	42.64 43.16 43.57 50.26 47.18 46.04	730.23 739.21 746.34 862.26 808.95 788.99	+06.69 +07.99 +09.04 +25.97 +18.19 +15.27

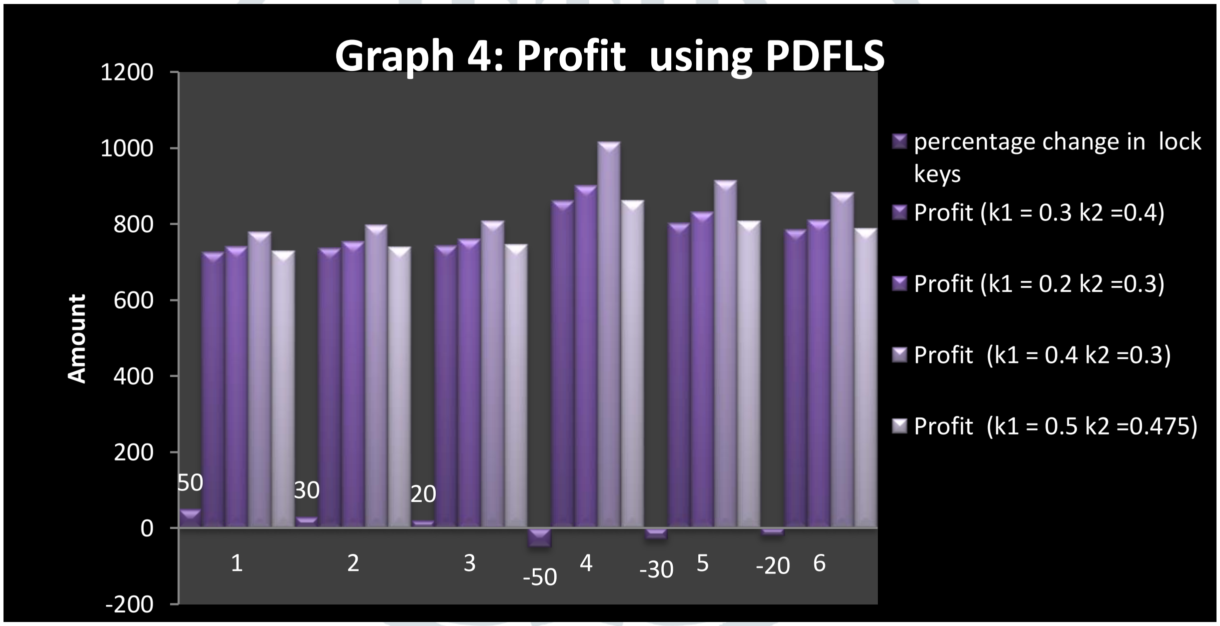
From table 2 it is observed that the increase in profit changes from 6.26% to 48.52%. for all the set of change in lock keys and there is rapid increase in the profit when the parameter values decreases to 50%. The values in table 2 are depicted in the following graphs



The order quantity for the various values of lock keys are depicted in graph 2. Graph 2 shows that for +50% , +30% and +20% of all the lock keys in column 1 of table 2, the order quantity is minimum when compared to the order quantity obtained for -50%, -30% and -20%. For all +50%, +30% and +20% of change in lock keys, the values obtained for order quantity are more or less same values.



Graph 3 depicts the demand of an item for various values of lock keys. Also from graph 3 it is observed that for +50% , +30% and +20% of all the lock keys in column 1 of table 2, the demand is minimum when compared to the demand obtained for -50%, -30% and -20%.



Graph 4 depicts the profit for various values of lock keys. Also from graph 4 it is observed that for +50% , +30% and +20% of all the lock keys in column 1 of table 2, the profit is minimum when compared to the demand obtained for -50%, -30% and -20%.

8. Conclusion

In inventory management, demand plays a vital role and it cannot be predicted in critical situation which occurs in unpredictable time. But this research work shows that by using pentagonal dense fuzzy lock rule, demand can be approximately predicted and the decision can be changed at any time even at the critical time by changing the key factors. Also the values of profit and the units of quantity to be ordered can be updated during the production cycle. The observation from this research work is, profit in inventory model can be optimized using pentagonal dense fuzzy lock. It gives optimum profit when compared to crisp, general fuzzy and pentagonal dense fuzzy rule. Also pentagonal dense fuzzy lock with double keys can be able to maximize profit instead of Pentagonal dense fuzzy lock with single key.

Research Challenge solved in this paper:

This paper solved the problem faced by decision maker while taking the decision in desperate situations. It also provides the effect of changes of lock keys which helped the decision maker to control the loss due to demand and made profit in critical circumstances occurred in inventory problems. Using Pentagonal dense fuzzy lock, the demands can be approximately predicted. Using this PDFS and PDFLS , risk can be assessed under disaster management. This situations may be created and evaluated by means of fuzzy set for n=0, then its distortions may be observed after the occurrences of several frequencies of disaster n=1,2,3... In this situation PDFLS can be utilised to predict the final risk.

Scope of this paper for future work

The membership function of Pentagonal dense fuzzy and dense fuzzy lock can be extended to hexagonal dense fuzzy and hexagonal dense fuzzy lock and also it can be extended to non symmetry PDFS and PDFLS. Like TDFS and TDFLS[8,9], PDFS and PDFLS can be used in medical diagnosis, water purification and Game theory .

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