JETIR.ORG
ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

# The New Integral "DIT Transform" and its Results

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***Abstract:***

In this study, a new integral transform called the DIT transform is introduced. We present the essential properties and proved some useful results including derivative properties and the double convolution theorem.

**(Keywords):**

Double convolution theorem, Integral transform.

## 1 INTRODUCTION:

Integral transform methods are one of the most important methods which have been used recently to solve partial differential equations. Therefore, many phenomena in mathematical physics, engineering and sciences fields can be modeled by mathematical equations written in terms of partial differential equations 1-8. The integral transforms enable us to transform the differential equations in algebraic equations and obtain the exact solution of the partial differential equations. Many scientist and researcher have made a great effort to develop these methods, and they apply them to solve large modern problems in mathematics. For instance, we have the Fourier transform methods and Laplace transform methods 9-16.

Recently, double Laplace transform is used extensively to solve partial differential equations with unknown functions of two variables which obtained good results compared to numeric methods 17-21. Moreover, there exist in literature further extensions of double Laplace transform like double Sumudu transform, 22-27, double Shehu transform,28 double Elzaki transform,29 and double Ramadan group integral transform 30.

In the current study, we introduce double transform in two dimensional spaces. It is called the New Integral DIT Transform. After presenting the definition of the new integral DIT transform for function of two variables, we proved the basic properties concern the existing conditions and the inverse of the DIT transform. Furthermore, we provided the DIT transform of some known functions. Later on, we establish new results relative to the partial differential derivatives and the double convolution theorem.

## 2 A New Integral DIT Transform

In this section, we introduce a novel concept of double transform in two dimensional spaces called the DIT Transform. We proved the basic properties, existing conditions and the inverse of the New General DIT transform.

### 2.1 Definition:

**2.1 Definition:** Let f(x, t) be a function of two variables x and t where $x, t \geq 0, p(s) \neq 0, q(r) \neq 0$ and $\varphi$(s) and $\psi$(r) are the transform functions for x and t, respectively. We define a New General DIT Transform of the function $f(x,t)$ denoted by $F_D(s,r)$ is defined by

$$F_D(s,r) = T^2\{f(x,t),(s,r)\} = p(s)q(r)\int_0^\infty \int_0^\infty e^{-\varphi(s)x} e^{-\psi(r)t} f(x,t)\,dx\,dt \qquad \text{-------------} (1)$$

The inverse of the DIT Transform is given by

$$f(x,t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \frac{1}{p(s)} e^{\varphi(s)y}\varphi'(s)\,ds\, \frac{1}{2\pi i}\int_{\beta-i\infty}^{\beta+i\infty} \frac{1}{q(r)} e^{\psi(r)y}\psi'(r)\,dr\, F_D(s,r) \qquad \text{----------------} (2)$$

Where $\alpha$ and $\beta$ are real constants.

### 2.2 Existence Condition

Let for all $x, t \geq 0$ the function f(x, t) is said to be of exponential order for some $l > 0, k > 0$ if there exists a constant $M > 0$

such that $|f(x,t)| \leq Me^{lx+kt}$

**2.3 Theorem:** If the function f(x, t) is piecewise continuous and satisfies $|f(x,t)| \leq Me^{lx+kt}$ where M is positive constant, then the New General Double Integral Transform exist for all $\varphi(s) > l, \psi(r) > k$

**Proof:** Since $\|T_2\{f(x,t),(s,r)\}\| = \left|p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt\right|$

$$\leq p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}|f(x,t)|dxdt$$

$$\leq p(s)q(r)\ \int_0^\infty \int_0^\infty Me^{lx+kt}e^{-\varphi(s)x}e^{-\psi(r)t}dxdt$$

$$\leq p(s)q(r)M \int_0^\infty e^{-(\varphi(s)-l)x}dx \int_0^\infty e^{-(\psi(r)-k)t}dt$$

$$\leq Mp(s)q(r)\frac{e^{-(\varphi(s)-l)x}}{-(\varphi(s)-l)}\Big|_0^\infty \frac{e^{-(\psi(r)-k)t}}{-(\psi(r)-k)}\Big|_0^\infty$$

$$\leq \frac{M\,p(s).q(r)}{\varphi(s)-l.\ \psi(r)-k}$$

### 3 DIT Transform of Some Special Functions

In this section we introduce the DIT Transform of some Functions.

**3.1 Property :** Let $f(x,t) = 1\ x > 0, t > 0$ then $F_D(s,r) = T^2\{f(x,t),(s,r)\} = \frac{p(s)\ q(r)}{\varphi(s)\ \ \psi(r)}$

**Proof :** $F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt$

Put $f(x,t) = 1$

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}\ 1\ dxdt$$

$$= p(s)q(r)\ \int_0^\infty e^{-\varphi(s)x}dx \int_0^\infty e^{-\psi(r)t}dt$$

$$= p(s)q(r)\frac{e^{-\varphi(s)x}}{-\varphi(s)}\Big|_0^\infty \frac{e^{-\psi(r)t}}{-\psi(r)}\Big|_0^\infty$$

$$= \frac{p(s)q(r)}{\varphi(s)\psi(r)}$$

**3.2 Property :** Let $f(x,t) = x.t\ \ x > 0, t > 0$ then $F_D(s,r) = T^2\{f(x,t),(s,r)\} = \frac{p(s)q(r)}{\varphi(s)^2\psi(r)^2}$

**Proof:** $F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt$

Put $f(x,t) = x.t$

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}x.t\ dx\ dt$$

$$= p(s)q(r)\ \int_0^\infty e^{-\varphi(s)x}x\ dx \int_0^\infty e^{-\psi(r)t}t\ dt$$

$$= p(s)\ q(r)\left\{\left[x\frac{e^{-\varphi(s)x}}{-\varphi(s)}\Big|_0^\infty - \int_0^\infty \frac{e^{-\varphi(s)x}}{-\varphi(s)}dx\right]\left[t\frac{e^{-\psi(r)t}}{-\psi(r)}\Big|_0^\infty - \int_0^\infty \frac{e^{-\psi(r)t}}{-\psi(r)}dt\right]\right\}$$

$$= p(s)q(r)\left\{\frac{1}{\varphi(s)}\left[\frac{e^{-\varphi(s)x}}{-\varphi(s)}\Big|_0^\infty\right]\frac{1}{\psi(r)}\left[\frac{e^{-\psi(r)t}}{-\psi(r)}\Big|_0^\infty\right]\right\}$$

$$= \frac{p(s)q(r)}{\varphi(s)^2\psi(r)^2}$$

**3.3 Property:** If $f(x,t) = x^\alpha t^\beta$ then

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\varphi(s)^{\alpha+1}\psi(r)^{\beta+1}} \text{ for all } \alpha > -1, \beta > -1$$

**Proof:** $F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt$

Put $f(x,t) = x^\alpha t^\beta$

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\ \int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}x^\alpha t^\beta\ dx\ dt$$

$$= p(s)q(r)\ \int_0^\infty e^{-\varphi(s)x}x^\alpha\ dx \int_0^\infty e^{-\psi(r)t}t^\beta\ dt$$

$$= p(s)\ q(r)\left\{\left[x^\alpha\ \frac{e^{-\varphi(s)x}}{-\varphi(s)}\Big|_0^\infty - \int_0^\infty \alpha x^{\alpha-1}\frac{e^{-\varphi(s)x}}{-\varphi(s)}dx\right]\left[t^\beta\frac{e^{-\psi(r)t}}{-\psi(r)}\Big|_0^\infty - \int_0^\infty \beta t^{\beta-1}\ \frac{e^{-\psi(r)t}}{-\psi(r)}dt\right]\right\}$$

$$= p(s)\ q(r)\frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\varphi(s)^{\alpha+1}\psi(r)^{\beta+1}} \text{ for all } \alpha > -1, \beta > -1$$

**3.4 Property :** If $f(x,t) = e^{ax+bt}$ then

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = \frac{p(s)q(r)}{\varphi(s)-a\ \psi(r)-b} \text{ for all } \varphi(s) > a, \psi(r) > b$$

**Proof:** $F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\int_0^\infty\int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt$

Put $f(x,t) = e^{ax+bt}$

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\int_0^\infty\int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}e^{ax+bt}\,dx\,dt$$

$$= p(s)q(r)\int_0^\infty e^{-\varphi(s)x}e^{ax}\,dx\int_0^\infty e^{-\psi(r)t}e^{bt}\,dt$$

$$= p(s)q(r)\int_0^\infty e^{-[\varphi(s)-a]x}\,dx\int_0^\infty e^{-[\psi(r)-b]t}\,dt$$

$$= p(s)q(r)\left\{\frac{e^{-[\varphi(s)-a]x}}{-[\varphi(s)-a]}\Big|_0^\infty \frac{e^{-[\psi(r)-b]t}}{-[\psi(r)-b]}\Big|_0^\infty\right\}$$

$$= \frac{p(s)\,q(r)}{(\varphi(s)-a)(\psi(r)-b)} \text{ for all } \varphi(s) > a, \psi(r) > b$$

**3.5 Property :** If $f(x,t) = e^{i(ax+bt)} = \cos(ax+bt) + i\sin(ax+bt)$ then

$$F_D(s,r) = T_2\{f(x,t),(s,r)\} = \frac{p(s)q(r)}{\varphi(s)-a\ \psi(r)-b} \text{ for all } \varphi(s) > a, \psi(r) > b$$

**Proof:** $F_D(s,r) = T_2\{f(x,t),(s,r)\} = p(s)q(r)\int_0^\infty\int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x,t)dxdt$

Put $f(x,t) = e^{i(ax+bt)}$

$$F_D(s,r) = T^2\{f(x,t),(s,r)\} = p(s)q(r)\int_0^\infty\int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}e^{i(ax+bt)}\,dx\,dt$$

$$= p(s)q(r)\int_0^\infty e^{-\varphi(s)x}e^{iax}\,dx\int_0^\infty e^{-\psi(r)t}e^{ibt}\,dt$$

$$= p(s)q(r)\int_0^\infty e^{-[\varphi(s)-ia]x}\,dx\int_0^\infty e^{-[\psi(r)-ib]t}\,dt$$

$$= p(s)q(r)\left\{\frac{e^{-[\varphi(s)-ia]x}}{-[\varphi(s)-ia]}\Big|_0^\infty \frac{e^{-[\psi(r)-ib]t}}{-[\psi(r)-ib]}\Big|_0^\infty\right\}$$

$$= \frac{p(s)q(r)}{[\varphi(s)-ia][\psi(r)-ib]}$$

$$= p(s)q(r)\frac{[\varphi(s)+ia][\psi(r)+ib]}{[\varphi(s)-ia][\varphi(s)+ia]\ [\psi(r)-ib][\psi(r)+ib]}$$

$$= p(s)\,q(r)\frac{\varphi(s)\,\psi(r)+ib\varphi(s)+ia\psi(r)+i^2ab}{[\varphi(s)^2-(ia)^2][\psi(r)^2-(ib)^2]}$$

$$= p(s)\,q(r)\frac{[\varphi(s)\psi(r)-ab]+i[b\varphi(s)+a\psi(r)]}{[\varphi(s)^2+(a)^2][\psi(r)^2+(b)^2]}$$

Consequently

$$F_D\{\cos(ax+bt)\} = p(s)q(r)\frac{[\varphi(s)\psi(r)-ab]}{[\varphi(s)^2+(a)^2][\psi(r)^2+(b)^2]}$$

$$F_D\{\sin(ax+bt)\} = p(s)q(r)\frac{[b\,\varphi(s)+a\,\psi(r)]}{[\varphi(s)^2+(a)^2][\psi(r)^2+(b)^2]}$$

**3.6 Property :** If $f(x,t) = \cosh(ax+bt)$ then $F_D\{\cosh(ax+bt)\} = p(s)q(r)\frac{[\varphi(s)\psi(r)+ab]}{[\varphi(s)^2-(a)^2][\psi(r)^2-(b)^2]}$

**3.7 Property :** If $f(x,t) = \sinh(ax+bt)$ then $F_D\{\sinh(ax+bt)\} = p(s)\,q(r)\frac{[b\varphi(s)+a\psi(r)]}{[\varphi(s)^2-(a)^2][\psi(r)^2-(b)^2]}$

## 4 PARTIAL DIFFERENTIAL DERIVATIVES OF DIT TRANSFORM

Now, we present some results related to the new general DIT transform of partial derivatives

### 4.1 Theorem: [Derivatives' Properties]

Let $F_D(s,r)$ be the general DIT transform of the function f (x, t) and let $F_G(0,r)$ be the general transform of the function f (0, t). Then

**4.1.1** $T_2\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \varphi(s)F_D(s,r) - p(s)F_G(0,r)$

**Proof :** By using Fubini's theorem, we get

$$T_2 \frac{\partial f(x,t)}{\partial x} = p(s)q(r) \int_0^\infty \int_0^\infty e^{-\varphi(s)x} e^{-\psi(r)t} \frac{\partial f(x,t)}{\partial x} dxdt$$

$$= q(r)\int_0^\infty e^{-\psi(r)t} \left(p(s)\int_0^\infty e^{-\varphi(s)x} \frac{\partial f(x,t)}{\partial x} dx\right) dt$$

From Jafari, 31, we have

$$T_x \frac{\partial f(x,t)}{\partial x} = \varphi(s)F_G(s,t) - p(s)f(0,t) \quad \text{------------------- (3)}$$

Therefore, by applying the general transform with respect to t for Equation (3), we find

$$T_2\left\{\frac{\partial f(x,t)}{\partial x}\right\} = \varphi(s)\left(q(r)\int_0^\infty F_G(s,t)e^{-\psi(r)t}dt\right) - p(s)\left(q(r)\int_0^\infty f(0,t)e^{-\psi(r)t}dt\right)$$

$$= \varphi(s)\left(F_D(s,r)\right) - p(s)F_G(0,r)$$

**4.2.2** $T_2\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\} = \varphi^2(s)F_D(s,r) - \varphi(s)p(s)F_G(0,r) - p(s)\frac{\partial F_G(0,r)}{\partial x}$

$$T_2\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\} = p(s)q(r)\int_0^\infty \int_0^\infty \frac{\partial^2 f(x,t)}{\partial x^2} e^{-(\varphi(s)x+\psi(r)t)} dxdt$$

$$= q(r)\int_0^\infty e^{-\psi(r)t}\left(p(s)\int_0^\infty \frac{\partial^2 f(x,t)}{\partial x^2} e^{-\varphi(s)x} dx\right) dt$$

From Jafari 31

$$T_x\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\} = \varphi^2(s)F_G(s,t) - \varphi(s)p(s)f(0,t) - p(s)\frac{\partial f(0,t)}{\partial x} \quad \text{-------------(4)}$$

Once again, by taking general transform with respect to t for Equation (4), it yields that

$$T_2\left\{\frac{\partial^2 f(x,t)}{\partial x^2}\right\} = \varphi^2(s)F_D(s,r) - \varphi(s)p(s)F_G(0,r) - p(s)\frac{\partial F_G(0,r)}{\partial x}$$

Furthermore, we establish the same results for the second variable t as follows.

**4.2 Theorem:** Let $F_D(s,r)$, $F_G(s,0)$ be the DIT transform and the general transform of the functions f (x, t) and f (x, 0) respectively. Then,

**4.2.1** $T_2\left\{\frac{\partial f(x,t)}{\partial t}\right\} = \psi(r)F_D(s,r) - q(r)F_G(s,0)$

**4.2.2** $T_2\left\{\frac{\partial^2 f(x,t)}{\partial t^2}\right\} = \psi^2(r)F_D(s,r) - \psi(r)q(r)F_G(s,0) - q(r)\frac{\partial F_G(s,0)}{\partial t}$

**4.3 Theorem: (Heaviside Function)**

Let $T_2\{f(x,t),(s,r)\}$ exist and $T_2\{f(x,t),(s,r)\} = F_D(s,r)$ then

$T_2(f(x-\delta,t-\varepsilon)H(x-\delta,t-\varepsilon) = e^{-\varphi(s)\delta}e^{-\psi(r)\varepsilon}F(s,r)$

Where $H(x-\delta,t-\varepsilon)$ is the Heaviside unit step function defined as

$$H(x-\delta,t-\varepsilon) = \begin{cases} 1, & x > \delta, t > \epsilon \\ 0, & \text{otherwise} \end{cases}$$

Proof: Using the definition of new general Double Integral Transform, we find

$$T_2[f(x-\delta,t-\varepsilon)H(x-\delta,t-\varepsilon)] = p(s)q(r)\int_0^\infty \int_0^\infty e^{-\varphi(s)x}e^{-\psi(r)t}f(x-\delta,t-\varepsilon)H(x-\delta,t-\varepsilon)dxdt$$

$$= p(s)q(r)\int_0^\infty \int_0^\infty e^{-[\varphi(s)x+\psi(r)t]} f(x-\delta,t-\varepsilon)dxdt \quad \text{------------- (5)}$$

Putting $x-\delta=\rho$ and $t-\varepsilon=\tau$ in equation (5) we obtain,

$$T_2[f(x-\delta,t-\varepsilon)H(x-\delta,t-\varepsilon)]$$

$$= p(s)q(r)\int_0^\infty \int_0^\infty e^{-[\varphi(s)(\delta+\rho)]} e^{-[\psi(r)(\varepsilon+\tau)]}f(\rho,\tau)d\rho d\tau \quad \text{-------------------- (6)}$$

Thus equation (6) can be simplified into

$$T_2[f(x-\delta,t-\varepsilon)H(x-\delta,t-\varepsilon)] = e^{-\varphi(s)\delta}e^{-\psi(r)\varepsilon}\left(p(s)q(r)\int_0^\infty \int_0^\infty e^{-\varphi(s)\rho}e^{-\psi(r)\tau}f(\rho,\tau)d\rho d\tau\right)$$

$$= e^{-\varphi(s)\delta}e^{-\psi(r)\varepsilon}F_D(s,r)$$

## 5 Double Convolution Result

### 5.1 Theorem:

Let $\mathbf{T_2[f(x,t)]}$ and $\mathbf{T_2[g(x,t)]}$ exist and $\mathbf{T_2[f(x,t)] = F_D(s,r), T_2[g(x,t)] = G_D(s,r)}$ then the new general Double Integral Transform of the convolution of f and g is

$$\mathbf{T_2[f(x,t) ** g(x,t)] = \frac{1}{p(s)q(r)} F_D(s,r).G_D(s,r)}$$

Where $\mathbf{f(x,t) ** g(x,t) = \int_0^\infty \int_0^\infty f(x-\delta, t-\tau) g(\rho,\tau) d\rho d\tau}$

and the symbol ** denotes the double convolution with respect to x and t

**Proof:** Using the definition of new General Double Integral Transform, we obtain

$$\mathbf{T_2[f(x,t) ** g(x,t)] = p(s)\,q(r) \int_0^\infty \int_0^\infty e^{-\varphi(s)x}\, e^{-\psi(r)t} [f(x,t) ** g(x,t)] dxdt}$$

$$\mathbf{= p(s)q(r) \int_0^\infty \int_0^\infty e^{-\varphi(s)x} e^{-\psi(r)t} \left[\int_0^\infty \int_0^\infty f(x-\delta, t-\tau)\ g(\rho,\tau) d\rho d\tau\right] dxdt} \quad \text{-------------(7)}$$

Using the Heaviside unit step function equation (7) can be written as

$$\mathbf{T_2[f(x,t) ** g(x,t)]}$$

$$\mathbf{= p(s)q(r) \int_0^\infty \int_0^\infty e^{-\varphi(s)x} e^{-\psi(r)t} \left[\int_0^\infty \int_0^\infty f(x-\delta, t-\tau) H(x-\delta, t-\varepsilon)\ g(\rho,\tau) d\rho d\tau\right] dxdt} \quad \text{---(8)}$$

Equation (8) can be written as

$$\mathbf{T_2[f(x,t) ** g(x,t)] = \int_0^\infty \int_0^\infty g(\rho,\tau)\, d\rho d\tau \left(p(s)q(r) \int_0^\infty \int_0^\infty e^{-[\varphi(s)(x+\rho)]}\, e^{-[\psi(r)(t+\tau)]} f(x-\delta, t-\tau) H(x-\delta, t-\varepsilon) dxdt\right)}$$

--------------- (9)

$$\mathbf{= \int_0^\infty \int_0^\infty g(\rho,\tau)\, d\rho d\tau e^{-[\varphi(s)\rho]} e^{-[\psi(r)\tau]} F(s,r)}$$

$$= \mathbf{F(s,r) \int_0^\infty \int_0^\infty e^{-[\varphi(s)\rho]}\ e^{-[\psi(r)\tau]} g(\rho,\tau) d\rho d\tau}$$

$$= \mathbf{\frac{1}{p(s)q(r)} F_D(s,r).\,G_D(s,r)}$$

## 6 CONCLUSION

Inspired by the new general integral transform in one dimension, we introduced a novel transform called a new general double integral transform in two dimensional spaces. This new transform collects and implies the known double Laplace transforms in the positive quadrant plane. Practically, we proved some essential properties related to the presented double transform such as derivative properties and the double convolution theorem.

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