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Fractional Differential Operators of Laplace Transforms Involving Multivariable I –Function

Ram Niwas Meghwal
Department of Mathematics, Government College,
sujangarh (INDIA)
(meghwalramniwas@gmail.com)
Dr. K.G. Bhadana
Department of Mathematics, SPC Government College, Ajmer (INDIA)

Abstract
The object of this paper is to solve an integral equation of convolution form having H-function of two variable as it's kernel. Some known results are obtained as special cases
In the present paper we use a fractional differential operator and to study the Multivariable I- function with Laplace Transform. In this paper we establish two Multiplication formulas for the Multivariable I-function. We determine some new and known results for the above function, a number of known results for other simple function follows as special case of our results.
Also using convolution of an integral equation with I-Function as its kernels. Kumbhat R.K. Khan Arif M [14] & I-Function of one variable as Saxena [9] we get new important results

Keywords - integral equation, convolution, , Fractional differential operator, Multivariable H-function. I-Function

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1. Definition and Introduction

The following definition and results will be required in this paper

The Laplace Transform if

$$F(P) = L[f(t);p] = \int_0^\infty e^{-pt} f(t)dt \;, \quad \mathrm{Re}(p) > 0 \qquad \ldots(1.1)$$

Then F(p) is called the Laplace transform of f(t) with parameter p and is represented by F(p) = f (t) Erdelyi [(3) pp.129-131]

$$L[f(t);p] = F(P) \text{ then } L[e^{-at} f(t)] = F(p+a) \qquad \ldots(1.2)$$

And if $f(0) = f^{'}(0) = f^{"}(0) = \ldots.. = f^{m-1}(0) = 0 \quad , \; f^{n}(t)$

Is continuous and differential, then

$$L[f^{n}(t);p] = P^{n} F(p) \qquad \ldots.(1.3)$$

(i) If

$$L[f_1(t)] = F_1(p) \text{ then } L[f_2(t)] = F_2(p)$$

Then convolution theorem for Laplace transform is

$$L\{\int_0^1 f_1(t)\, f_2(t-u)du\} = L\{f_1(t)\}L\{f_2(t)\} = F_1(p).F_2(p) \qquad \ldots(1.4)$$

(ii) The H-Function Defined by Saxena and kumbhat [1] is an extension of Fox's H-Function on specializing the parameters , H-Function can be reduced to almost all the known special function as well as unknown

The Fox's H-Function of one variable is defined and represented in this Paper as follows [see Srivastava et al [2] ,pp 11-13]

$$H[x] = H_{P,Q}^{M,N}\left[x \Big/ {}^{(a_j,\alpha_j)_{1,P}}_{(b_j,\beta_j)_{1,Q}} \right] = \frac{1}{2\pi\omega} \int_{\theta=N-1} \theta(\xi) x^{\xi} d\xi \qquad \ldots(1.5)$$

$$\theta(\xi) = \frac{\prod_{i=1}^{n} \Gamma b_j - \beta_j \xi \prod_{j=1}^{N} \Gamma 1 - a_j - \alpha_j \xi}{\prod_{i=M=1}^{Q} \Gamma 1 - b_j + \beta_j \xi \prod_{j=N+1}^{P} \Gamma a_j - \alpha_j \xi} \qquad \ldots(1.6)$$

For condition of the H-Function of one variable (1.5) and on the contour L we refer to srivastava et al [2]

(V) The H-Function of two variable occurring in this paper is defined and represented as follows [see Srivastava et al [2] ,pp 83-85]

$$H[x,y] = H_{p_1,q_1,p_2,q_2,p_3,q_3}^{0,n_1,m_2,n_2,m_3,n_3}\left[{}^{x}_{y} \Big/ {}^{(a_j,\alpha_j,A_j)_{1,p_1}\ (c_j,z_j)_{1,p_2}\ (e_j,E_j)_{1,p_3}}_{(b_j,\beta_j,B_j)_{1,q_1}\ (d_j,\delta_j)_{1,q_2}\ (f_j,F_j)_{1,q3}} \right]$$

$$= -\frac{1}{4\pi^2} \int_{L_1} \int_{L_2} \phi_1(\xi,\eta) \psi_2(\xi)\, \psi_3(\eta) x(\xi) y(\eta) d\xi\, d\eta \qquad \ldots(1.7)$$

Where

$$\phi_1(\xi,\eta) = \prod_{j=1}^{n_1} \Gamma(1 - a_j + \alpha_j \xi + A_j \eta)$$

$$\times \left[\prod_{j=n_1+1}^{p_1} \Gamma a_j - \alpha_j \xi - A_j \eta \prod_{j=1}^{q_1} \Gamma 1 - b_j - \beta_j \xi + B_j \eta \right] \qquad \ldots(1.8)$$

Where the $\psi_2(\xi)$ and $\psi_3(\eta)$ are defined as (1.6) and for conditions of existence etc. of the $H(x,y)$ we refer to srivastava et al [2]

The fractional derivative of special function of one and more variables is important such as in the evaluation of series,[10,15] the derivation of generating function [12,chap.5] and the solution of differential equations [4,14;chap-3] motivated by these and many other avenues of applications, the fractional differential operators $D^{n}_{k,\alpha,x}$ and ${}_{\alpha}D^{\mu}_{x}$ are much used in the theory of special function of one and more variables .

We use the fractional derivative operator defined in the following manner [8]

$$D^{n}_{k,\alpha,x}(x^{\mu})=\prod_{r=0}^{n-1}[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}]x^{\mu+nk} \qquad \text{... (1.9)}$$

Whereα≠μ+1 andα and k are not necessarily integers

(vi) Let be complex numbers,and let x ∈ = (0, ∞) Follwing Saigo [7} Fractional integral (Re(α)>0 and derivative Re(α) < 0 of first kind of a function f(x) on are defined respectively in the forms:

(f) = $\frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\, 2F1\Big(\alpha+\beta\ ;$ …(1.10) Re(α)>0

(f) , 0< Re(α) +n < 1 (n= 1, 2, 3, …………..),

Where 2F1(a, b; c; z) is Gauss's hypergeometric function

Fractional integral Re(α)>0 and derivative Re(α) < 0 of second kind a function f(x) on are given by:

= $\frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\, t^{\alpha-\beta}\, 2F_1\Big(\alpha+\beta\ ,\ ;$ ….(1.11) Re(α)>0

= (f) , 0< Re(α) +n < 1 (n= 1, 2, 3, …),

Let α, β,η and λ be complex numbers. Then there hold the following formulae. . the R.H.S. has a definite meaning

= …..(1.12)

provided that Re(α)> max [0, Re(β-η)] — 1

(vii) The I-Function which was recently introduced by saxena [9] is an extension of fox'S H-function. on Specializing the parameters,I-Function can bereduced to almost all the known special function as wellas unknowns. The I-Function of one variable is further studied by so many researchers notably as vaishya, jain, and verma [13], Sharma and shrivastava [11] , Sharma and Tiwari [12] ,Nair [10] with certain properties series summation, integration etc.

Saxena [9] represent and define the I-Function of one variable as follows

[Z] = $I^{m,n}_{p_i;\ q_i\ ;\ r}\left[Z \,\middle|\, \begin{matrix} (a \\ \\ (b_j \end{matrix}\right]$ ……(1.13)

=

Where

= $\frac{\prod_{J=1}^{m}\left[b_j - \beta_j s\right.}{\sum_{J=1}\left[\prod_{J=m+1}^{q_i}\Gamma(1-bji+\right.}$ ……(1.14)

The following definition and results will be required in this paper

2. Main Result

Result I

$$D^{\mu}_{\eta}\ \{x^{k+\lambda s}\ L\ \{e^{-nt}\ \ t^{h}\ I^{m,n}_{pi,qi,r}\ \left[zt^{k}\Big|^{(aj\ \ \alpha j)\ \ldots\ldots\ldots(aji,\alpha ji)_{pi}}_{(bj,\beta j)\ldots\ldots\ldots(bji,\beta ji)_{qi}}\right],p$$

$$=\frac{x^{k-u}}{(P+n)^{1+h}}\ \ I^{m\ \ \ n+2}_{p;+2\ \ q;+1,r}$$

$$\left[zx^{\lambda}\,(\rho+\eta)^{-k}\ \middle|\ \begin{matrix}(-k,\lambda)\ (-h,k)\ (aj,\alpha j)\ \ldots\ldots\ (aji,\alpha ji)_{pi}\\ \vdots \\ (-k+u,\lambda)\ (bj,\beta j)\ \ldots\ldots\ (bji,\beta ji)_{qi}\end{matrix}\right]$$

Provided (in addition to the appropriate convergence and existence conditions)that

$\lambda, \mu > 0$ $\quad Re(1+\propto) > 0$ $Re(\beta) > 0$ and $Re(p) > 0$ $1 \geq \lambda > a$

Result II

$$D^{\eta}_{K,\alpha,x}\left\{x^{\mu}\ L\left\{e^{-nt}\ t^{\eta}\ I^{m,\eta}_{p;,q;r}\left\{\left[zx^{k}\ \middle|\ \begin{matrix}(c\\ \vdots \\ (b_j\end{matrix}\right.\right.\right.\right. =$$

$$\frac{\lceil 1+n}{(p+n)^{1+n}}\ x^{u+nk^1}\ I^{m,\eta+1}_{p;,+n^1\ \ q;+n^1r}\left[zx^{k}\ \middle|\ \begin{matrix}(-u-rk^1,k)_{r=0,n-1}\ (a_j\ \alpha_j)\ldots\ldots(a_{ji}\ \alpha_{ji})p_i\\ \vdots \\ (-u-rk^1+\alpha,k)_{r=0,n-1}\,(b_j\ \beta_j)\ldots\ldots(b_{ji},\beta_{ji})_{qi}\end{matrix}\right]$$

Provided (in addition to the appropriate convergence and existence conditions)that

$\lambda, \mu > 0$ $\quad Re(a) > 0$ $Re(\beta) > 0$

$$\mathrm{Re}\left(\alpha+\lambda\frac{b_j}{\beta_j}\right) > 0\ \mathrm{Re}\left(\beta+\mu\frac{d_j}{\delta_j}\right) > 0 \quad j = 1, 2 \ldots. m \quad k = 1, 2 \ldots. g$$

$$|\arg z_1| < \tfrac{1}{2}\pi\Delta_1 \qquad |\arg z_2| < \tfrac{1}{2}\pi\Delta_2 \quad \Delta_1\ \Delta_2 > 0$$

$$\Delta_2 = \sum_{1}^{M_2} d_j - \sum_{M_2+1}^{Q_2} d_j + \sum_{1}^{N_2} c_j - \sum_{N_2+1}^{P_2} c_j$$

Result III

$$D^{\eta}_{k,\alpha,x}\left\{x^{u}\int_0^1 x^{\alpha-1}\ (1-x)^{\beta-1}\ I^{m,n}_{pi,qi,r}\left[zx^{\lambda}\ \middle|\ \begin{matrix}(aj,\alpha j)\ldots\ldots(aji,\alpha ji)_{pi}\\ \vdots \\ (bj,\beta j)\ldots\ldots(bji,\beta ji)_{qi}\end{matrix}\right]\right\}$$

$$= \lceil\beta\ I^{m\ \ n+n+1}_{p;n+1,\ \ q;n+1;r}\left[z\ \middle|\ \begin{matrix}(1-u-\alpha-nk,\lambda)\ (aj,\alpha j)\ldots\ldots(aji,\alpha ji)_{pi}\ \ (1-u-\alpha-rk,\lambda)_{g=0,n-1}\\ \vdots \\ (1-u-\alpha-\beta-nk,\lambda)\ (bj,\beta j)\ldots\ldots(bji,\beta ji)_{qi}\ \ (1-u-rk,\lambda)_{g=0\ \ n-1}\end{matrix}\right]$$

Provided (in addition to the appropriate convergence and existence conditions)that

$\lambda, \mu > 0$ $\quad Re(a) > 0$ $Re(\beta) > 0$

$$\mathrm{Re}\left(\alpha+\lambda\frac{b_j}{\beta_j}\right) > 0\ \mathrm{Re}\left(\beta+\mu\frac{d_j}{\delta_j}\right) > 0 \quad j = 1, 2 \ldots. m \quad k = 1, 2 \ldots. g$$

$$|\arg z_1| < \tfrac{1}{2}\pi\Delta_1 \qquad |\arg z_2| < \tfrac{1}{2}\pi\Delta_2 \quad \Delta_1\ \Delta_2 > 0$$

$$\Delta_2 = \sum_{1}^{M_2} d_j - \sum_{M_2+1}^{Q_2} d_j + \sum_{1}^{N_2} c_j - \sum_{N_2+1}^{P_2} c_j$$

Proof I First Taking by mellin barnes type contour integral for I- function for one variable and then convolution of laplace transform for H-function.

Then useing by mellin barnes type contour integral for I- function for multivariable and then convolution of laplace transform for H-function

We then apply the formula [7, p.67 eq.4.4.4]

$$D_x^{\mu}\left(x^{\lambda}\right)=\frac{\Gamma 1+\lambda}{\Gamma 1+\lambda-\mu}x^{\lambda-\mu}, \quad \left[\operatorname{Re}(\lambda)>-1\right]$$

then we get required result

Proof II First Taking by mellin barnes type contour integral for I- function for one variable and then convolution of laplace transform for H-function. Then using by mellin barnes type contour integral for I- function for multivariables and then convolution of laplace transform for H-function Now Apply the fractional derivative operator defined in the following manner [8]

$$D_{k,\alpha,x}^{n}(x^{\mu})=\prod_{r=0}^{n-1}[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}]\mathrm{x}^{\mu+nk}$$

then we get required result

Proof III First Taking Fractional integral (Re(α)>0 and derivative Re(α) < 0 of first kind of a function f(x) on are defined (1.10)

Then using then using Euler's first integral (Beta function) formula

We use the fractional derivative operator defined in the following manner [8]

$$D_{k,\alpha,x}^{n}(x^{\mu})=\prod_{r=0}^{n-1}[\frac{\sqrt{\mu+rk+1}}{\sqrt{\mu+rk-\alpha+1}}]\mathrm{x}^{\mu+nk}$$

then we get required result

3. Conclusion

From this Paper we get some many solution of integral equation of convolution from having H – Function of one or more veriables. Specializing the parameters of the multivariable I-function, we can obtain a large number of news and knowns fractional derivatives involving various special functions of one and several variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics. The result derived in this paper is of general character and may prove to be useful in several interesting situations appearing in the literature of sciences

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