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Title:"ON CHARACTERIZATION OF BITOPOLOGICAL SPACES AND IT'S PROPERTIES"

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Abstract

Our main aim is to derive modified version of Separation axioms in bitopological space. Levine[6] introduced the concept of generalized closed sets in topological spaces and initiated the notion of $T_{1/2}$- spaces in unital topology which is properly placed between T_0–space and T_1-spaces by defining $T_{1/2}$–space in which every generalized closed set is closed. We extend R_0-topological spaces introduced by Pervin[3]. The properties of R_0-topological spaces and results have been obtained in various contexts. McCarty[7] also introduced the notion of R_1-topological space which is independent of both T_0 and T_1 but strictly weaker than T_2. Bitopological forms of these concepts have appeared in the definitions of pairwise R_0 and pairwise R_1 spaces given by Mrsevic. J.C. Kelly[1] studied quasi metrics and showed that a quasi-metric p on $X \neq \phi$ gives rise in a natural way to another quasi metric p* called conjugate of p, defined by $p^*(x, y) = p(y, x)$ for all $x, y \in X$.

Key words: (Separation Axiom, Bitopological space, Hausdorff spaces, regular spaces, open set, Close set, and quasi metrics)

1.1 Introduction

The separation axioms of topological spaces are denoted by T**.** Since separation axioms are terms of generalized closed sets. The first separation axiom between T_0 and T_1 was introduced by Aull and Thron[2]. The separation axioms are where the axioms for Hausdorff spaces, regular spaces and normal spaces. Separation axioms and closed sets in topological spaces have been very useful in the study of certain objects in digital topology. We to introduce modified and new separation axioms called pairwise

Q* T_i spaces (I = 0 , 1 , 2 , 3, 4), pairwise Q* normal, pairwise Q* regular, pairwise Q* US, pairwise Q* KC space, pairwise urysohn space, pairwise Q*$T_{7/8}$, pairwise Q* $T_{1\ 1/2}$, pairwise Q* $T_{1\ 2/3}$, pairwise Q* $T_{2\ 1/2}$ space, pairwise Q* $T_{4\ 1/2}$ space and pairwise Q*$T_{5\ 1/2}$ space in bitoplogical spaces. The concept of generalized closed sets and generalized open sets was first introduced by Levine[6] in topological space. The new class of sets called semi generalized closed sets and obtained various properties. The study of generalized semi closed sets was initiated. The new class of sets called g closed sets in general topological spaces is studies their properties. Throughout this paper, (X, τ) represents a nonempty topological space on which separation axioms are assumed unless explicitly stated.

The concept of closed sets is a fundamental object in general topology. Levine[6] introduced the generalized closed sets in a single topological space in order to extend many of the important properties of closed sets to a larger family. A set A of a topological space (X, τ) is called generalized closed set (g-closed) if cl (A) ⊆ U whenever A⊆ U and U is open in X.

1.2 Theorem

In a bitopological space (X, τ_1, τ_2) the following statements are equivalent:

i) (X, τ_1, τ_2) is pairwise Q*- R_0.

ii) For any τ_i – Q* closed set F and a point x ∉ F, there exists a U ∈ Q* O(X, τ_2) such that x ∉ U and F ⊂ U for I, j = 1, 2 and I ≠ j.

iii) For any τ_i – Q* closed set F and a point x ∉ F, τ_j – Q* cl ({x}) ∩ F = ϕ, for j = 1, 2 and I ≠ j.

Proof:

Case i) ⇒ ii): Let F be a τ_j – Q* closed set F and a point x ∉ F. Then by i) τ_j – Q* cl ({ x }) ⊂ X – F, where I , j = 1, 2 and I ≠ j.

Let U = X – τ_j – Q* cl ({x}) then U ∈ Q* O(X, τ_2) and also F ⊂U and x ∉ U.

Case ii) ⇒ iii): Let F be a τ_j – Q* closed set F and a point x ∉ F. we suppose that the given conditions hold. Since

U∈ Q* O(X, τ_2), U ∩τ_j – Q* cl ({x}) = ϕ ...(1)

Then F ∩ τ_j – Q* cl ({x}) = ϕ, where i, j = 1 , 2 and I ≠ j.

Case- iii) ⇒i) : Let G ∈ Q* O(X, τ_i) and x ∈ G. Now X – G is τ_j – Q*closed and x ∉ X – G. By iii), τ_j – Q* cl ({x})∩ (X – G) = ϕ and hence τ_j – Q* cl ({ x })⊂ G for I , j = 1, 2and I ≠j.

Therefore, the space (X, τ_1, τ_2) is pairwise Q* - R_0.

1.3 Theorem

If (X, τ_1, τ_2) is pairwise Q* - R_1, then it is pairwise Q* - R_0.

Proof

Let us consider that (X, τ_1, τ_2) is pairwise Q* - R_1. Let U be a τ_i – Q* open set and x ∈ U. If y ∉ U, then y ∈ X – U and x ∉ τ_i – Q* cl ({y}).

Therefore, for each point y ∈ X – U, τ_j – Q* cl ({x}) ≠ τ_i – Q* cl ({y}).

Since (X, τ_1, τ_2) is pairwise Q*- R_1, there exist a τ_i – Q* open set U_y and a τ_j – Q* open set V_y such that τ_j – Q* cl ({x}) ⊂ U_y, τ_i – Q* cl ({y}) ⊂ V_y and U_y ∩ V_y= ϕ, where I, j = 1, 2 and I ≠ j.

Let A = ∪ {V_y /y ∈ X – U}, then X – U ⊂ A, x ∉ A and A is τ_j – Q*open set. Therefore, τ_j – Q*- cl ({x}) ⊂ X – A ⊂ U. Hence (X, τ_1, τ_2) is pairwise Q* -R_0.

The converse of theorem is not true in general. The space (X, τ_1, τ_2) because is pairwise Q* -R_0 but not pairwise Q*-R_1.

1.4 Theorem

The product of an arbitrary family of pairwise Q* T_0 space is pairwise Q* T_0.

Proof

Let (X, τ_1, τ_2) =$\prod_{\alpha\epsilon\Delta}(X_\alpha , \tau_{1\alpha} , \tau_{2\alpha})$, where τ_1 and τ_2 are the product topologies on X generated by $\tau_{1\alpha}$ and $\tau_{2\alpha}$ respectively and X = $\prod_{\alpha\epsilon\Delta} X_\alpha$. Let x = ($x_\alpha$) and y = ($y_\alpha$) be two distinct points of X. Hence, $x_\alpha \neq y_\alpha$ for some α ∈ Δ.

But $(X_\lambda, \tau_{1\lambda}, \tau_{2\lambda})$ is pairwise Q* T_0, there exist either a $\tau_{1\lambda}$- Q* open set U_α containing x_λ but not y_λ or a $\tau_{2\lambda}$- Q* open set V_α containing y_λ but not x_λ.

Let us derive $U = \prod_{\lambda \neq} (X_\lambda \times U_\alpha)$ and $V = \prod_{\lambda \neq} (Y_\lambda \times V_\alpha)$. Then U is τ_1- Q* open and V is τ_2- Q* open. Also, U contains x but not y. Hence, the theorem is proved.

1.5 Theorem

Let f: X→Y be a bijection, pairwise Q* continuous and Y is pairwise Q* T_0 space, then X is a pairwise Q* T_0 space.

Proof

Let f: X→Y be a bijection, pairwise Q* continuous map and Y is a pairwise Q* T_0 space that X is a pairwise Q* T_0 space.

Let $x_1, x_2 \in X$ with $x_1 \neq x_2$. Since f is a bijection, there exists $y_1, y_2 \in Y$ with $y_1 \neq y_2$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$.

$$\Rightarrow x_1 = f^{-1}(y_1) \text{ and } x_2 = f^{-1}(y_2) \qquad \ldots(2)$$

Since Y is pairwise Q* T_0space, there exists a set which is either τ_i – Q* open or τ_j – Q* open set M in X such that $y_1 \in M$ and $y_2 \notin M$. Since f is pairwise Q* continuous map, $f^{-1}(M)$ is a pairwise Q* open set in Y. we get $y_1 \in M \Rightarrow f^{-1}(y_1) \in f^{-1}(M) \Rightarrow x_1 \in f^{-1}(M)$ and $x_2 \notin f^{-1}(M)$. Hence for any two distinct points y_1, y_2 in Y, there exists pairwise Q* open set $f^{-1}(M)$ in Y such that $x_1 \notin f^{-1}(M)$ and $x_2 \notin f^{-1}(M)$. Now Y is a pairwise Q* T_0 space. Hence, the theorem is proved.

1.6 Theorem

If a space (X, τ_1, τ_2) is pairwise Q* - T_2, then it is pairwise Q* -R_1.

Proof

Let (X, τ_1, τ_2) be pairwise Q* -T_2.

Then for any two distinct points x, y of X, their exist a τ_i – Q* open set U and a τ_j – Q* open set V such that $x \in U$, $y \in V$ and $U \cap V = \phi$ where I, j = 1, 2 and I ≠ j. If (X, τ_1, τ_2) is pairwise Q* -T_1, then $\{x\} = \tau_j$ – Q* cl({x}) and $\{y\} = \tau_i$- Q* cl ({y}) and thus τ_i – Q* cl({x}) ≠ τ_j- Q* cl({y}), where I, j = 1, 2 and I ≠ j.

Thus, for any distinct pair of points x, y of X such that τ_i – Q* cl({x}) ≠ τ_j – Q* cl({y}) where I, j = 1, 2 and I ≠ j, there exist a τ_i – Q* open set U and τ_j – Q* open set V such that x ∈ V, y ∈ U and U ∩ V = ϕ where I, j = 1, 2 and I ≠ j. Hence, (X, τ_1, τ_2) is pairwise Q* - R_1.

(i) The converse of the theorem (Every pairwise Q* T_2 space is pairwise Q* T_1 space) is not true in general i.e.) pairwise Q* T_1 space is not pairwise Q* T_2 space.

(ii) If a bitopological space X pairwise Q* T_i, then it is pairwise Q* T_{I-1}. I = 1, 2.

1.7 Theorem

Every pairwise Q* completely normal space is pairwise Q* normal.

Proof

Let X be a pairwise Q* completely normal bitopological space. Now A be a τ_i – Q* closed set and B be a τ_j – Q* closed set such that A ∩ B =ϕ. Then

$$T_i - Q^* \, cl(A) \cap B = A \cap B = \phi$$

$$\text{and } A \cap \tau_j - Q^* \, cl(B) = A \cap B = \phi \qquad \ldots(3)$$

By complete Q* normality, there exists a τ_j – Q* open set U and a τ_i – Q* open set V such that A ⊂ U, B ⊂ V, U ∩ V =ϕ. Since X is pairwise Q* normal. Hence, the theorem is proved.

1.8 References

1.	J.C. Kelly, (1955 and 1963)	General Topology D. Van Non-standard New York and Bitopological spaces, Proc. London Math. Soc., 13, 71-81.
2.	C. E. Aull and W. J. Thron, (1962)	Separation axioms between T_0 and T_1, Indag. Math 24, 26-37.
3.	Pervin, W. J. (1967)	Connectedness in bitopoligcal space, Duke Math. J V 34 (367-392)
4.	Patty, C. W. (1967)	Bitopological Space Duke Math J. V34 (387-392)
5.	Cullen, H. F., (1968)	"Separation Axioms" in Introduction to General Topology. Boston, MA: Heath, pp. 99-140.

6.	Levine, N., (1970)	Generalized closed sets in topology. Rend. Circ. Mat. Palermo., 19(2): 89-96
7.	McCarty, G., (1970)	Topology, an Introduction with Application to Topological Groups. New York: McGraw-Hill, 1967.Willard, S. "The Separation Axioms." §13 in General Topology. Reading, MA: Addison-Wesley, pp. 85-92.
8.	E.G. Manes, (1974)	Compact Hausdorff objects, General Topology Appl. 4, 341-360.
9.	Joshi, K. D. , (1983)	"Separation Axioms" in Introduction to General Topology. New Delhi, India: Wiley, pp. 159-188.
10.	P.Bhattacharyya and B.K.Lahiri, (1987)	Semi-generalized closed sets in Topology, Indian J. Math. 29 (3), 375-382.
11.	H.Maki, R.Devi and K.Balachandran, (1994)	Associated topologies of generalized α closed sets and α generalized closed sets, Mem. Fac. Sci., Kochi Uni. (Math), 15, 51-63.
12.	J. Brazas, (2011)	The topological fundamental group and free topological groups, Topology Appl. 158 779–802.
13.	Pauline Mary Helan, M.Veronica Vijayan, Ponnuthai Selvarani, (2012)	g closed sets in topological spaces, IJMA ,3(30), , 1-15.
14.	R. Lakehal, Lj. D.R. Kocinac and D. Seba, (2021)	Neighbourhood star selection properties in bitopological spaces, Filomat. 35(1), 339—351.