



Approximate Floating Point Divider Design

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Abstract : Approximate computing is a promising solution to design faster and more energy efficient systems, which provide an adequate quality for a variety of functions. Division, in particular floating point division is one of the most important operations in multimedia applications, which has been implemented less in hardware due to its significant cost and complexity. Approximation is nothing but approximating the output by reducing the constraints in it. There are several different strategies for approximate computing like Approximating the circuits, Approximating the software, Approximating the storage etc. This project uses Approximating the Circuit strategy. Approximating the circuits includes approximating the adders subtractors, multipliers, dividers etc. In this project, we will approximate the floating point divider using the concept of Approximate computing. The focus of this work will be on implementing a Verilog-based Approximate divider for floating point divider. Both accuracy and power will be calculated. We expect an improvement in accuracy and reduction in power.

IndexTerms - Approximate computing, Accuracy, Power efficient, Floating point representation, Division.

I. INTRODUCTION

In today's world of technologies increasing usage of energy consumption by computer systems is a serious problem that needs to be addressed. Computer systems will be processing massive amounts of data on this requires more power. Making the models energy efficient will save a lot of power. Simply said, energy efficiency is nothing but using less energy for doing the same function, in other words, eliminating energy waste. The power consumption of current consumer electronics has become a crucial design concern as the speed, mobility and compactness of these products have increased.

Numerous approaches have been introduced in the past to improve energy efficiency across circuit, architectural, and system abstraction levels. An approach to decrease energy and power usage in the system model involves approximating the hardware. This method of approximation is suggested for designing the IEEE754 floating point divider. The utilization of floating point blocks has grown in various applications, including arithmetic architecture, due to their low production cost and ease of use. In arithmetic digital design, the floatingpoint divider is considered a highly complex logic block.

Current sensory data algorithms tend to be statistical in nature, which makes them able to perform approximate computations. In embedded devices, precision would be replaced by energy efficiency as approximate computing continues to gain popularity due to its low energy consumption. Attention has lately focused on the design of approximation units, e.g. adder and multiplier. But one of the key operations in multimedia applications is division, especially with regard to floating point divisions.

In recent work it has been attempted to speed up the division in order to allow approximation. In this study [1], they propose FPAD, a novel approximation floating point divider based on the basic mathematical method of breaking down the mantissa division into a sequence of shifts and addition operations. A wide range of services is also offered by them. The FPAD is designed with a variety of errors and tradeoffs with the PDP. At each level of the proposed divider, a limit is set for inaccuracy. To increase the different areas, power, and delay, the number of adders at each level may be changed.

In this research [2], they are offering a revolutionary adjustment of the approximation coefficient that splits floating point data in an efficient and precise way. CADE removes the costly mantissa division and replaces it with a single subtraction between two input operands mantissa. The tuning procedure consists of defining the amount of error, then correcting it by using both input mantissa's initial N bits. The CADE may be configured to vary degrees of precision depending on the N value. According to our findings, CADE is the first floating point divider to include a new GPU knob for configuring the amount of approximation at runtime based on the application/user accuracy requirements.

In this paper, the main aim is to reduce the error and power consumption of the floating point divider. By studying different algorithms for floating point division and the main brief from it is the floating point dividers mantissa consumes more power and energy and is the main block that is responsible for the obtained error rate. This leads to finding a different algorithm for floating point division, which has fewer errors, power and energy than the current ones. In order to reduce the error rate, approximate calculations are performed. Since the mantissa is the main block of implementation so we primarily focused on mantissa block and performed different operations to analyze the error, power, utilization and delay of system. The aim of this work is to approximate the floating point divider, analyze and compare the power, accuracy and error rates of the proposed model.

II. METHODOLOGY

2.1 IEEE-754 FLOATING POINT REPRESENTATION

In order to separate decimal and floating point values, the floating point representation of a number represents where the point should be indicated. In order to allocate the values mentioned, a specific number of bits is allocated to the representation. It's where the float point has to float rather than being a fixed point. This is one of the main differences in representation by fixed and float points. For floating point, we use the IEEE-754 format because it contains various set formats and operation modes that are widely used in a number of hardware applications.

IEEE-754 floating point representation can be represented in two different precisions SINGLE PRECISION [32 bit] and DOUBLE PRECISION [64 bit]. The three main components of IEEE-754 floating point representation are SIGN BIT, EXPONENT and MANTISSA. Sign bit is a single bit value that represents whether the given number is positive or negative. If the bit value is '0' then the number is said to be positive number. Exponent bits are the bits that represent both positive and negative values that describe where the floating point (or decimal point) needs to be placed. Represented as power of 2. Mantissa bits are the bits that describe the floating point numbers of binary values, always positive. This part holds the main bits of the number.

The precision mainly differs by its size i.e. the bit size. In single precision the total float point representation is a 32-bit representation. As said earlier they contain sign, mantissa and exponents bits. This contains 1 sign bit, '23' mantissa bits and '8' exponent bits. This precision is also called binary 32. In the 24-bit mantissa, the MSB (most significant bit) is always '1' so this bit need not to be stored. So though theoretically the mantissa is a 24-bit value but technically mantissa is a 23-bit value. So, this MSB '1' is called as "Hidden bit". As the exponent is an 8-bit value the range of value it can hold is 2^{-126} to 2^{127} .

In double precision the total floating point representation is a 64-bit representation. The total bit size of double precision is twice that of single precision, that is why this is called double precision. As said earlier they contain sign, mantissa and exponent bits. The below diagram depicts the distribution of the three in a 64-bit single precision representation. This contains '1' sign bit, '52' mantissa bits and '11' exponent bits as shown. This precision is also called as binary64. In the 53-bit mantissa, the MSB (most significant bit) is always '1' so this bit need not to be stored. So though theoretically the mantissa is a 53-bit value but technically mantissa is a 52-bit value. So, this MSB '1' is called as "Hidden bit". As the exponent is an 11-bit value the range of value it can hold is 2^{-1022} to 2^{1023} .

For converting a given decimal number into a floating point representation we need to follow certain steps. Firstly, we need to convert the decimal number into binary number then normalize the binary value resulting in an exponent value (in integer) and mantissa value (in binary). In the given formula, S represents the sign bit, M represents the mantissa and E represents the exponent value. We use a formula as shown

$$\text{Number} = (-1)^s * (1.M) * 2^{(E - 127)}$$

2.2 FLOATING POINT DIVISION

Let us consider two numbers X & Y, floating point representation of these numbers is as follows: $X = (-1)^{S_x} * (1.M_x) * 2^{(E_x - 127)}$ and $Y = (-1)^{S_y} * (1.M_y) * 2^{(E_y - 127)}$. If we divide the two equations X/Y the resultant is $Z = (-1)^{S_z} * (M_x / M_y) * 2^{(E_y - E_x)}$.

The following are few steps to perform generalized floating point division:

Step-1: Output of sign bit is XOR of the input sign bits i.e. $S_z = S_x \oplus S_y$.

Step-2: Exponent is $E_z = (E_y - E_x) + \text{bias}$. [for single precision the bias value is +127].

Step-3: Mantissa Division $M_z = (M_x) / (M_y)$. This is the general binary division of the two input mantissas.

Normalization: This is the case where the final output is normalized. We need to normalize by moving the mantissa towards the left and reducing the final exponent value. This step is not mandatory, if required we need to perform this operation.

Percentage Error:

$$\text{Percentage Error} = (\text{Actual value} - \text{Obtained value}) / \text{Obtained value} * 100\%$$

III. PROPOSED ALGORITHMS

IEEE 754 floating point number $(X_{31}, X_{30}, \dots, X_0)$ consists of 3 parts i.e. sign(S_{31}), exponent($E_{30}, E_{29}, \dots, E_{23}$) and Mantissa ($M_{22}, M_{21}, \dots, M_0$). The operation that we perform is floating point division.

3.1 ALGORITHM-1: DIVIDE & SUBTRACT

Figure-3.1 shows a block diagram for the proposed algorithm -1. This illustrates the whole system architecture in which operations are carried out for each block.

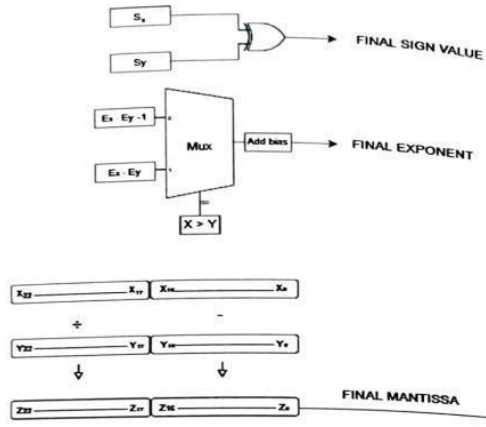


Figure-1: Block diagram of Algorithm-1

Sign Bit: The sign bit operator of the divider output is "Exor". We perform Exor operation of both the inputs i.e. $S_z = S_x \text{ XOR } S_y$ gives the sign bit value of the divider output.

Exponent Bit: The exponent bit is deducted by comparing the input mantissas. During this comparison we will be having two cases -

CASE 1: $M_x < M_y$: In this case we subtract the two input exponent values and then again subtract 1 from the obtained value. And the key point here is we should always add the bias to the exponent to get the stored value. So, the final exponent value of the divider output is $E_z = E_x - E_y - 1$.

Case 2: $M_x > M_y$: In this case we just need to subtract the two input exponent values. And the bias value which is + 127 should be added to the exponent to get the stored value. So, the final exponent value of the divider output is $E_z = E_x - E_y$

Mantissa Bit: As mentioned earlier in the notation of mantissa bits, the mantissa bits are $(M_{22}, M_{21}, \dots, M_0)$. In the proposed algorithm, we consider the first eight input bits are chosen for division and the rest of the 15 bits are chosen for subtraction. This is done to reduce the hardware and power consumption. The 8-bit division is done by considering the hidden bit also. As the division operation consumes a lot of power and indeed requires a lot of hardware to implement the 32-bit division. So, the final operation is $(M_{z22}M_{z21} \dots M_{z0}) = (M_{x22}M_{x21} \dots M_{x17}) / (M_{y22}M_{y21} \dots M_{y17}) + ((M_{x16}M_{x15} \dots M_{x0}) - (M_{y16}M_{y15} \dots M_{y0}))$. The final divider outputs are obtained from the proposed "Algorithm 1".

3.2 ALGORITHM-2: DIVIDE & ALTERNATE '10'

Figure-3.2 shows a block diagram for the proposed algorithm -1. This illustrates the whole system architecture in which operations are carried out for each block.

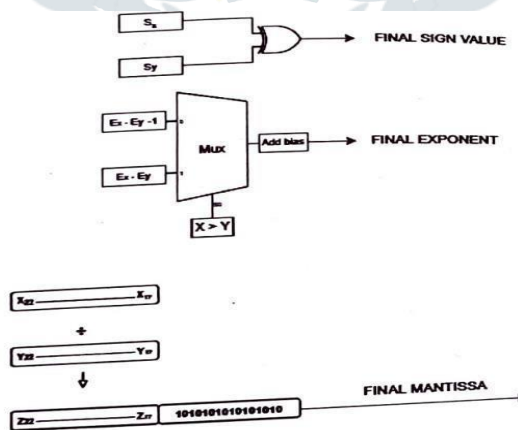


Figure-2: Block diagram of Algorithm-2

Sign Bit: The sign bit operator of the divider output is "Exor". We perform Exor operation of both the inputs i.e. $S_z = S_x \text{ XOR } S_y$ gives the sign bit value of the divider output.

Exponent Bit: The exponent bit is deducted by comparing the input mantissa. During this comparison we will be having two cases -

CASE 1: $M_x < M_y$: In this case we subtract the two input exponent values and then again subtract 1 from the obtained value. And the key point here is we should always add the bias to the exponent to get the stored value. So, the final exponent value of the divider output is $E_z = E_x - E_y - 1$.

Case 2: $M_X > M_Y$: In this case we just need to subtract the two input exponent values. And the bias value which is +127 should be added to the exponent to get the stored value. So, the final exponent value of the divider output is $E_Z = E_X - E_Y$.

Mantissa Bit: As mentioned earlier in the notation of mantissa bits, the mantissa bits are $(M_{22}, M_{21}, \dots, M_0)$. In the proposed algorithm, we consider the first eight input bits to be chosen for division and the rest of the 15 bits are not considered for any of the operation. The 8-bit division is done by considering the hidden bit also. This is valid as the output of the divider is mostly based on the first few MSB values of the mantissa. And for the final result the rest of the bits i.e. the bits left after storing the division output are made alternate '10' (1010101...). This is done to reduce the hardware and power consumption. As the division operation consumes a lot of power and indeed requires a lot of hardware to implement the 32-bit division. So, the final operation is $M_{Z22}M_{Z21} \dots M_{Z0} = (M_{X22}M_{X21} \dots M_{X17}) / (M_{Y22}M_{Y21} \dots M_{Y17}) + "1010101010101010"$. The final divider outputs are obtained from the proposed "Algorithm 2".

33 ALGORITHM-3: DIVIDE & ZERO

Figure-3.3 shows a block diagram for the proposed algorithm -1. This illustrates the whole system architecture in which operations are carried out for each block.

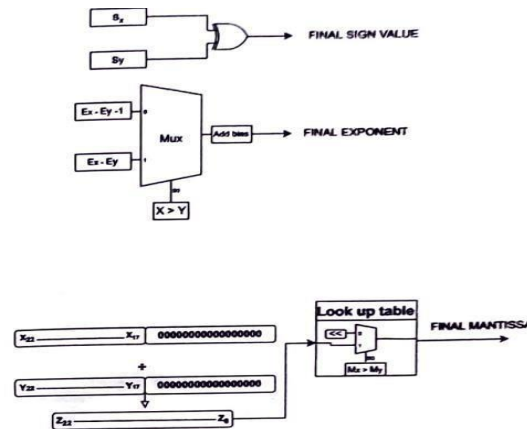


Figure-3: Block diagram of Algorithm-3

Sign Bit: The sign bit operator of the divider output is "Exor". We perform Exor operation of both the inputs i.e. $S_Z = S_X \text{ XOR } S_Y$ gives the sign bit value of the divider output.

Exponent Bit: The exponent bit is deducted by comparing the input mantissa. During this comparison we will be having two cases -

CASE 1: $M_X < M_Y$: In this case we subtract the two input exponent values and then again subtract 1 from the obtained value. And the key point here is we should always add the bias to the exponent to get the stored value. So, the final exponent value of the divider output is $E_Z = E_X - E_Y - 1$.

Case 2: $M_X > M_Y$: In this case we just need to subtract the two input exponent values. And the bias value which is +127 should be added to the exponent to get the stored value. So, the final exponent value of the divider output is $E_Z = E_X - E_Y$.

Mantissa Bit: As mentioned earlier in the notation of mantissa bits, the mantissa bits are $(M_{22}, M_{21}, \dots, M_0)$. In the proposed algorithm, we consider the first eight input bits to be chosen for division and the rest of the 15 bits are not considered for any of the operation. The 8-bit division is done by considering the hidden bit also. This is valid as the output of the divider is mostly based on the first few MSB values of the mantissa. And for the final result the rest of the bits i.e. the bits left after storing the division output are made zeros (000000...). This is done to reduce the hardware and power consumption. As the division operation consumes a lot of power and indeed requires a lot of hardware to implement the 32-bit division. We then introduced a look up table into the algorithm to provide operational range without requiring more time and large gate count which results in reducing the cost of operation. This look up table includes a comparison of mantissa for further operation.

We have two cases for mantissa operation-

CASE-1 $M_x < M_y$: In this case we use the output from the previous block as it is without any further operation and consider this as the final mantissa of the divider output.

CASE-2 $M_x > M_y$: In this case we use a shift operator to the output obtained from the previous block i.e. we shift the number 1 bit towards the left, and then this output is considered as the final mantissa of the divider output.

So, the final operation is

$$(M_{Z22}M_{Z21} \dots M_{Z0}) = (M_{X22}M_{X21} \dots M_{X17}) / (M_{Y22}M_{Y21} \dots M_{Y17}) + "0000000000000000"$$

The final divider outputs are obtained from the proposed "Algorithm 3".

IV. RESULTS & ANALYSIS

4.1 ERROR ANALYSIS

Error analysis is analyzing how much percentage error rate has been obtained for each algorithm. As said earlier we have three different proposed algorithms and two existing algorithms for comparison.

	Algorithm-1	Algorithm-2	Algorithm-3
Percentage Error Rate	6.0%	5.9%	0.36%

Table-1: Error rates of three proposed algorithms

According to the above table, Algorithm 3 gives a very low percentage error rate among all algorithms and actually makes an appreciable improvement in accuracy. Although the error rates of all three algorithms should be taken into account, since each algorithm has an error rate below 10%, in order to produce a high performance model, it is necessary to have minimal errors. So, in our case the algorithm-3 which is given an error rate of 0.36% for N=8 bits (bit size) will definitely be the best model. The error rate is also reduced by increasing the number of bits, but this increases hardware and power consumption due to a division block. For both error rate and power consumption, it would therefore be reliable to restrict the bit size to '8'.

4.2 COMPARISON OF POWER, UTILIZATION & DELAY b/w THE THREE PROPOSED ALGORITHMS

The corresponding reports of the three algorithms after simulation are given below.

	Algorithm-1	Algorithm-2	Algorithm-3
Power Report	25.403W	13.808W	11.274W
Device Utilization Report	92 LUT's	76 LUT's	85 LUT's
Delay Report	14.061ns	12.457ns	9.264ns

Table-2: Comparison of three algorithms

4.3 POWER CONSUMPTION ANALYSIS

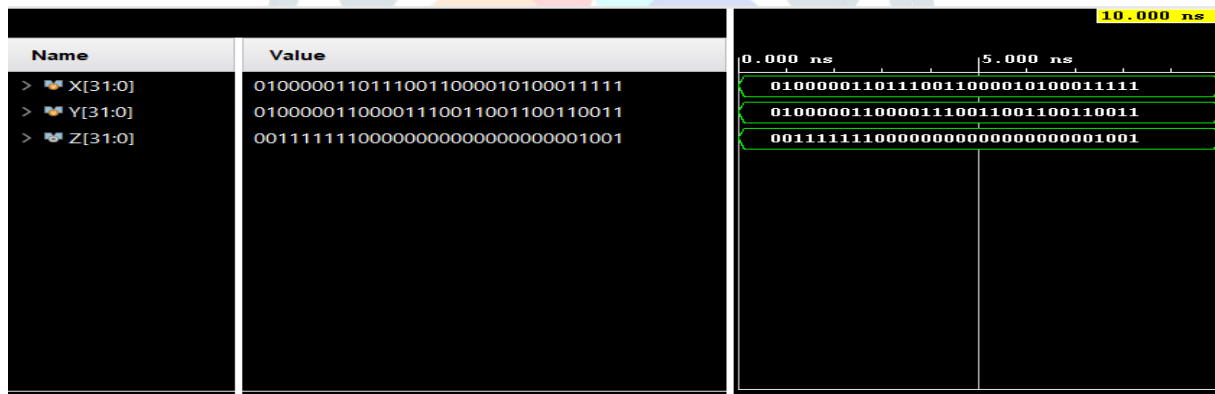


Figure-4: Output obtained from proposed divider design

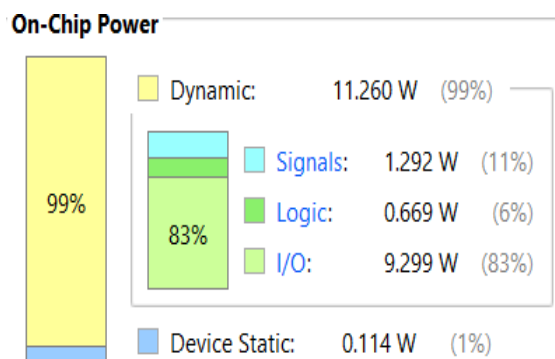


Figure-5: Power consumption by proposed divider design

The final output of the approximate divider is shown in fig-4.1. This output is represented in binary format. The output is approximated giving a high accuracy and less error rate. For the power analysis, fig-4.2 is the power consumption summary of the proposed approximate divider which is 11.260W and this power is split into two categories: dynamic and static. The static and dynamic power distributions are 99% and 1% respectively. Because the dynamic power is more than the static power, we can say that the device has high performance. Because of the leakage current, this device has an extremely low static power consumption. The existing system power consumption is 33.356W which is high. And our design uses power of 11.260W that indicates that we have reduced the power consumption to more than 50%, which is highly reliable.

V. CONCLUSION

In this paper, we proposed a novel method for design of approximate division algorithm. Our method has been shown to perform better in terms of error rate, accuracy and energy consumption compared to many existing methods such as CADE, FPAD etc. When compared to the existing floating point division. We have observed a 50% decrease in power consumption while maintaining the error percentage at 1%. This can further be done by reducing the size of mantissa input selection for division. This can be considered as the final approximate algorithm for floating point divider.

VI. REFERENCES

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