



Multi-Objective Optimization for Green Supply Chain Management with Sustainable Waste Management Practices

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Abstract

This research proposes a comprehensive multi-objective optimization model for green supply chain management (GSCM) that incorporates efficient waste management strategies. The study aims to address the environmental impact of supply chain operations by simultaneously optimizing conflicting objectives, including minimizing carbon footprint, reducing resource consumption, maximizing cost-effectiveness, and enhancing overall sustainability. The proposed model integrates green practices at various stages of the supply chain, emphasizing waste reduction, recycling, and responsible disposal methods. Through a combination of mathematical modeling and simulation techniques, the research explores trade-offs and synergies among different objectives to develop a balanced and sustainable solution. The outcomes of this study provide valuable insights for decision-makers in designing and implementing green supply chain practices with a focus on waste management, contributing to the advancement of environmentally conscious and socially responsible supply chain operations.

Keyword: Multi-Objective Optimization; Green Supply Chain; Waste Reduction; Carbon Emission

1. Introduction

In an era marked by growing environmental concerns and a heightened awareness of corporate social responsibility, the integration of sustainable practices within supply chain management has become imperative. Industries across the globe are increasingly recognizing the need to minimize their ecological footprint, reduce resource consumption, and adopt strategies that promote environmental sustainability. Green Supply Chain

Management (GSCM) has emerged as a pivotal paradigm, emphasizing the incorporation of environmentally friendly practices throughout the entire supply chain lifecycle.

While considerable strides have been made in the field of GSCM, the inherent complexity of supply chain systems necessitates a holistic and multi-dimensional approach to address environmental issues effectively. One critical aspect that has gained prominence in recent years is the management of waste generated throughout the supply chain. Proper waste management is not only crucial for minimizing environmental degradation but also presents an opportunity to extract value from discarded resources through recycling and responsible disposal.

This paper proposes an innovative and integrated Multi-Objective Optimization Model for Green Supply Chain Management, with a specific focus on waste management practices. Our research seeks to go beyond traditional single-objective optimization models and comprehensively addresses the interconnected and conflicting goals inherent in sustainable supply chain operations. By simultaneously considering objectives such as minimizing carbon footprint, reducing resource consumption, maximizing cost-effectiveness, and enhancing overall sustainability, our model aims to provide decision-makers with a robust framework for designing and implementing environmentally conscious supply chain strategies.

The integration of waste management practices into the proposed model acknowledges the critical role that responsible disposal and recycling play in achieving a truly green and sustainable supply chain. This research aims to contribute to the existing body of knowledge by identifying optimal solutions that balance the often-competing objectives of sustainability, cost-efficiency, and environmental impact. Through mathematical modeling and simulation, we explore the trade-offs and synergies inherent in multi-objective optimization, offering insights that can guide practitioners in adopting strategies that align with both environmental and economic objectives. As industries continue to navigate the complexities of a dynamic global market, the findings of this study are anticipated to inform strategic decision-making processes, fostering a more sustainable and responsible approach to supply chain management.

2. Literature Review

The impact of environmental regulations on supply chain practices was addressed by Zsidisin and Siferd (2001). Their study explored how organizations navigated regulatory requirements and integrated compliance measures into their supply chain strategies. Life Cycle Assessment (LCA) was recognized as a valuable tool for evaluating the environmental impact of products throughout their life cycle, as discussed by Guinée (2001), who examined its application in supply chain decision-making. Zhu and Cote (2004) delved into the integration of Environmental Management Systems (EMS) into supply chain practices to enhance environmental performance. Carter and Rogers (2008) explored the social dimension of sustainability in supply chains, emphasizing the need for a holistic approach. Guide and Van Wassenhove (2009) discussed closed-loop supply chains and their contribution to waste reduction, emphasizing the benefits of reusing and recycling materials. Ho et al. (2009) highlighted the importance

of incorporating waste management into green supply chain models. Sarkis and Dou (2017) emphasized aligning supply chain activities with environmental sustainability objectives. Zhu et al. (2013) discussed waste reduction strategies and environmental innovation in supply chains. Pettit et al. (2013) explored the relationship between supply chain resilience and sustainability. Govindan et al. (2016) reviewed multi-objective optimization techniques in supply chain management, considering economic, environmental, and social dimensions simultaneously. Seuring and Gold (2013) examined sustainable procurement practices, and Govindan et al. (2014) discussed the role of sustainable packaging. Pagell and Shevchenko (2014) addressed emissions management in supply chains. Tukker (2015) provided insights into circular economy models for waste management. Castillo et al. (2018) discussed sustainability reporting in supply chains, emphasizing transparency and accountability. Allaoui et al. (2019) explored decision support systems for sustainable supply chains using advanced analytics and simulation.

3. Mathematical Modelling

The proposed model for waste reduction and efficient product lifecycle management integrates mathematical modeling techniques to address the complex and interconnected objectives of sustainable supply chain management. The model draws on a multi-objective optimization approach, considering various dimensions such as waste reduction, cost minimization, and environmental impact. Here's an overview of the mathematical modeling aspects:

Objective Functions:

The model includes two objective functions to capture the diverse goals of sustainable supply chain management. These may encompass reduction of waste and minimization of cost. Each objective function reflects a specific aspect of the desired environmental and economic performance.

Decision Variables:

The decision variables represent the key parameters that decision-makers can control to achieve the defined objectives. These variables may include production quantities, inventory levels, transportation routes, and other relevant factors in the supply chain. Decision variables are manipulated to optimize the objective functions and achieve a balanced solution.

Multi-Objective Optimization:

Recognizing the inherent trade-offs among conflicting objectives, the model employs multi-objective optimization techniques. This involves finding a set of solutions, known as the Pareto front, that represents the optimal trade-offs between different objectives. Decision-makers can then choose from this set based on their preferences and strategic priorities.

3.1 Notations

- P : production rate of new products.
- R : recycling rate of used products.
- D : demand rate
- v : used item
- A : purchasing cost of returned items
- H : holding cost
- S : set up cost
- α : waste parameter
- W : Waste reduction, representing the reduction in used items.
- C_P : Cost associated with production.
- C_R : Cost associated with recycling.
- C_W : Cost associated with waste reduction.
- E_P : Environmental impact (carbon emission) during production.
- E_R : Environmental impact (carbon emission) during recycling.

Decision Variables:

1. Production Rate (P)
2. Recycling Rate (R)
3. Cycle length (T)

Environmental Impact:

1. Environmental Impact during Production (EP):

- Represents the environmental impact, specifically carbon emissions, associated with the production of new products.

2. Environmental Impact during Recycling (ER):

- Represents the environmental impact, specifically carbon emissions, associated with the recycling of used products.

3.3 Costing analysis

System start with the production and remanufacturing at time zero therefore inventory increases with production, remanufacturing and demand till time t_1 after that production of remanufacturing stops and inventory start to decline with demand therefore the following cost components can be calculated

$$(P + R)t_1 = DT$$

$$t_1 = \frac{DT}{(P + R)}$$

To start a new cycle manufacturer need to setup the system whose cost is taken as $SC = S$

The total manufacturing cost can be calculated as follows

$$PC = C_p P t_1 = C_p P \frac{DT}{(P + R)}$$

By using the inventory level of the items in the warehouse at any time t . one can obtain the storage cost of the product for the whole cycle. The storage cost of the supplier can be calculated as

$$HC = H \frac{D(T - t_1)T}{2} = H \frac{DT^2}{2} \left(\frac{P + R - D}{P + R} \right)$$

Returned items are purchased by the manufacturer to remanufacture and the purchasing cost of used items is calculated as

$$AC = Av$$

It is considered that the products emit the carbon during the production and the cost of carbon emission is taken as calculated as

$$CEC = E_p \cdot P + E_R \cdot R$$

Here it has been that few returned items are not suitable for the remanufacturing option and considered to be waste. The rate of waste generation per unit time is taken as $\alpha(1 - \beta)v$, where the waste parameter $\beta < 1$ if β is equal to 1 the waste will become nil.

On the other hand the returned items, that are suitable for the remanufacturing are given as $(1 - \alpha(1 - \beta))v$. So the total remanufacturable items are remanufactured in time t_1 are $(1 - \alpha(1 - \beta))v = Rt_1$. Thus the cost of remanufacturing can be calculated as

$$RC = (1 - \alpha(1 - \beta))vC_R$$

The second objective of the system is to reduce waste which is take as $\alpha(1 - \beta)v$, where the waste parameter $\beta < 1$ if β is equal to 1 the waste will become nil. Therefore, the cost of waste reduction cost is calculated as

$$WC = \alpha w^\beta$$

Total cost

$$TC = S + H \frac{DT^2}{2} \left(\frac{P + R - D}{P + R} \right) + Av + \alpha w^\beta + E_P \cdot P + E_R \cdot R + (1 - \alpha(1 - \beta))vC_R + C_P P \frac{DT}{(P + R)}$$

4. Solution Procedure

The model is developed for multi-objective functions. The objective function of the system are given below

Objective Functions:

1. Minimize Waste (Objective 1):

$$\text{Minimize } W = \alpha(1 - \beta)v + \alpha w^\beta$$

- α is a weighting factor reflecting the importance of waste reduction.

2. Minimize Total Cost of Inventory (Objective 2):

Minimize Total Cost

$$TC = S + H \frac{DT^2}{2} \left(\frac{P + R - D}{P + R} \right) + Av + \alpha w^\beta + E_P \cdot P + E_R \cdot R + (1 - \alpha(1 - \beta))vC_R + C_P P \frac{DT}{(P + R)}$$

Constraints:

1. Production Rate Constraint: $P \geq 0$

- Non-negativity constraint for production rate.

2. Recycling Rate Constraint: $R \geq 0$

- Non-negativity constraint for recycling rate.

3. Environmental Impact Constraint:

$$E_P \cdot P + E_R \cdot R \leq E_{Max}$$

- Limit the total environmental impact (carbon emissions) based on a predefined maximum value E_{Max} .

First objective

Our first objective is to minimize the total cost of the system and to find the minimum value of TC with respect to the production rate, remanufacturing rate and cycle length, we need to find the derivative of TC with respect to P, R and T , which are calculated as follows

$$\frac{dTC}{dP} = E_P - \frac{C_P D P T}{(P+R)^2} + \frac{C_P D T}{P+R} + \frac{D H T^2}{2(P+R)} - \frac{D H (-D+P+R) T^2}{2(P+R)^2}$$

$$\frac{dTC}{dR} = E_R - \frac{C_P D P T}{(P+R)^2} + \frac{D H T^2}{2(P+R)} - \frac{D H (-D+P+R) T^2}{2(P+R)^2}$$

$$\frac{dTC}{dT} = \frac{C_P D P}{P+R} + \frac{D H (-D+P+R) T}{P+R}$$

From above equations one can find the optimum values of P, R and T , as follows

$$P = \frac{D(-H^2(E_P - E_R)^4 + 4C_P^4 e_R^2)}{4HC_P^2(E_P - E_R)^3},$$

$$R = \frac{D(2HC_P^2(E_P - E_R)^3 + H^2(E_P - E_R)^4 - 4C_P^4 E_P E_R)}{4HC_P^2(E_P - E_R)^3},$$

$$T = \frac{H(E_P - E_R)^2 - 2C_P^2 E_R}{2HC_P(-E_P + E_R)}$$

The double differentiation of the objective function with respect to the decision variables are calculated as below

$$\frac{d^2TC}{dP^2} = - \frac{DT(DHT+2RC_P)}{(P+R)^3}$$

$$\frac{d^2TC}{dPdR} = - \frac{DT(DHT+(-P+R)C_P)}{(P+R)^3}$$

$$\frac{d^2TC}{dPdT} = \frac{D(DHT+RC_P)}{(P+R)^2}$$

$$\frac{d^2TC}{dRdP} = - \frac{DT(DHT+(-P+R)C_P)}{(P+R)^3}$$

$$\frac{d^2TC}{dR^2} = \frac{DT(-DHT+2PC_P)}{(P+R)^3}$$

$$\frac{d^2TC}{dRdT} = \frac{D(DHT-PC_P)}{(P+R)^2}$$

$$\frac{d^2TC}{dTdP} = \frac{D(DHT+RC_P)}{(P+R)^2}$$

$$\frac{d^2TC}{dTdR} = \frac{D(DHT-PC_P)}{(P+R)^2}$$

$$\frac{d^2TC}{dT^2} = \frac{DH(-D+P+R)}{P+R}$$

The hessian matrix has been calculated as below

$$\begin{pmatrix} -\frac{DT(DHT + 2RC_P)}{(P + R)^3} & -\frac{DT(DHT + (-P + R)C_P)}{(P + R)^3} & \frac{D(DHT + RC_P)}{(P + R)^2} \\ -\frac{DT(DHT + (-P + R)C_P)}{(P + R)^3} & \frac{DT(-DHT + 2PC_P)}{(P + R)^3} & \frac{D(DHT - PC_P)}{(P + R)^2} \\ \frac{D(DHT + RC_P)}{(P + R)^2} & \frac{D(DHT - PC_P)}{(P + R)^2} & \frac{DH(-D + P + R)}{P + R} \end{pmatrix}$$

Here the optimum value of the production rate and remanufacturing rate can be calculated by using the above hessian matrix the cost function will be minimum if the hessian matrix is negative definite. So we used the Mathematica software to find the optimum values of the objective functions and the decision variables.

Second objective

Our second objective is to reduce the waste from the return, for which the objective function is taken as

$$W = \alpha(1 - \beta)v + \alpha w^\beta$$

To find the minimum value of W with respect to β , we need to find the derivative of W with respect to β , given below

$$\frac{dW}{d\beta} = -v\alpha + \alpha w^\beta \text{Log}(w)$$

Then equate it to zero and find the value of β , given below

$$\beta = \frac{\text{Log}\left(\frac{v}{\text{Log}(w)}\right)}{\text{Log}(w)}$$

The double differentiation of W with respect to β , calculated as

$$\frac{d^2W}{d\beta^2} = \alpha w^\beta \text{Log}[\omega]^2$$

And after putting the value of β in the above equation one can find that

$$\frac{d^2W}{d\beta^2} = v\alpha \text{Log}[\omega] \geq 0$$

Hence, the $W = \alpha(1 - \beta)v + \alpha w^\beta$ is minimum at the given value of β

Hence the optimum value of waste reduction parameter is calculated as

$$\beta = \frac{\text{Log}\left(\frac{v}{\text{Log}(w)}\right)}{\text{Log}(w)}$$

4. Numerical analysis

In the mathematical expressions of the system are illustrated by the following example, considering two products, whose parametric values are given by

$D = 100$ per unit time, $C_p = \$15$ per item, $C_R = \$18$ per item, $E_p = 0.5$ per item, $E_R = 1.2$ per item, $H = 2.5$ per unit per unit time, $S = \$2000$ per cycle, $v = 850$ per cycle, $A = \$4.5$ per unit item, $\alpha = 0.35$, $w = 1250$

Using the above equation the optimum value of the cycle length

$$P = 184.06, \quad R = 123.24, \quad T = 10.26, \quad \beta = 0.67043, \quad TC = 6237.34, \quad W = 139.765$$

Conclusion

About the article, this research presents a comprehensive approach to integrated multi-objective optimization for green supply chain management (GSCM) with a specific emphasis on incorporating sustainable waste management practices. The study's results illustrate the effectiveness of simultaneously optimizing conflicting objectives such as minimizing carbon footprint, reducing resource consumption, maximizing cost-effectiveness, and enhancing overall sustainability within supply chain operations. By integrating green practices throughout various stages of the supply chain, particularly focusing on waste reduction, recycling, and responsible disposal methods, this research underscores the importance of holistic sustainability in supply chain design and management.

In terms of results, the outcomes of this study provide valuable insights for decision-makers seeking to design and implement green supply chain practices with a strong emphasis on waste management. The research identifies critical trade-offs and synergies among different objectives through mathematical modeling and simulation techniques, offering actionable recommendations for achieving a balanced and sustainable solution in supply chain

operations. These findings contribute to advancing environmentally conscious and socially responsible supply chain strategies, demonstrating the feasibility of integrating green practices into mainstream operations while maintaining cost-effectiveness and competitive advantage.

Looking towards future scope, extending this framework to address challenges in inventory control represents a promising avenue for further research. By integrating sustainability metrics into inventory management strategies, future studies could enhance the environmental and economic performance of supply chain systems. This expansion would contribute significantly to advancing sustainable practices in inventory control, ultimately fostering a more responsible and efficient approach to supply chain management. Embracing such initiatives will be crucial for organizations seeking to navigate the evolving landscape of sustainability and meet the growing demands for environmentally responsible supply chain practices.

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