JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

CERTAIN INTEGRAL FORMULAE INVOLVING WITH MULTI-INDEX MITTAG LEFFLER FUNCTION

Sunil Kumar[1] andKrishna Gopal Bhadana[2]

[1, 2] Department of Mathematics, S.P.C. Government College, Ajmer

Maharshi Dayanand Saraswati University, Ajmer, Rajasthan-305009, India

Abstract: The present paper, we aim to establishing certain integral formulae involving the multi-index Mittag Leffler function. The obtained results are in the form of hypergeometric which are made with the help of Hadamard product. We have also derived some other interesting formulaeas special cases of our main results.

2010 Mathematics Subject Classification: 26A33, 33C45, 33C20

Keywords and Phrases:hypergeometric function, multi-index Mittag Leffler function, Lavoie-Trottier,MacRobertand Edwardintegrals.

1. Introduction

For the generalized multi-index Mittag Leffler function is defined by Saxena and Nishimoto in the following summation form

$$E^{\delta,\mu}_{(A_j,B_j)_m}(x) = \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m} \Gamma(A_j\, k + B_j)} \frac{x^k}{k!}; \quad (m \in \mathbb{N})(1.1)$$

where and $\sum$.

For the generalized multi-index Mittag Leffler function reduce into the generalized Mittag Leffler function given by Shukla and Prajapati and defined as

$$E^{\delta,\mu}_{A,B}(x) = \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\Gamma(Ak+B)} \frac{x^k}{k!} (1.2)$$

where $A, B, \delta \in C,\ \mathcal{R}e(A) > ($ and and is the wellknownPohhammer symbol.

investigate the generalized Mittag Leffler function for

$$E^{\delta}_{A,B}(x) = \sum_{k=0}^{\infty} \frac{(\delta)_k}{\Gamma(A\,k+B)} \frac{x^k}{k!}; (1.3)$$

where .

investigate the generalized Mittag Leffler function for

$$E_{A,B}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(A\,k+B)}; (1.4)$$

where .

Now the generalized hypergeometric function in terms of Pochhammer symbol is expressed as follows with p and q ( )

$${}_pF_q\left[\begin{matrix}\alpha_1,\ldots,\alpha_p; \\ \beta_1,\ldots,\beta_q;\end{matrix}\; x\right] = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k,\ldots,(\alpha_p)_k}{(\beta_1)_k,\ldots,(\beta_q)_k} \frac{x^k}{k!} (1.5)$$

where with are positive integers.

For and the definition of Fox-Wright function is defined as below ( )

$${}_p\Psi_q\left[\begin{matrix}(\alpha_i, A_i)_{1,p} \\ (\beta_j, B_j)_{1,q}\end{matrix}\; \middle|\; x\right] = \sum_{\kappa=0}^{\infty} \frac{\Gamma(\alpha_1 + A_1\kappa),\ldots,\Gamma(\alpha_p + A_p\kappa)}{\Gamma(\beta_1 + B_1\kappa),\ldots,\Gamma(\beta_q + B_q\kappa)} \frac{x^\kappa}{\kappa!} (1.6)$$

where A and for all values of the under the condition if .

2. Preliminaries and Definitions

The following Euler type integral formulais introduced by Lavoie-Trottier and defined as

$$\int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1} d\xi = \left(\frac{4}{9}\right)^{\nu} \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \qquad (2.1)$$

where .

The following next Euler type integral formula is introduced by MacRobert and defined as

$$\int_0^1 \xi^{\nu-1}(1-\xi)^{\epsilon-1}\,[C\xi + D(1-\xi)]^{-\nu-\epsilon} d\xi = \frac{1}{C^{\nu}D^{\epsilon}} \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \qquad (2.2)$$

where and are non zero constant with the expression where .

The following Euler type double integral formula is introduced by Edward and defined as

$$\int_0^1 \int_0^1 \xi^{\epsilon} (1-\xi)^{\nu-1} (1-\zeta)^{\epsilon-1} (1-\xi\,\zeta)^{1-\epsilon-\nu} d\xi\, d\zeta = \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \qquad (2.3)$$

where and .

Let and are two analytic functions with their radii of convergence and respectively, then their Hadamard product is given by the following power series

$$f * g(z) = g * f(z) = \sum_{k=0}^{\infty} C_k\, D_k\, z^k ; \quad (|z| < R\,) \qquad (2.4)$$

where is the radius of convergence of the composite series.

3. Main Results

Theorem 3.1. Let be such that and the conditions is satisfied, then for the generalized multi-index Mittag Lefflerfunction the following integral formula holds true

$$\int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1} E^{\delta,\mu}_{(A_j,B_j)_m}(X)\; d\xi$$

$$= \left(\frac{4}{9}\right)^{\nu} \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \;{}_2F_1\left[\begin{matrix}\epsilon, 1\\ \nu+\epsilon\end{matrix}\middle|\; \theta\right] \qquad (3.1)$$

where

Proof. we refer to the left hand side of equation as the sign then making the use of equation in equation we have

$$I_1 \equiv \int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1} \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)} \frac{(1-\xi)^{2k}\left(1-\frac{\xi}{4}\right)^{k}\theta^k}{k!} d\xi$$

After interchanging the order of integration and summation under the theorem's condition

$$I_1 \equiv \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)} \frac{\theta^k}{k!} \int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon+2k-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon+k-1} d\xi$$

By using equation and after some simplification, we get

$$I_1 \equiv \left(\frac{4}{9}\right)^{\nu} \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)} \frac{\theta^k}{k!} \frac{\Gamma(\nu)\,\Gamma(\epsilon+k)}{\Gamma(\nu+\epsilon+k)}$$

$$I_1 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \left(\frac{4}{9}\right)^{\upsilon} \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)} \frac{\theta^k}{k!} \frac{(\epsilon)_k (1)_k}{(\nu+\epsilon)_k\; k!}$$

Now apply Hadamard product*i.e.* $\sum_{k=0}^{\infty} C_k\, z$

$$I_1 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \left(\frac{4}{9}\right)^{\nu} E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \;{}_2F_1\left[\begin{matrix}\epsilon, 1\\ \nu+\epsilon\end{matrix}\middle|\; \theta\right]$$

Theorem 3.2. Let be such that and the conditions is satisfied, then for the multi-index Mittag Lefflerfunction the following integral formula holds true

$$\int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1} E^{\delta,\mu}_{(A_j,B_j)_m}(Y)\,d\xi$$

$$= \left(\frac{4}{9}\right)^{\nu}\frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \ {}_2F_1\left[\begin{matrix}\nu,1\\ \nu+\epsilon\end{matrix}\middle|\ \theta\right] \qquad (3.2)$$

where

Proof. we refer to the left hand side of equation as the sign then making the use of equation in equation we have

$$I_2 \equiv \int_0^1 \xi^{\nu-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\xi^k\left(1-\frac{\xi}{3}\right)^{2k}\theta^k}{k!}\,d\xi$$

After interchanging the order of integration and summation under the theorem's condition

$$I_2 \equiv \sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\int_0^1 \xi^{\nu+k-1}(1-\xi)^{2\epsilon-1}\left(1-\frac{\xi}{3}\right)^{2\nu+2k-1}\left(1-\frac{\xi}{4}\right)^{\epsilon-1}\,d\xi$$

By using equation and after some simplification, we get

$$I_2 \equiv \left(\frac{4}{9}\right)^{\nu}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\frac{\Gamma(\nu+k)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon+k)}$$

$$I_2 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)}\left(\frac{4}{9}\right)^{\upsilon}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\frac{(\nu)_k(1)_k}{(\nu+\epsilon)_k\,k!}$$

Now apply Hadamard product*i.e.* $\sum_{k=0}^{\infty} C_k\, z^{k}$

$$I_2 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)}\left(\frac{4}{9}\right)^{\nu} E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \ {}_2F_1\left[\begin{matrix}\nu,1\\ \nu+\epsilon\end{matrix}\middle|\ \theta\right]$$

Theorem 3.3. Let be such that and the conditions is satisfied, then for the multi-index Mittag Lefflerfunction the following integral formula holds true

$$\int_0^1 \xi^{\nu-1}(1-\xi)^{\epsilon-1}\left[C\xi+D(1-\xi)\right]^{-\nu-\epsilon} E^{\delta,\mu}_{(A_j,B_j)_m}(Z)\,d\xi$$

$$= \frac{1}{C^{\nu}D^{\epsilon}}\frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4CD}\right) * \ {}_3F_2\left[\begin{matrix}\nu,\epsilon,1\\ \frac{\nu+\epsilon}{2},\frac{\nu+\epsilon+1}{2}\end{matrix}\middle|\ \frac{\theta}{4CD}\right] \qquad (3.3)$$

where

Proof. First we refer to the left hand side of equation as the sign then making the use of equation in equation we have

$$I_3 \equiv \int_0^1 \xi^{\nu-1}(1-\xi)^{\epsilon-1}[C\xi+D(1-\xi)]^{-\nu-\epsilon}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\xi^k(1-\xi)^k\,\theta^k}{[C\xi+D(1-\xi)]^{2k}\,k!}d\xi$$

After interchanging the order of integration and summation under the theorem's condition

$$I_3 \equiv \sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\int_0^1 \xi^{\nu+k-1}(1-\xi)^{\epsilon+k-1}[C\xi+D(1-\xi)]^{-\nu-\epsilon-2k}\,d\xi$$

By using equation and after some simplification, we get

$$I_3 \equiv \frac{1}{C^{\nu}D^{\epsilon}}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\frac{1}{C^kD^k}\frac{\Gamma(\nu+k)\,\Gamma(\epsilon+k)}{\Gamma(\nu+\epsilon+2k)}$$

$$I_3 \equiv \frac{1}{C^{\nu}D^{\epsilon}}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\frac{1}{C^kD^k}\frac{(\nu)_k(\epsilon)_k}{(\nu+\epsilon)_{2k}}$$

$$I_3 \equiv \frac{1}{C^{\nu}D^{\epsilon}}\frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\left(\frac{\theta}{C\,D}\right)^k}{k!}\frac{(\nu)_k(\epsilon)_k(1)_k}{2^{2k}\left(\frac{\nu+\epsilon}{2}\right)_k\left(\frac{\nu+\epsilon+1}{2}\right)_k\,k!}$$

Now apply Hadamard product*i.e.* $\sum_{k=0}^{\infty} C_k\, z^{k}$

$$I_3 \equiv \frac{1}{C^{\nu}D^{\epsilon}}\frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)}E_{(A_j,B_j)_m}^{\delta,\mu}\left(\frac{\theta}{4CD}\right) * {}_3F_2\left[\begin{matrix}\nu,\epsilon,1\\ \frac{\nu+\epsilon}{2},\frac{\nu+\epsilon+1}{2}\end{matrix}\middle|\frac{\theta}{4CD}\right]$$

Theorem 3.4. Let be such that and the conditions is satisfied, then for the multi-index Mittag Lefflerfunction the following integral formula holds true

$$\int_0^1\int_0^1 \xi^{\epsilon}\,(1-\xi)^{\nu-1}(1-\zeta)^{\epsilon-1}(1-\xi\,\zeta)^{1-\epsilon-\nu}E_{(A_j,B_j)_m}^{\delta,\mu}(W)\;d\xi\;d\zeta$$

$$=\frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)}E_{(A_j,B_j)_m}^{\delta,\mu}\left(\frac{\theta}{4}\right) * {}_3F_2\left[\begin{matrix}\epsilon,\nu,1\\ \frac{\nu+\epsilon}{2},\frac{\nu+\epsilon+1}{2}\end{matrix}\middle|\frac{\theta}{4}\right] \qquad (3.4)$$

where

Proof. we refer to the left hand side of equation as the sign then making the use of equation in equation we have

$$I_4 \equiv \int_0^1\int_0^1\frac{\xi^{\epsilon}(1-\xi)^{\nu-1}(1-\zeta)^{\epsilon-1}}{(1-\xi\,\zeta)^{\epsilon+\nu-1}}\sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\xi^k(1-\xi)^k(1-\zeta)^k\theta^k}{(1-\xi\,\zeta)^{2k}\;\;k!}\;d\xi\,d\zeta$$

After interchanging the order of integration and summation under the theorem's condition

$$I_4 \equiv \sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\int_0^1\int_0^1\frac{\xi^{\epsilon+k}(1-\xi)^{\nu+k-1}(1-\zeta)^{\epsilon+k-1}}{(1-\xi\,\zeta)^{\epsilon+\nu+2k-1}}\;d\xi\,d\zeta$$

By using equation and after some simplification, we get

$$I_4 \equiv \sum_{k=0}^{\infty}\frac{(\delta)_{\mu k}}{\prod_{j=1}^{m}\Gamma(A_j\,k+B_j)}\frac{\theta^k}{k!}\frac{\Gamma(\epsilon+k)\,\Gamma(\nu+k)}{\Gamma(\epsilon+\nu+2k)}$$

$$I_4 \equiv \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m} \Gamma(A_j\, k + B_j)} \frac{\theta^k}{k!} \frac{(\epsilon)_k (\nu)_k}{(\nu+\epsilon)_{2k}}$$

$$I_4 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} \sum_{k=0}^{\infty} \frac{(\delta)_{\mu k}}{\prod_{j=1}^{m} \Gamma(A_j\, k + B_j)} \frac{\theta^k}{k!} \frac{(\epsilon)_k (\nu)_k (1)_k}{2^{2k} \left(\frac{\nu+\epsilon}{2}\right)_k \left(\frac{\nu+\epsilon+1}{2}\right)_k \; k!}$$

Now apply Hadamard product*i.e.* $\sum_{k=0}^{\infty} C_k\, z^k$

$$I_4 \equiv \frac{\Gamma(\nu)\,\Gamma(\epsilon)}{\Gamma(\nu+\epsilon)} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4}\right) * \; {}_3F_2\left[\begin{matrix}\epsilon, \nu, 1 \\ \frac{\nu+\epsilon}{2}, \frac{\nu+\epsilon+1}{2}\end{matrix} \middle| \; \frac{\theta}{4}\right]$$

4. Special Cases

On taking in we get

$$\int_0^1 (1-\xi)\left(1-\frac{\xi}{3}\right) E^{\delta,\mu}_{(A_j,B_j)_m}(X)\; d\xi =$$

$$\left(\frac{4}{9}\right) E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \; {}_2F_1\left[\begin{matrix}1,1 \\ 2\end{matrix} \middle| \; \theta\right] \quad (4.1)$$

where

On taking in we get

$$\int_0^1 (1-\xi)\left(1-\frac{\xi}{3}\right) E^{\delta,\mu}_{(A_j,B_j)_m}(Y)\; d\xi =$$

$$\left(\frac{4}{9}\right) E^{\delta,\mu}_{(A_j,B_j)_m}(\theta) * \; {}_2F_1\left[\begin{matrix}1, \; 1 \\ 2\end{matrix} \middle| \; \theta\right] \quad (4.2)$$

where

On taking in we get

$$\int_0^1 (1-\xi)^{\epsilon-1} \left[C\xi + D(1-\xi)\right]^{-\epsilon-1} E^{\delta,\mu}_{(A_j,B_j)_m}(Z)\; d\xi$$

$$= \frac{1}{C\; D^{\epsilon}} \frac{1}{\epsilon} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4CD}\right) * \; {}_3F_2\left[\begin{matrix}\epsilon, 1, 1 \\ \frac{\epsilon+1}{2}, \frac{\epsilon+2}{2}\end{matrix} \middle| \; \frac{\theta}{4CD}\right] \quad (4.3)$$

where

On taking in we get

$$\int_0^1 \xi^{\upsilon-1} \left[C\xi + D(1-\xi)\right]^{-\upsilon-1} E^{\delta,\mu}_{(A_j,B_j)_m}(Z)\, d\xi$$

$$= \frac{1}{C^{\upsilon} D} \frac{1}{\upsilon} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4CD}\right) * \; {}_3F_2\left[\begin{matrix}\upsilon, 1, 1 \\ \frac{\upsilon+1}{2}, \frac{\upsilon+2}{2}\end{matrix} \middle| \; \frac{\theta}{4CD}\right] \quad (4.4)$$

where

On taking in we get

$$\int_0^1 \int_0^1 \xi^{\epsilon} (1-\zeta)^{\epsilon-1}(1-\xi\,\zeta)^{-\epsilon} E^{\delta,\mu}_{(A_j,B_j)_m}(W)\; d\xi\, d\zeta$$

$$= \frac{1}{\epsilon} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4}\right) * \;{}_3F_2\left[\begin{matrix}\epsilon,1,1\\ \frac{\epsilon+1}{2}, \frac{\epsilon+2}{2}\end{matrix}\middle| \frac{\theta}{4}\right] (4.5)$$

where

On taking in we get

$$\int_0^1 \int_0^1 \xi\, (1-\xi)^{\upsilon-1}(1-\xi\,\zeta)^{-\upsilon} E^{\delta,\mu}_{(A_j,B_j)_m}(W)\, d\xi\, d\zeta$$

$$= \frac{1}{\upsilon} E^{\delta,\mu}_{(A_j,B_j)_m}\left(\frac{\theta}{4}\right) * \;{}_3F_2\left[\begin{matrix}\upsilon,1,1\\ \frac{\upsilon+1}{2}, \frac{\upsilon+2}{2}\end{matrix}\middle| \frac{\theta}{4}\right] (4.6)$$

where

REFERENCES

1. A. Wiman (1905). Uber den fundamental satz in der theorie der funcktionen$E_{\alpha}(x)$, *Acta Math.*, Vol. 29, pp. 191-201

2. A. Erdelye, W. Magnus, F. Oberhettinger and F. G. Tricomi (1953). Higher transcedental functions, Vol. I, McGraw-Hill, New York, London

3. A. A. Kilbas (2005). Fractional calculus of generalized Wright function, *Fractional Calculus and Applied Analysis*, Vol. 8, pp. 113-126

4. A. K. Shukla and J. C. Prajapati (2007). On a generalization of Mittag Leffler function and its properties, J. Math. Ann. Appl.,Vol. 336(2), 797-811.

5. E. D. Rainville (1960). Special functions, Chelsea Publication Co., New York

6. E. M. Wright (1940). The asymptotic expansion of the generalized hypergeometric function II, Proc. London Math. Soc., Vol. 46 (2), pp.389-408

7. F. Oberhettinger (1974). Tables of Mellin Transforms, Springer, New York

8. H. M. Srivastava (1972). A contour integral involving Fox's H-function, Indian J. Math. Vol. 14,pp. 1-6

9. J. Edward (1922). A treatise on the integral calculus, Vol. II, Chelsea Publishing Company, New York

10. J. L. Lavoie and G. Trottier (1969). On the sum of certain Appell's series, Ganita, Vol. 20 pp. 43–46

11. R. K. Saxena and R. K. Parmar (2017). Fractional integration and differention of the generalized Mathieu series, axioms, MDPI

12. R. K. Saxena and K. Nishimoto (2010). *N*-Fractional calculus of generalized Mittag Leffler functions, J. Frac. Calc.,Vol. 37, 43-52

13. T. Pohlen (2009). The hadamard product and universal power series, Ph.D Thesis, Universitat Trier, Germany

14. T. M. MacRobert (1961). Beta functions formulae and integrals involving E-function, Math. Annalen., Vol. 142, pp. 450-452

15. T. R. Prabhakar (1971). A singular integral equation with a generalized Mittag Leffler function in the kernel, *Yokohama Math. J.*, Vol. 19, pp. 7-15

16. V. S. Kiryakova (2000). Multiple (multiindex) Mittag-Leffler functions and relations to generalized fractional calculus, *J. Comput. Appl. Math.*, Vol. 118, pp. 241-259