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ON NEW MAPPINGS IN TOPOLOGICAL SPACES.

Dr. Arun Kumar Gali
Associate professor
Department of Mathematics
Government First Grade College Sector no-43,Navanagar
Bagalkote-587103, Karnataka State , INDIA.

Abstract

In this paper a new class of homeomorphism called minimal Generalized regular homeomorphism in topological spaceshomeomorphism and maximal Generalized regular homeomorphism in topological spaces homeomorphism are introduced and investigated and during this process some properties of the new concepts have been studied.

Keywords: Maximal open set, Minimal open set, Maximal Homeomorphism, Minimal homeomorphism.

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1. INTRODUCTION:

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)$[$M_aC(X)$].The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000, M.Sheik john [4] introduced and studied weakly homeomorphism in topological spaces and in the year 2008 B.M.Ittanagi [5] introduced and studied minimal open sets and maps in topological spaces and minimal homeomorphism and maximal homeomorphism in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre[6] [7] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces.In the year 2014 [8] Vivekananda Dembre, Manjunath Gowda and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly continuous functions in topological spaces. In the year 2014 [9] Vivekananda Dembre and Jeetendra Gurjar introduced and studied minimal weakly and maximal weakly open maps in topological spaces.

Definition 1.1[1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

Definition 1.2[2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

Definition 1.3[3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

Definition 1.4[3]:A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

Definition 1.5[4]: Let X and Y be the topological spaces. A bijective function $f : X \rightarrow Y$ is called weakly homeomorphism if both f and f^{-1} are weakly continuous.

Definition 1.6[5]: Let X and Y be the topological spaces. A bijective function $f : X \rightarrow Y$ is called

(i) Minimal homeomorphism if both f and f^{-1} are minimal continuous maps.
(ii) Maximal homeomorphism if both f and f^{-1} are maximal continuous maps.

Definition 1.7 [5]: Let X and Y be the topological spaces. A map $f : X \rightarrow Y$ is called

(i) Minimal continuous function if for every minimal open set N in Y, $f^{-1}(N)$ is an open set in X.

(ii)Maximal continuous function if for every maximal open set N in Y, $f^{-1}(N)$ is an open set in X.

(iii)Minimal open map for every open set U of X, f(U) is minimal open set in Y.

(iv)Maximal open map for every open set U of X, f(U) is maximal open set in Y.

(v)Minimal closed map for every closed set F of X, f(F) is minimal closed set in Y.

(vi)Maximal closed map for every closed set F of X, f(F) is maximal closed set in Y.

(vii)Minimal-Maximal open map for every minimal open set N of X, f(N) is maximal open set in Y.

(viii)Maximal-Minimal openmap for every maximal open set N of X,f(N) is minimal open set in Y.

(ix)Minimal-Maximal continuous if for every minimal open set N in Y, $f^{-1}(N)$ is a maximal open set in X.

(x) Maximal-Minimal continuous if for every maximal open set N in Y, $f^{-1}(N)$ is a minimal open set in X.

Definition 1.8 [6]: Let X and Y be the topological spaces. A map $f : X \rightarrow Y$ is called

(i) Minimal weakly continuous if for every minimal weakly open set N in Y,$f^{-1}(N)$ is an open set in X.
(ii) Maximal weakly continuous if for every maximal weakly open set N in Y,$f^{-1}(N)$ is an open set in X.

Definition 1.9 [7]: A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is ∅ or U.

Definition 1.10 [8]: A proper non-empty weakly closed subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

Definition 1.11 [9]: Let X and Y be topological spaces.

(i) A map $f : X \longrightarrow Y$ is called minimal weakly open map for every open set U of X , f(U) is minimal weakly open set in Y.

(ii) A map $f : X \longrightarrow Y$ is called maximal weakly open map for every open set U of X , f(U) is maximal weakly open set in Y.

(iii) A map $f : X \longrightarrow Y$ is called minimal weakly closed map for every closed set F of X , f(F) is minimal weakly closed set in Y.

(iv) A map $f : X \longrightarrow Y$ is called maximal weakly closed map for every closed set F of X , f(F) is maximal weakly closed set in Y.

2. MINIMAL GENERALIZED REGULARHOMEOMORPHISM IN TOPOLOGICAL SPACESAND MAXIMAL REGULAR WEAKLY HOMEOMORPHISM.

Definition 2.1: A bijection function $f : X \longrightarrow Y$ is called

(i) Minimal Generalized regular homeomorphism in topological spaces homeomorphism if both f and f^{-1} are minimal regular weakly continuous maps.

(ii) Maximal Generalized regular homeomorphism in topological spaces homeomorphism if both f and f^{-1} are maximal Generalized regular homeomorphism in topological spacescontinuous maps.

Theorem 2.2: Every homeomorphism is minimal Generalized regularhomeomorphism in topological spaceshomeomorphism but not conversely.

Proof: Let $f : X \longrightarrow Y$ be a homeomorphism. Now f and f^{-1} are continuous maps then f and f^{-1} are minimal Generalized regularhomeomorphism in topological spacescontinuous as every continuous map is minimal Generalized regular homeomorphism in topological spaces continuous. Hence f is a minimal Generalized regular homeomorphism in topological spaceshomeomorphism.

Example 2.3 : Let X=Y={a,b,c} be with τ = { X, φ, {a},{a,b}} and μ = { Y, φ, {a},{b},{a,b} } then $f : X \longrightarrow Y$ be a function defined by f(a)=a, f(b)=a and f(c)=c, then f and f^{-1} are minimal Generalized regular homeomorphism in topological spacescontinuous maps then f is a minimal Generalized regular homeomorphism in topological spaces homeomorphism but it is not a homeomorphism. Since f is not continuous map for the open set {b} in Y ; f^{-1} ({b}) = b which is not open set in X.

Theorem 2.4: Every homeomorphism is maximal Generalized regular homeomorphism in topological spaces homeomorphism but not conversely.

Proof: Let $f : X \longrightarrow Y$ be a homeomorphism. Now f and f^{-1} are continuous maps then f and f^{-1} are maximal Generalized regularhomeomorphism in topological spaces continuous as every continuous map is minimal Generalized regular homeomorphism in topological spaces continuous. Hence f is a minimal Generalized regularhomeomorphism in topological spaceshomeomorphism.

Example 2.5: Let X = Y = {a,b,c} be with τ = { X, φ, {a,b}} and μ = { Y, φ, {a},{b},{a,b} } then f: X $\rightarrow$ Y be identity function then f and f^{-1} are maximal Generalized regularhomeomorphism in topological spacescontinuous maps then f is a maximal Generalized regular homeomorphism in topological spaces homeomorphism but it is not a homeomorphism. Since f is not a continuous map for the open set {b} in Y ; f^{-1} ({b}) = b which is not open set in X.

Remark 2.6: Minimal Generalized regular homeomorphism in topological spaces homeomorphism and maximal Generalized regular homeomorphism in topological spaceshomeomorphism are independent of each other.

Example 2.7: In example 2.3 f is minimal Generalized regularhomeomorphism in topological spaceshomeomorphism but it is not maximal Generalized regularhomeomorphism in topological spaceshomeomorphism. In example 2.5 f is maximal Generalized regular homeomorphism in topological spaces homeomorphism but it is not minimal Generalized regular homeomorphism in topological spaceshomeomorphism.

Theorem 2.8: Let f : X $\rightarrow$ Y be a bijective and minimal Generalized regular homeomorphism in topological spacescontinuous then the following statements are equivalent.

(i) f: X $\rightarrow$ Y is minimal Generalized regular homeomorphism in topological spaceshomeomorphism.

(ii) f is minimal Generalized regular homeomorphism in topological spaces open map.

(iii) f is maximal Generalized regular homeomorphism in topological spaces closed map.

Proof :

(i) $\rightarrow$ (ii) : Let N be any minimal Generalized regular homeomorphism in topological spaces open set in X ; by assumption $(f^{-1})^{-1}$ (N) is an open set in Y. But $(f^{-1})^{-1}$ (N) = f (N) is an open set in Y ; therefore f is a minimal Generalized regular homeomorphism in topological spacesopen map.

(ii) $\rightarrow$ (iii) : Let F be any maximal Generalized regular homeomorphism in topological spaces closed set in X ; then X-F is a minimal Generalized regularhomeomorphism in topological spacesopen set in X ; by assumption f (X-F) is an open set in Y. But f (X-F) = Y- f (F) is an open set in Y ; therefore f(F) is a closed set in Y. Hence f is a maximal Generalized regularhomeomorphism in topological spacesclosed map.

(iii) $\rightarrow$ (i): Let N be any minimal Generalized regular homeomorphism in topological spacesopen set in X ; then X-N is a maximal Generalized regularhomeomorphism in topological spacesclosed set in X ; by assumption f (X-N) is an closed set in Y. But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is closed set in Y ; therefore (f^{-1}) (N) is an open set in Y. Hence f^{-1} : Y $\rightarrow$ X is a minimal Generalized regularhomeomorphism in topological spaceshomeomorphism and similarly f is minimal Generalized regularhomeomorphism in topological spaceshomeomorphism.

Theorem 2.9: Let f : X $\rightarrow$ Y be a bijective and maximal Generalized regular homeomorphism in topological spacescontinuous then the following statements are equivalent.

(i) f^{-1}:X $\rightarrow$ Y is maximal Generalized regularhomeomorphism in topological spaceshomeomorphism.

(ii) f is maximal Generalized regular homeomorphism in topological spacesopen map.

(iii) f is minimal Generalized regular homeomorphism in topological spacesclosed map.

Proof : (i) $\rightarrow$ (ii) : Let N be any maximal Generalized regular homeomorphism in topological spaces open set in X ; by assumption $(f^{-1})^{-1}$ (N) is an open set in Y. But $(f^{-1})^{-1}$ (N) = f (N) is an open set in Y ; therefore f is a minimal Generalized regular homeomorphism in topological spacesopen map.

(ii) $\longrightarrow$ (iii): Let F be any minimal Generalized regular homeomorphism in topological spaces closed set in X ; then X-F is a maximal Generalized regular homeomorphism in topological spaces open set in X ; by assumption f (X-F) is an open set in Y. But f (X-F) = Y- f (F) is an open set in Y ; therefore f(F) is a closed set in Y. Hence f is a maximal Generalized regular homeomorphism in topological spaces closed map.

(iii) $\longrightarrow$ (i): Let N be any maximal Generalized regular homeomorphism in topological spaces open set in X ; then X-N is a minimal Generalized regular homeomorphism in topological spaces closed set in X ; by assumption f (X-N) is an closed set in Y. But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is closed set in Y ; therefore (f^{-1}) (N) is an open set in Y. Hence f^{-1} : Y $\longrightarrow$ X is a minimal Generalized regular homeomorphism in topological spaceshomeomorphism and similarly f is minimal Generalized regular homeomorphism in topological spaces homeomorphism.

Definition 2.10: Let X and Y be the topological spaces. A map f : X $\longrightarrow$ Y is called

(i)Minimal-Maximal Generalized regular homeomorphism in topological spaces continuous if for every minimal Generalized regular homeomorphism in topological spaces open set N in Y, f^{-1}(N) is a maximal Generalized regular homeomorphism in topological spaces open set in X.

(ii) Maximal-Minimal Generalized regular homeomorphism in topological spaces continuous if for every maximal Generalized regular homeomorphism in topological spacesopen set N in Y, f^{-1}(N) is a minimal Generalized regular homeomorphism in topological spaces open set in X.

(iii) Minimal-Maximal Generalized regular homeomorphism in topological spaces open map for every minimal Generalized regular homeomorphism in topological spacesopen set N of X, f(N) is maximal Generalized regular homeomorphism in topological spacesopen set in Y.

(iv) Maximal-Minimal Generalized regularhomeomorphism in topological spacesopen map for every maximal Generalized regularhomeomorphism in topological spacesopen set N of X, f(N) is minimal Generalized regularhomeomorphism in topological spacesopen set in Y.

(v) Minimal-Maximal Generalized regularhomeomorphism in topological spacesclosed map for every minimal Generalized regularhomeomorphism in topological spacesclosed set F of X, f(F) is maximal Generalized regularhomeomorphism in topological spacesclosed set in Y.

(vi) Maximal-Minimal Generalized regularhomeomorphism in topological spacesclosed map for every maximal regular weaklyclosed set F of X, f(F) is minimal Generalized regularhomeomorphism in topological spacesclosed set in Y.

Definition 2.11: A bijection f : X $\longrightarrow$ Y is called

(i) min - max Generalized regular homeomorphism in topological spaces homeomorphism if both f and f^{-1} are min - max Generalized regularhomeomorphism in topological spacescontinuous maps.

(ii) max- min Generalized regularhomeomorphism in topological spaces homeomorphism if both f and f^{-1} are max - min Generalized regular homeomorphism in topological spacescontinuous maps.

Theorem 2.12: Every min-max Generalized regularhomeomorphism in topological spaces homeomorphism is minimal Generalized regular homeomorphism in topological spaces homeomorphism but not conversely.

Proof : : Let f : X $\longrightarrow$ Y be a homeomorphism. Now f and f^{-1} are continuous maps then f and

f^{-1} are min-max Generalized regular homeomorphism in topological spaces continuous as every continuous map is min-max Generalized regular homeomorphism in topological spacescontinuous. Hence f is a min-max Generalized regular homeomorphism in topological spaceshomeomorphism.

Example 2.13: In example 2.3 f is a minimal Generalized regular homeomorphism in topological spaces homeomorphism but it is not a min-max Generalized regular homeomorphism in topological spaces homeomorphism. Since f is not a min-max Generalized regular homeomorphism in topological spaces continuous map for the minimal Generalized regular homeomorphism in topological spacesopen set {b} in Y, f^{-1}(b) = {b} which is not a maximal Generalized regular homeomorphism in topological spacesopen set in X.

Theorem 2.14: Every max-min Generalized regular homeomorphism in topological spaces homeomorphism is maximal Generalized regular homeomorphism in topological spaces homeomorphism but not conversely.

Proof : Let $f : X \longrightarrow Y$ be a homeomorphism. Now f and f^{-1} are continuous maps then f and f^{-1} are max-min Generalized regular homeomorphism in topological spacescontinuous as every continuous map is max-min Generalized regular homeomorphism in topological spaces continuous. Hence f is a min-max Generalized regular homeomorphism in topological spaces homeomorphism.

Theorem 2.15: Let $f : X \longrightarrow Y$ be a bijective and min-max Generalized regular homeomorphism in topological spaces continuous map then the following statements are equivalent.

(i) f is minimal - maximal Generalized regular homeomorphism in topological spaceshomeomorphism.
(ii) f is minimal - maximal Generalized regular homeomorphism in topological spacesopen map.
(iii) f is maximal - minimal Generalized regular homeomorphism in topological spacesclosed map.

Proof :

(i) $\longrightarrow$ (ii) : Let N be any minimal Generalized regular homeomorphism in topological spacesopen set in X ; by assumption $(f^{-1})^{-1}$ (N) is an open set in Y. But $(f^{-1})^{-1}$ (N) = f (N) is a maximal Generalized regular homeomorphism in topological spacesopen set in Y ; therefore f is a min-max Generalized regular homeomorphism in topological spacesopen map.

(ii) $\longrightarrow$ (iii) : Let F be any maximal Generalized regular homeomorphism in topological spacesclosed set in X ; then X-F is a minimal Generalized regular homeomorphism in topological spaces open set in X ; by assumption f (X-F) is a maximal Generalized regular homeomorphism in topological spaces open set in Y. But f (X-F) = Y- f (F) is a maximal Generalized regular homeomorphism in topological spacesopen set in Y ; therefore f(F) is a minimal Generalized regular homeomorphism in topological spacesclosed set in Y. Hence f is a max-min Generalized regular homeomorphism in topological spacesclosed map.

(iii) $\longrightarrow$ (i) : Let N be any minimal Generalized regular homeomorphism in topological spaces open set in X ; then X-N is a maximal Generalized regularhomeomorphism in topological spacesclosed set in X ; by assumption f (X-N) is a minimal Generalized regular homeomorphism in topological spaces closed set in Y. But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is a minimal Generalized regular homeomorphism in topological spaces open set in Y ; therefore (f^{-1}) (N) is a maximal regular weaklyopen set in Y. Hence $f^{-1}: Y \longrightarrow X$ is a minimal-maximal Generalized regularhomeomorphism in topological spaceshomeomorphism and similarly f is min-max Generalized regular homeomorphism in topological spaceshomeomorphism.

Theorem 2.16: Let f : X $\longrightarrow$ Y be a bijective and min-max Generalized regular homeomorphism in topological spaces continuous map then the following statements are equivalent.

(i) f is max - min Generalized regular homeomorphism in topological spaceshomeomorphism.

(ii) f is max - min Generalized regular homeomorphism in topological spacesopen map.

(iii) f is min - max Generalized regular homeomorphism in topological spacesclosed map.

Proof : (i) $\longrightarrow$ (ii) : Let N be any maximal Generalized regular homeomorphism in topological spacesopen set in X ; by assumption $(f^{-1})^{-1}$ (N) is an open set in Y. But $(f^{-1})^{-1}$ (N) = f (N) is a minimal Generalized regular homeomorphism in topological spacesopen set in Y ; therefore f is a max - min Generalized regular homeomorphism in topological spacesopen map.

(ii) $\longrightarrow$ (iii) : Let F be any minimal Generalized regular homeomorphism in topological spacesclosed set in X ; then X-F is a maximal Generalized regular homeomorphism in topological spacesopen set in X ; by assumption f (X-F) is a minimal Generalized regularhomeomorphism in topological spacesopen set in Y.
But f (X-F) = Y- f (F) is a minimal Generalized regular homeomorphism in topological spacesopen set in Y ; therefore f(F) is a minimal Generalized regularhomeomorphism in topological spacesclosed set in Y. Hence f is a max-min Generalized regularhomeomorphism in topological spacesclosed map.

(iii) $\longrightarrow$ (i) : Let N be any minimal Generalized regular homeomorphism in topological spaces open set in X ; then X-N is a maximal Generalized regularhomeomorphism in topological spacesclosed set in X ; by assumption f (X-N) is a minimal Generalized regularhomeomorphism in topological spacesclosed set in Y.
But f (X-N) = (f^{-1}) (X - N) = Y - (f^{-1}) (N) is a minimal Generalized regularhomeomorphism in topological spacesopen set in Y ; therefore (f^{-1}) (N) is a maximal regular weaklyopen set in Y. Hence f^{-1}:Y $\longrightarrow$ X is a minimal-maximal Generalized regularhomeomorphism in topological spaceshomeomorphism and similarly f is min-max Generalized regularhomeomorphism in topological spaceshomeomorphism.

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