



# TWO-DIMENSIONAL MUTUALISM MODEL: INTERACTIONS BETWEEN PLANTS AND POLLINATORS

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## ABSTRACT

Two-dimensional models describing the interaction between plants and pollinators play a critical role in understanding the dynamics of mutualistic relationships in ecological systems. These models typically involve equations representing the population dynamics of plants and pollinators, considering factors such as growth rates, carrying capacities, and interaction coefficients. By simulating the dynamics of plant and pollinator populations over time, these models provide insights into mutualistic benefits, population trends, and stability of the relationship between two species. This shows applications of two-dimensional models in studying plant-pollinator interactions, highlighting their importance in ecological research and conservation.

**Keywords: Populations dynamics, mutualistic benefits, population trends, stability.**

## 1. INTRODUCTION

The intricate relationship between plants and pollinators forms the backbone of the terrestrial ecosystem, influencing biodiversity, ecosystem stability, and agricultural productivity. Understanding the dynamics of this mutualistic interaction is essential for conservation efforts, ecosystem management, and sustainable agriculture. In recent years, mathematical modeling has emerged as a powerful tool for exploring the complexities of plant-pollinator interactions.

This introduces a two-dimensional mutualism model to investigate the dynamics between plant pollinators in spatially heterogeneous environments. Building upon classical ecological theory and mathematical modeling techniques, our model aims to capture the spatial dynamics and population interactions inherent in mutualistic networks. The primary objective is to explore how spatial heterogeneity influences the stability and resilience of plant-pollinator communities. By extending traditional one-dimensional models to two dimensions, we can better represent the spatial distribution

of resources, habitat fragmentation, and dispersal dynamics, which are crucial factors shaping mutualistic interactions in real-world ecosystems.

**A Two-Dimensional Model of Plant-Pollinator Interactions by Thompson and Smith (2017):** This study presents a two-dimensional model of mutualistic interactions between plants and pollinators using logistic equations. The model explores how population growth and resource availability influence the dynamics of mutualistic networks. By incorporating logistic equations, the researchers examine how carrying capacity and density-dependent processes affect the stability of mutualistic relationships.

## 2. RESEARCH METHOD

To simulate the dynamics of plant and pollinator populations in a two-dimensional landscape, we employ Euler's method, a numerical approximation technique widely used in mathematical modeling. Euler's method allows us to discretize differential equations describing population dynamics iteratively estimate population dynamics and iteratively estimate population sizes over time and space.

## 3. MUTUAL BENEFACTION EQUATION

We will similarly approach the problem as the Lotka-Volterra competition model by first assuming two species  $N_1$  and  $N_2$  that logically can't survive in the absence of each other. The adjusted equations governing mutual benefaction are denoted by

$$\frac{dN_1}{dt} = r_1 N_1 \left[ 1 - \frac{(N_1 - \alpha N_2)}{K_1} \right] \text{ and}$$

$$\frac{dN_2}{dt} = r_2 N_2 \left[ 1 - \frac{(N_2 - \beta N_1)}{K_2} \right]$$

Where  $\alpha$  and  $\beta$  clump many mechanisms together into phenomenological entities which determine the strength of mutualistic benefaction from each species.

The above equations represent a logistic growth model with mutualistic interactions between two species, where  $N_1$  represents the population size of plant species,  $N_2$  represents the population size of pollinator species,  $r_1$  represents the intrinsic growth rate of plant species,  $r_2$  represents the intricate growth of pollinator species,  $\alpha$  is the coefficient representing the effect of pollinators on plants and  $\beta$  is the coefficient representing the effect of plants on pollinators,  $K_1$  is the carrying capacity of the environment for plants and  $K_2$  is the carrying capacity of the environment for pollinators.

## 4. EULER'S METHOD FOR SIMULATING PLANT-POLLINATOR INTERACTIONS

We'll use Euler's method to numerically solve this equation and calculate the population dynamics of plants over a specified period. Now let us consider the parameters

$$N_1 = \text{Plants}$$

$$N_2 = \text{Pollinators}$$

$$r_1 = 0.1$$

$$r_2 = 0.05$$

$$\alpha = 0.2$$

$$\beta = 0.01$$

$$K_1 = 1000$$

$$K_2 = 500$$

$$N_1(0) = 100$$

$$N_2(0) = 50$$

Let's proceed with calculating the population dynamics of plants and pollinators using the provided equations and parameters. We'll use a time step of 1 unit for simplicity. Using Euler's method to calculate the population dynamics of plants and pollinators over some time of 5-time units.

$$N_1(t+1) = N_1(t) + r_1 N_1(t) \left[ 1 - \frac{(N_1(t) - \alpha N_2(t))}{K_1} \right]$$

$$N_2(t+1) = N_2(t) + r_2 N_2(t) \left[ 1 - \frac{(N_2(t) - \beta N_1(t))}{K_2} \right]$$

At t=0,

$$N_1(1) = N_1(0) + r_1 N_1(0) \left[ 1 - \frac{(N_1(0) - \alpha N_2(0))}{K_1} \right]$$

$$N_1(1) = 100 + 0.1(100) \left[ 1 - \frac{(100 - 0.2(50))}{1000} \right]$$

$$N_1(1) = 109.1$$

$$N_2(1) = N_2(0) + r_2 N_2(0) \left[ 1 - \frac{(N_2(0) - \beta N_1(0))}{K_2} \right]$$

$$N_2(1) = 50 + 0.05(50) \left[ 1 - \frac{(50 - 0.01(100))}{500} \right]$$

$$N_2(1) = 52.255$$

At t=1,

$$N_1(2) = N_1(1) + r_1 N_1(1) \left[ 1 - \frac{(N_1(1) - \alpha N_2(1))}{K_1} \right]$$

$$N_1(2) = 109.1 + 0.1(109.1) \left[ 1 - \frac{(109.1 - 0.2(52.255))}{1000} \right]$$

$$N_1(2) = 118.9337394$$

$$N_2(2) = N_2(1) + r_2 N_2(1) \left[ 1 - \frac{(N_2(1) - \beta N_1(1))}{K_2} \right]$$

$$N_2(2) = 52.255 + 0.05(52.255) \left[ 1 - \frac{(52.255 - 0.01(109.1))}{500} \right]$$

$$N_2(2) = 54.60041496$$

At t=2,

$$N_1(3) = N_1(2) + r_1 N_1(2) \left[ 1 - \frac{(N_1(2) - \alpha N_2(2))}{K_1} \right]$$

$$N_1(3) = 118.9337394 + 0.1(118.9337394) \left[ 1 - \frac{(118.9337394 - 0.2(54.60041496))}{1000} \right]$$

$$N_1(3) = 225.0210107$$

$$N_2(3) = N_2(2) + r_2 N_2(2) \left[ 1 - \frac{(N_2(2) - \beta N_1(2))}{K_2} \right]$$

$$N_2(3) = 54.60041496 + 0.05(54.60041496) \left[ 1 - \frac{(54.60041496 - 0.01(118.9337394))}{500} \right]$$

$$N_2(3) = 57.040254368$$

At t=3,

$$N_1(4) = N_1(3) + r_1 N_1(3) \left[ 1 - \frac{(N_1(3) - \alpha N_2(3))}{K_1} \right]$$

$$N_1(4) = 225.0210107 + 0.1(225.0210107) \left[ 1 - \frac{(225.0210107 - 0.2(57.040254368))}{1000} \right]$$

$$N_1(4) = 242.7163713587$$

$$N_2(4) = N_2(3) + r_2 N_2(3) \left[ 1 - \frac{(N_2(3) - \beta N_1(3))}{K_2} \right]$$

$$N_2(4) = 57.040254368 + 0.05(57.040254368) \left[ 1 - \frac{(57.040254368 - 0.01(225.0210107))}{500} \right]$$

$$N_2(4) = 59.57974328$$

At t = 4,

$$N_1(5) = N_1(4) + r_1 N_1(4) \left[ 1 - \frac{(N_1(4) - \alpha N_2(4))}{K_1} \right]$$

$$N_1(5) = 242.7163713587 + 0.1(242.7163713587)$$

$$\left[ 1 - \frac{(242.7163713587 - 0.2(59.57974328))}{1000} \right]$$

$$N_1(5) = 261.38610438392$$

$$N_2(5) = N_2(4) + r_2 N_2(4) \left[ 1 - \frac{(N_2(4) - \beta N_1(4))}{K_2} \right]$$

$$N_2(5) = 59.57974328 + 0.05(59.57974328)$$

$$\left[ 1 - \frac{(59.57974328 - 0.01(242.7163713587))}{500} \right]$$

$$N_2(5) = 62.218216842164$$

At  $t = 5$ ,

$$N_1(6) = N_1(5) + r_1 N_1(5) \left[ 1 - \frac{(N_1(5) - \alpha N_2(5))}{K_1} \right]$$

$$N_1(6) = 261.38610438392 + 0.1(261.38610438392)$$

$$\left[ 1 - \frac{(261.38610438392 - 0.2(62.218216842164))}{1000} \right]$$

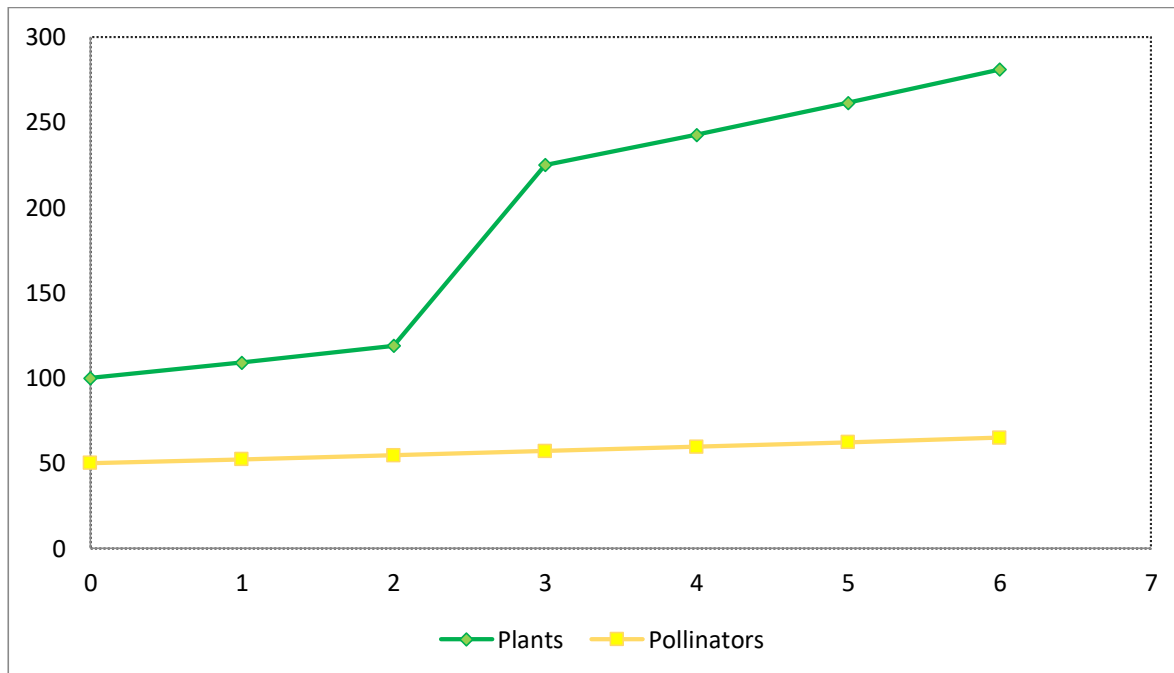
$$N_1(6) = 281.010770481265$$

$$N_2(6) = N_2(5) + r_2 N_2(5) \left[ 1 - \frac{(N_2(5) - \beta N_1(5))}{K_2} \right]$$

$$N_2(6) = 62.218216842164 + 0.05(62.218216842164)$$

$$\left[ 1 - \frac{(62.218216842164 - 0.01(261.38610438392))}{500} \right]$$

$$N_2(6) = 64.958280010892$$



## 5. DISCUSSION AND RESULTS

The Mutualistic relationship between plants and pollinators is given by:

**Positive population Growth:** Both plants and pollinators experience positive population growth over the observed period (from  $t = 0$  to  $t = 5$ ). This suggests that the mutualistic interaction between the two species is beneficial, with each species contributing to the growth and sustainability of the other.

**Different Growth Rates:** The population growth rates of plants and pollinators may differ, as indicated by their respective population sizes at  $t = 5$ . In this case, the population of plants has increased more significantly compared to the population of pollinators, potentially reflecting variations in reproductive rates, resource availability, or ecological dynamics between the two species.

**Steady Increase in Plant Population:** The estimated population of plants continues to increase steadily over time, reaching a value of approximately 281 at  $t = 5$ . This suggests that the mutualistic relationship between plants and pollinators is supporting the growth and expansion of the plant population, possibly through enhanced pollination and reproductive success.

**Moderate Increase in pollinator population:** While the population of pollinators also increases over the observed time, the rate of increase appears to be more moderate compared to that of plants. This may indicate that the availability of resources or other factors constrain the growth of the pollinator population to some extent, relative to the growth of the plant population.

**Sustainability of Mutualistic Relationship:** The observed Population dynamics suggest that the mutualistic relationship between plants and pollinators is sustainable over the observed period. However, further analysis and monitoring may be needed to assess the long-term stability and resilience of the mutualistic interaction under changing environmental conditions or other perturbations.

## 6. CONCLUSION

Euler's method provides insights into the population dynamics of plants and pollinators within a mutualistic relationship, highlighting positive growth trends and potential differences in growth rates between the two species. These findings contribute to our understanding of the dynamics and sustainability of mutualistic interactions in ecological systems.

## REFERENCE

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