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Stability analysis of triple-diffusive convection in a non-Newtonian fluid layer

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Abstract : The onset of convective instability is analyzed in a triply diffusive Maxwell fluid layer in which density depends on three stratifying agents having different molecular diffusivities. The modified Darcy-Brinkman-Maxwell model is used for the momentum equation. Two problems have been analyzed mathematically. In the first problem, a sufficient condition is derived for the validity of the principle of the exchange of stabilities(PES). Further, when the complement of this condition holds good, oscillatory motions of neutral or growing amplitude can exist. Thus as a second problem bounds for the complex growth rate are also obtained. Further established that the results are consistently valid for any combination of rigid and/or free boundaries.

Keywords: Instability, triple diffusive convection, Maxwell fluid.

INTRODUCTION:

Buoyancy driven convection in a fluid layer investigated by Bénard and by Rayleigh in the early 20th century, known as Rayleigh-Bénard convection, remains a topic of active research for more than a century. In many situations both temperature and some dissolved substance or any two dissolved substances possessing different diffusivities may contribute in opposite senses to the buoyancy gradient leading to instability in a fluid layer and the process is often referred to as double diffusive convection. Double diffusive convection is studied extensively, both experimentally and theoretically, because of its wide range of applications in many fields of science and engineering (for details see Chen and Johnson [1]). Excellent reviews on the development of this subject are reported by Turner [2], Huppert and Turner [3] and Platten and Legros [4].

The presence of three different diffusing components having different molecular diffusivities is expected in many naturally occurring and industrial fluid systems which lead to convective instabilities known as triple diffusive convection. Examples of such systems include the solidification of molten alloys, Earth core, geothermally heated lakes, sea water, oil reservoir engineering, magmas and their laboratory models. Griffiths [5] was the first to investigate theoretically the linear stability of triple diffusive convection in a horizontally unbounded fluid layer while Griffiths [6, 7] and Turner [2] reported the related experimental works. Coriell et al. [8] and Noulty and Leaist [9] presented explicit situations in which triple diffusive convection has useful significance. In their seminal paper, Pearlstein et al. [10] performed a comprehensive study on the linear stability of triple diffusive fluid layer and captured the physics of the onset of convection. Terrones and Pearlstein [11] generalized the linear stability analysis to an arbitrary number of components in a horizontal fluid layer. Moroz [12] considered the linear instability problem originally discussed by Griffiths [5]. Ryzhkov and Shevtsova [13] Lopez et al. [14] discovered the result of rigid boundaries on convective instability in a triply diffusive fluid layer. The effects of cross-diffusion on the onset of convective instability in a horizontally unbounded triply cross-diffusive fluid layer were investigated by Terrones [15]. Straughan and Walker [16] analyzed a variety of aspects of penetrative convection in a triply diffusive fluid layer, while Straughan and Tracey [17] considered multicomponent convection-diffusion with internal heating or cooling in a fluid layer. Recently, Shivakumara and Naveen Kumar [18] investigated linear and weakly nonlinear triple diffusive convection in a layer of couple stress fluid.

Majority of the investigations on triple diffusive convection have been dealt with Newtonian fluids. But it is an established fact that the hypothesis of a Newtonian fluid turns out to be inadequate in describing rheologically complex fluid flows occurring in many contexts such as polymer solutions, melts and paints which involve more than two diffusing components. To predict the flow of such fluids use of alternative non-Newtonian models are inevitable and viscoelastic fluid models which account for elastic and memory effects are one among them. Copious literature is available on linear and nonlinear Rayleigh-Bénard convection of viscoelastic fluids (Rosenblat [19], Li and Khayat [20]). The establishment of the nonoccurrence of every slow oscillatory motions, which may be neutral or unstable, implies the validity of the PES. The validity of this principle in stability problems eliminates unsteady terms from linearized perturbation equations. Pellew and Southwell [21] proved the PES validity for the classical Rayleigh-Bernard instability problem. Guptaet al.[22] andPrakash et al. [23-28] investigated on the principle of the exchange of stabilities ofmulti diffusive convection in the presence of rotation, magnetic field. Prakash et al. [23] investigated on the characterization of magnetohydrodynamic triple diffusive convection. Contrary to the stationary onset observed in Newtonian fluids, the onset of convection in viscoelastic fluids was found to be oscillatory depending on the fluid elasticity. Mardones et al. [29] investigated the onset of convection in a binary-viscoelastic Oldroyd-B fluid layer. Malashetty et al. [30] discussed double diffusive convection in a viscoelastic fluid layer. Recently, Hirata et al. [31] examined convective and absolute nature of instabilities in Rayleigh–Bénard–Poiseuille mixed convection for viscoelastic fluids. Recently, multi-diffusive convection in a non-Newtonian fluid saturated porous and nonporous domain problems are studied [32-45].

Zhao et al. [46] investigated the linear instability of triply diffusive convection in a Maxwell fluid-saturated porous layer. Prakash et al. [25] derived a sufficient condition for the occurrence of stationary convection for triply diffusive fluid systems and also obtained upper bounds. Prakash et al. [28] investigated the limitations of linear growth rates for triply diffusive convection in a Newtonian fluid-saturated porous medium. In the present work, we have analyzed the onset of triple diffusive convection in a Maxwell fluid layer by proving theorems for the validity of PES and also upper bounds on the complex growth rate of oscillatory motions. The results obtained are shown to be uniformly valid for any combination of rigid and free boundaries.

MATHEMATICAL FORMULATION AND ANALYSIS:

We consider an incompressible Maxwell fluid layer of depth d in which the density depends on three different stratifying agents having different molecular diffusivities (temperature T and solute concentrations $S_i, i=1,2$). The lower and upper boundaries of the fluid layer $z=0$ and $z=d$ are held at constant temperatures T_L and $T_U(<T_L)$, respectively while species concentration of ith component is held at fixed values S_{iL} and $S_{iU}(<S_{iL})$, respectively. The stratifying agents are assumed to obey the following equation of state

$$\rho=\rho_0\left(1-\alpha_T\left(T-T_L\right)+\alpha_{S1}\left(S_1-S_{1L}\right)+\alpha_{S2}\left(S_2-S_{2L}\right)\right) \tag{1}$$

where α_T is the thermal expansion coefficient, α_{S1} and α_{S2} are the solute analogs of α_T and ρ_0 is the reference density. Under the Boussinesq approximation, the conservation of mass, momentum, energy and solute concentrations are

$$\nabla\cdot\vec{q}=0 \tag{2}$$

$$\frac{\partial\vec{q}}{\partial t}+\left(\vec{q}\cdot\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_0}\right)+\frac{1}{\rho_0}\nabla\cdot\underset{\sim}{\tau}+\frac{\rho}{\rho_0}\vec{g} \tag{3}$$

$$\frac{\partial T}{\partial t}+\left(\vec{q}\cdot\nabla\right)T=\kappa_T\nabla^2 T \tag{4}$$

$$\frac{\partial S_1}{\partial t}+\left(\vec{q}\cdot\nabla\right)S_1=\kappa_{S1}\nabla^2 S_1 \tag{5}$$

$$\frac{\partial S_2}{\partial t}+\left(\vec{q}\cdot\nabla\right)S_2=\kappa_{S2}\nabla^2 S_2 \tag{6}$$

where $\vec{q}=\left(u,v,w\right)$ is the velocity, p is the pressure, $\underset{\sim}{\tau}$ is the stress tensor, μ is the fluid viscosity, $\vec{g}$ is the acceleration due to gravity, κ_T is the thermal diffusivity, κ_{S1} and κ_{S2} are the solute analogs of κ_T. The constitutive equation for an Maxwell fluid is (Rosenblat [19] and Bird et al. [47])

$$\underset{\sim}{\tau}+\lambda_1\left(\frac{\partial\underset{\sim}{\tau}}{\partial t}+\left(\vec{q}\cdot\nabla\right)\underset{\sim}{\tau}-\left(\nabla\vec{q}\right)^T\underset{\sim}{\tau}-\underset{\sim}{\tau}\left(\nabla\vec{q}\right)\right)=\mu\underset{\sim}{A} \tag{7}$$

where $\underset{\sim}{A}=\nabla\vec{q}+\left(\nabla\vec{q}\right)^T$ is the rate-of-strain tensor, μ is the fluid viscosity, λ_1 is the relaxation time. The constitutive equation considered includes Stokes's law adopted in the theory of Newtonian viscous fluid flows as a special case for $\lambda_1=0$. The basic state is quiescent and the gradients of the stratifying agents are constant and vertical. Thus

$$\vec{q}_b=0,\ \underset{\sim}{\tau}_b=0,\ T_b=T_L-\beta_1 z,\ S_{ib}=S_{iL}-\beta_i z\left(i=1,2\right),\ \rho_b=\rho_0\left(1+\left(\alpha_T\beta-\alpha_{S1}\beta_1-\alpha_{S2}\beta_2\right)z\right) \tag{8}$$

$$p_b/\rho_0 = P_b = P_0 - \rho_0\, g\left(z + \left(\alpha_T\beta - \alpha_{S1}\beta_1 - \alpha_{S2}\beta_2\right)z^2/2\right) \tag{9}$$

where the subscript b denotes the basic state, p_0 is the pressure at $z=0$, $\Delta T = T_L - T_U$, $\Delta S_i = S_{iL} - S_{iU}\ (i=1,2)$, $\beta = \dfrac{\Delta T}{d}$ and $\beta_i = \dfrac{\Delta S_i}{d}(i=1,2)$.

To study the instability of the system, we superimpose infinitesimal perturbations on the basic state which are of the form

$$\vec{q} = \vec{q}_b + \vec{q}',\ P = P_b + P',\ \rho = \rho_b + \rho',\ \underset{\sim}{\tau} = \underset{\sim}{\tau}_b + \underset{\sim}{\tau}',\ T = T_b + T',\ S_i = S_{ib} + S_i'\left(i=1,2\right) \tag{10}$$

where primes indicate infinitesimal perturbations. Equation (10) is substituted back into the governing equations and the linearized perturbation equations are

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0 \tag{11}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial u'}{\partial t} + \frac{\partial P'}{\partial x}\right) = \nu\ \nabla^2 u' \tag{12}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial v'}{\partial t} + \frac{\partial P'}{\partial y}\right) = \nu\ \nabla^2 v' \tag{13}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\partial w'}{\partial t} + \frac{\partial P'}{\partial z} - g\alpha_T T' + g\alpha_{S1}S_1' + g\alpha_{S2}S_2'\right) = \nu\ \nabla^2 w' \tag{14}$$

$$\frac{\partial T'}{\partial t} - \beta w' = \kappa_T \nabla^2 T' \tag{15}$$

$$\frac{\partial S_1'}{\partial t} - \beta_1 w' = \kappa_{S1}\nabla^2 S_1' \tag{16}$$

$$\frac{\partial S_2'}{\partial t} - \beta_2 w' = \kappa_{S2}\nabla^2 S_2' \tag{17}$$

The normal mode expansion of the unknown variables u', v', w', P', T' and $S_i'(i=1,2)$ is assumed in the form

$$F'(x,y,z,t) = F(z)\exp(i(k_x x + k_y y) + nt) \tag{18}$$

where n is the growth term, k_x and k_y are wave numbers in the x and y direction respectively.

Substituting Eq. (18) into Eqs. (11)-(17), thus becomes

$$ik_x u + ik_y v + \frac{\partial w}{\partial z} = 0 \tag{19}$$

$$\left(1+\lambda_1 n\right)\left(nu + ik_x P\right) = \nu\left(D^2 - k^2\right)u \tag{20}$$

$$\left(1+\lambda_1 n\right)\left(nv + ik_y P\right) = \nu\left(D^2 - k^2\right)v \tag{21}$$

$$\left(1+\lambda_1 n\right)\left(nw + \frac{\partial P}{\partial z} - g\alpha_T T + g\alpha_{S1}S_1 + g\alpha_{S2}S_2\right) = \nu\left(D^2 - k^2\right)w \tag{22}$$

$$nT - \beta w = \kappa_T\left(D^2 - k^2\right)T \tag{23}$$

$$nS_1 - \beta_1 w = \kappa_{S1}\left(D^2 - k^2\right)S_1 \tag{24}$$

$$nS_2-\beta_2 w=\kappa_{S2}\left(D^2-k^2\right)S_2 \tag{25}$$

where $k^2=k_x^2+k_y^2$. Eliminating u and v from Eq. (20) and (21) by multiplying Eq. (20) by ik_x and (21) by ik_y respectively, adding the resulting equations and using Eq. (19) and then eliminating P between this ensuing equation and Eq. (22), we get

$$\nu\left(D^2-k^2\right)^2 w-n\left(1+\lambda_1 n\right)\left(D^2-k^2\right)w=\left(1+\lambda_1 n\right)k^2\left[g\alpha_T T-g\alpha_{S1}S_1-g\alpha_{S2}S_2\right] \tag{26}$$

Equations (23)-(25) can also be written as

$$\left(D^2-k^2-\frac{n}{\kappa_T}\right)T=-\frac{\beta}{\kappa_T}w \tag{27}$$

$$\left(D^2-k^2-\frac{n}{\kappa_{S1}}\right)S_1=-\frac{\beta_1}{\kappa_{S1}}w \tag{28}$$

$$\left(D^2-k^2-\frac{n}{\kappa_{S2}}\right)S_2=-\frac{\beta_2}{\kappa_{S2}}w \tag{29}$$

Now using the following non-dimensional parameters

$$a_*=kd,\ z_*=z/d,\ \tau_{1*}=\kappa_{S1}/\kappa_T,\ \tau_{2*}=\kappa_{S2}/\kappa_T,\ \sigma_*=\nu/\kappa_T,\ p_*=nd^2/\nu,$$

$$R_*=g\alpha_T\beta d^4/\nu\kappa_T,\ R_{S1*}=g\alpha_{S1}\beta_1 d^4/\nu\kappa_T,\ R_{S2*}=g\alpha_{S2}\beta_2 d^4/\nu\kappa_T,\ D_*=d\frac{d}{dz}, \tag{30}$$

$$w_*=\frac{\beta d^2}{\kappa_T}w,\ T_*=T,\ S_{i*}=\frac{\beta}{\beta_i}S_i\ (i=1,2),\ \Lambda_{1*}=\lambda_1 d^2/\nu .$$

We can reduce Eqs. (26)-(29) to the following coupled ordinary differential equations in the non-dimensional form(after omitting the asterisks for simplicity)

$$\left(D^2-a^2\right)^2 w-\frac{p}{\sigma}\left(1+\Lambda_1 p\right)\left(D^2-a^2\right)w=\left(1+\Lambda_1 p\right)a^2\left(RT-R_{S1}S_1-R_{S2}S_2\right) \tag{31}$$

$$\left(D^2-a^2-p\right)T=-w \tag{32}$$

$$\left(D^2-a^2-\frac{p}{\tau_1}\right)S_1=-\frac{1}{\tau_1}w \tag{33}$$

$$\left(D^2-a^2-\frac{p}{\tau_2}\right)S_2=-\frac{1}{\tau_2}w \tag{34}$$

The linear coupled ordinary differential equations (31)-(34) are to be solved by using the following boundary conditions

$w=Dw=T=S_1=S_2=0$ at $z=0$ and $z=1$ (both the boundaries are rigid) (35)

$w=D^2w=T=S_1=S_2=0$ at $z=0$ and $z=1$ (both the boundaries are free) (36)

$w=Dw=T=S_1=S_2=0$ at $z=0$ (lower boundary is rigid)
and $w=D^2w=T=S_1=S_2=0$ at $z=1$ (upper boundary is free) (37)

$w=D^2w=T=S_1=S_2=0$ at $z=0$ (lower boundary is free) and $w = Dw = T = S_1 = S_2 = 0$ at $z = 1$ (upper boundary is rigid) (38)

where z is the rear known variable such that $0 \le z \le 1$, D is the differentiation with respect to z, $\sigma > 0$ is the Prandtl number, $\tau_1, \tau_2 > 0$ are the ratio of diffusivities, $R > 0$ is the thermal Rayleigh number, $R_{S1}, R_{S2} > 0$ are solute Rayleigh numbers, Λ_1 is the relaxation parameter, a^2 is the square of the wave number, $p = p_r + ip_i$ is the complex growth rate where p_r and p_i are real constants and as consequence the unknown variables $w(z) = w_r(z) + i\, w_i(z)$, $T(z) = T_r(z) + i\, T_i(z)$, $S_1(z) = S_{1r}(z) + i\, S_{1i}(z)$, $S_2(z) = S_{2r}(z) + i\, S_{2i}(z)$ are complex valued functions of the real known variable z. Further we note that Eqs. (31)-(38) describes an eigenvalue problem for p and govern triple diffusive convection in a Maxwell fluid layer for any combination of dynamically free and rigid boundaries.

Theorem 1. If (w, T, S_1, S_2, p), $p_r \ge 0$ with $R_{S1} > 0$, $R_{S2} > 0$, $\Lambda_1 > 0$, $0 < R < 2\pi^4 / \sigma \Lambda_1 \left(\pi^2 + a^2\right)$ and $p_r \ge 0$ is a solution of Eqs. (31)-(34) together with either of the boundary conditions Eqs. (35)-(38) and $\dfrac{\sigma R_{S1}}{2\tau_1^2 \pi^4} + \dfrac{\sigma R_{S2}}{2\tau_2^2 \pi^4} \le 1 - \dfrac{\sigma R \Lambda_1 \left(\pi^2 + a^2\right)}{2\pi^4}$ then $p_i = 0$. In particular $p_r = 0$ implies $p_i = 0$ if $\dfrac{\sigma R_{S1}}{2\tau_1^2 \pi^4} + \dfrac{\sigma R_{S2}}{2\tau_2^2 \pi^4} \le 1 - \dfrac{\sigma R \Lambda_1 \left(\pi^2 + a^2\right)}{2\pi^4}$.

Proof. Multiplying Eq. (31) by w^* (w^* is the complex conjugate of w) on both the sides and integrating over vertical range of z, we get

$$\int_0^1 w^* \left(D^2 - a^2\right)^2 w\, dz - \frac{p}{\sigma}\left(1 + \Lambda_1 p\right) \int_0^1 w^* \left(D^2 - a^2\right) w\, dz = \left(1 + \Lambda_1 p\right) a^2 \left[R \int_0^1 w^* T\, dz - R_{S1} \int_0^1 w^* S_1\, dz - R_{S2} \int_0^1 w^* S_2\, dz \right]. \tag{39}$$

Making use of Eqs. (33)-(38), we can write

$$R\, a^2 \int_0^1 w^* T\, dz = -R\, a^2 \int_0^1 T \left(D^2 - a^2 - p^*\right) T^*\, dz \tag{40}$$

$$R_{S1} a^2 \int_0^1 w^* S_1\, dz = -R_{S1} a^2 \tau_1 \int_0^1 S_1 \left(D^2 - a^2 - \frac{p^*}{\tau_1}\right) S_1^*\, dz \tag{41}$$

$$R_{S2} a^2 \int_0^1 w^* S_2\, dz = -R_{S2} a^2 \tau_2 \int_0^1 S_2 \left(D^2 - a^2 - \frac{p^*}{\tau_2}\right) S_2^*\, dz \tag{42}$$

Substituting Eqs. (40)-(42) in Eq. (39), we obtain

$$\int_0^1 w^*\left(D^2-a^2\right)^2 w\,dz-\frac{p}{\sigma}(1+\Lambda_1 p)\int_0^1 w^*\left(D^2-a^2\right)w\,dz=$$
$$(1+\Lambda_1 p)a^2\left(-R\int_0^1 T\left(D^2-a^2-p^*\right)T^*dz+R_{S1}\tau_1\int_0^1 S_1\left(D^2-a^2-\frac{p^*}{\tau_1}\right)S_1^*\,dz\right)+$$
$$(1+\Lambda_1 p)a^2R_{S2}\tau_2\int_0^1 S_2\left(D^2-a^2-\frac{p^*}{\tau_2}\right)S_2^*\,dz \tag{43}$$

Integrating various terms of Eq. (43) by using integration by parts for an suitable number of times and making use of either of the boundary conditions (35)-(38), it follows that

$$\int_0^1\left(\left|D^2w\right|^2+2a^2\left|Dw\right|^2+a^4\left|w\right|^2\right)dz+\frac{p}{\sigma}(1+\Lambda_1 p)\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz$$
$$=(1+\Lambda_1 p)a^2\left(R\int_0^1\left(\left|DT\right|^2+a^2\left|T\right|^2+p^*\left|T\right|^2\right)dz-R_{S1}\tau_1\int_0^1\left(\left|DS_1\right|^2+a^2\left|S_1\right|^2+\frac{p^*}{\tau_1}\left|S_1\right|^2\right)dz\right)-$$
$$(1+\Lambda_1 p)a^2R_{S2}\tau_2\int_0^1\left(\left|DS_2\right|^2+a^2\left|S_2\right|^2+\frac{p^*}{\tau_2}\left|S_2\right|^2\right)dz \tag{44}$$

Equating imaginary parts of both sides of Eq. (44) and cancelling $p_i(\neq 0)$ throughout from the imaginary part, we have

$$\frac{1}{\sigma}[1+2\Lambda_1 p_r]\int_0^1\left(\left|Dw\right|^2+a^2\left|w\right|^2\right)dz=R_{S1}a^2\int_0^1\left|S_1\right|^2dz+R_{S2}a^2\int_0^1\left|S_2\right|^2dz-Ra^2\int_0^1\left|T\right|^2dz+$$
$$\Lambda_1 a^2\left(R\int_0^1\left(\left|DT\right|^2+a^2\left|T\right|^2\right)dz-R_{S1}\tau_1\int_0^1\left(\left|DS_1\right|^2+a^2\left|S_1\right|^2\right)dz-R_{S2}\tau_2\int_0^1\left(\left|DS_2\right|^2+a^2\left|S_2\right|^2\right)dz\right) \tag{45}$$

Multiplying Eq. (32) by its complex conjugate and integrating the ensuing equation for a suitable number of times and using the boundary conditions on T namely, $T(0)=0=T(1)$, we get

$$\int_0^1\left(\left|D^2T\right|^2+2a^2\left|DT\right|^2+a^4\left|T\right|^2\right)dz+2p_r\int_0^1\left(\left|DT\right|^2+a^2\left|T\right|^2\right)dz+\left|p\right|^2\int_0^1\left|T\right|^2dz=\int_0^1\left|w\right|^2dz \tag{46}$$

given that $p_r\geq 0$, it follows from above equation is that

$$\int_0^1\left|DT\right|^2dz<\frac{1}{2a^2}\int_0^1\left|w\right|^2dz \tag{47}$$

similarly from Eqs. (33) and (34), by adopting the same procedure, we obtain

$$\int_0^1\left|DS_1\right|^2dz<\frac{1}{2a^2\tau_1^2}\int_0^1\left|w\right|^2dz\,, \tag{48}$$

$$\int_0^1\left|DS_2\right|^2dz<\frac{1}{2a^2\tau_2^2}\int_0^1\left|w\right|^2dz \tag{49}$$

respectively. We first note that since w,T,S_1 and S_2 satisfy $w(0)=0=w(1)$, $T(0)=0=T(1)$, $S_1(0)=0=S_1(1)$ and $S_2(0)=0=S_2(1)$ respectively, we have by Rayleigh-Ritz inequality [48]that

$$\pi^2\int_0^1\left|w\right|^2dz\leq\int_0^1\left|Dw\right|^2dz\,. \tag{50}$$

$$\pi^2 \int_0^1 |T|^2 \, dz \le \int_0^1 |DT|^2 \, dz \tag{51}$$

$$\pi^2 \int_0^1 |S_1|^2 \, dz \le \int_0^1 |DS_1|^2 \, dz \tag{52}$$

$$\pi^2 \int_0^1 |S_2|^2 \, dz \le \int_0^1 |DS_2|^2 \, dz \tag{53}$$

Utilizing inequalities (47)-(50) in inequalities (51)-(53) we obtain

$$\int_0^1 |DT|^2 \, dz < \frac{1}{2a^2\pi^2} \int_0^1 |Dw|^2 \, dz \tag{54}$$

$$\int_0^1 |T|^2 \, dz < \frac{1}{2a^2\pi^4} \int_0^1 |Dw|^2 \, dz \tag{55}$$

$$\int_0^1 |S_1|^2 \, dz < \frac{1}{2a^2\tau_1^2\pi^4} \int_0^1 |Dw|^2 \, dz \tag{56}$$

$$\int_0^1 |S_2|^2 \, dz < \frac{1}{2a^2\tau_2^2\pi^4} \int_0^1 |Dw|^2 \, dz \tag{57}$$

Given that $p_r \ge 0$ and utilizing inequalities (54)-(57) in Eq. (45), we get

$$\begin{aligned} &\frac{a^2}{\sigma} \int_0^1 |w|^2 \, dz + \Lambda_1 a^2 R_{S1}\tau_1 \int_0^1 \left(|DS_1|^2 + a^2 |S_1|^2 \right) dz + \Lambda_1 a^2 R_{S2}\tau_2 \int_0^1 \left(|DS_2|^2 + a^2 |S_2|^2 \right) dz \\ &\qquad + Ra^2 \int_0^1 |T|^2 \, dz + \frac{1}{\sigma} \left(1 - \frac{\sigma R \Lambda_1 \left(\pi^2 + a^2 \right)}{2\pi^4} - \frac{\sigma R_{S1}}{2\tau_1^2\pi^4} - \frac{\sigma R_{S2}}{2\tau_2^2\pi^4} \right) \int_0^1 |Dw|^2 \, dz < 0 \end{aligned} \tag{58}$$

the above equation clearly implies that

$$\frac{\sigma R_{S1}}{2\tau_1^2\pi^4} + \frac{\sigma R_{S2}}{2\tau_2^2\pi^4} > 1 - \frac{\sigma R \Lambda_1 \left(\pi^2 + a^2 \right)}{2\pi^4} \tag{59}$$

Hence, if $\frac{\sigma R_{S1}}{2\tau_1^2\pi^4} + \frac{\sigma R_{S2}}{2\tau_2^2\pi^4} \le 1 - \frac{\sigma R \Lambda_1 \left(\pi^2 + a^2 \right)}{2\pi^4}$ then we must have $p_i = 0$.

we complete the proof.

The vital content of theorem 1 from the physical point of view is that an arbitrary neutral or unstable mode of the system has the non-oscillatory character for the problem of triple diffusive convection in Maxwell fluid layer. In particular, PES is valid if

$$\frac{\sigma R_{S1}}{2\tau_1^2\pi^4} + \frac{\sigma R_{S2}}{2\tau_2^2\pi^4} \le 1 - \frac{\sigma R \Lambda_1 \left(\pi^2 + a^2 \right)}{2\pi^4}. \tag{60}$$

Special case: It follows from Theorem 1 that PES is valid for the triple diffusive convection($R_{S1} > 0$, $R_{S2} > 0$, $\Lambda_1 = 0$, $R > 0$) if $\frac{\sigma R_{S1}}{2\tau_1^2\pi^4} + \frac{\sigma R_{S2}}{2\tau_2^2\pi^4} \le 1$. (See Ref. [23]).

Clearly the complement of the above results implies the occurrence of oscillatory motions, therefore it is significant to derive the bounds for the complex rate oscillatory motions. we prove the following theorem in this direction.

Theorem 2. If $R_{S1} > 0, R_{S2} > 0, \Lambda_1 > 0, R < a^2/\sigma\Lambda_1$, $p_r \geq 0$, $p_i \neq 0$, then a necessary condition for the existence of a nontrivial solution (w, T, S_1, S_2, p) of Eqs. (31)-(34) together with either of the boundary conditions Eqs. (35)-(38) is that $|p|^2 < (R_{S1} + R_{S2})\sigma \Big/ \left(1 - \frac{\Lambda_1 R}{a^2}\sigma\right)$.

Proof. Given that $p_r \geq 0$ and rewriting the Eq. (45), we get

$$\begin{aligned}\frac{1}{\sigma}\int_0^1\left(|Dw|^2 + a^2|w|^2\right)dz + \Lambda_1 a^2\left(R_{S1}\tau_1\int_0^1\left(|DS_1|^2 + a^2|S_1|^2\right)dz + R_{S2}\tau_2\int_0^1\left(|DS_2|^2 + a^2|S_2|^2\right)dz\right)\\ + Ra^2\int_0^1|T|^2\,dz \leq \Lambda_1 a^2 R\int_0^1\left(|DT|^2 + a^2|T|^2\right)dz + R_{S1}a^2\int_0^1|S_1|^2\,dz + R_{S2}a^2\int_0^1|S_2|^2\,dz\end{aligned} \tag{61}$$

Now, multiplying Eq. (32) by its complex conjugate and integrating the ensuing equation for a suitable number of times and using the boundary conditions on T namely, $T(0) = 0 = T(1)$, we get

$$\int_0^1\left(|D^2T|^2 + 2a^2|DT|^2 + a^4|T|^2\right)dz + 2p_r\int_0^1\left(|DT|^2 + a^2|T|^2\right)dz + |p|^2\int_0^1|T|^2\,dz = \int_0^1|w|^2\,dz \tag{62}$$

given that $p_r \geq 0$, it follows from above equation is that

$$a^2\int_0^1\left(|DT|^2 + a^2|T|^2\right)dz < \int_0^1|w|^2\,dz \tag{63}$$

likewise from Eqs. (33) and (34), by adopting the same procedure, we obtain

$$\int_0^1|S_1|^2\,dz < \frac{1}{|p|^2}\int_0^1|w|^2\,dz \tag{64}$$

and $\int_0^1|S_2|^2\,dz < \frac{1}{|p|^2}\int_0^1|w|^2\,dz$ (65)

respectively.

given that $p_r \geq 0$ and utilizing inequalities (63)-(65) in Eq. (61) , we have

$$\begin{aligned}\frac{1}{\sigma}\int_0^1|Dw|^2\,dz + a^2\Lambda_1\left(R_{S1}\tau_1\int_0^1\left(|DS_1|^2 + a^2|S_1|^2\right)dz + R_{S2}\tau_2\int_0^1\left(|DS_2|^2 + a^2|S_2|^2\right)dz\right)\\ + Ra^2\int_0^1|T|^2\,dz + \left(\frac{a^2}{\sigma} - \Lambda_1 R - \frac{R_{S1}a^2}{|p|^2} - \frac{R_{S2}a^2}{|p|^2}\right)\int_0^1|w|^2\,dz < 0\end{aligned} \tag{66}$$

which obviously implies that

$$|p|^2 < (R_{S1} + R_{S2})\sigma \Big/ \left(1 - \frac{\Lambda_1 R}{a^2}\sigma\right) \tag{67}$$

This proves the theorem.

The above theorem states, from the physical point of sight, that the complex growth rate $p=(p_r, p_i)$ of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude, in a triple diffusive convection in a Maxwell fluid layer with one of the components as heat with diffusivity κ_T , must lie inside a semicircle in the right-half of the (p_r, p_i) plane whose centre is the origin and radius equals $|p|<\sqrt{(R_{S1}+R_{S2})\sigma \Big/ \left(1-\frac{\Lambda_1 R}{a^2}\sigma\right)}$. It is further proved that this result is uniformly valid for any combination of rigid and/or free boundaries.

Special case:

The following result may be obtained from the theorem 2 as a special case for triple diffusive convection ($R_{S1}>0, R_{S2}>0,\ \Lambda_1=0, R>0$), $|p|^2<(R_{S1}+R_{S2})\sigma$ (See Ref. [27]).

CONCLUSIONS

Linear instability analysis is used to investigate triple diffusive convection in Maxwell fluid layer. The mathematical analysis carried out here yields a sufficient condition for the validity of the principle of the exchange of stabilities in the present problem. Since the complement of this condition implies the occurrence of oscillatory motions, the upper bounds for the linear growth rate of an arbitrary neutral or unstable oscillatory perturbation of growing amplitude are obtained. It is further proved that the results are uniformly applicable to any combination of rigid and/or free boundaries.

Declarations

Conflict of interest The authors declares that he has no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by the author

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