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Achieving Resilience: Integrating Perfect Fault Coverage into k-out-of-n Systems with Cold Standby

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Abstract: When designing a system, reliability analysis is essential, but it is especially important when using redundancy and standby units to improve performance. Here, we examine k-out-of-n systems with cold standby and perfect fault coverage strategies to determine their reliability. In a k-out-of-n system with cold standby, you have k working units and n-k standby units that can be used to replace the failing ones. We take it as read that all units are interchangeable and can be fixed, with the fixed units being just as excellent as the original. Furthermore, a flawless fault coverage system is used to detect issues prior to beginning the repair procedure. If it works, the unit is fixed; if not, it is left unfixed. This investigation explores the features of such systems' dependability taking into account the statistical autonomy of states and the lack of an observing state. We also take into consideration the incredibly short repair wait times. We assess the dependability measures, including system availability, mean time to failure, and reliability, using mathematical modelling and simulation approaches. The study's results shed light on the efficiency and dependability of k-out-of-n systems that use cold standby and are enhanced by fault covering technologies. When it comes to real-world applications, these kinds of insights are vital for improving system design and maintenance tactics to ensure reliability.

Keywords: Reliability analysis, k-out-of-n systems, Cold standby, Perfect fault coverage, Redundancy.

I. Introduction

It is of the utmost importance in contemporary engineering to guarantee the dependability of systems, particularly in critical applications where failures might have significant repercussions due to their severity. Increasing dependability can be accomplished through the implementation of redundancy methods, such as the k-out-of-n system with cold standby, which is an effective strategy. A primary set of k operating units is included in this arrangement, and it is supplemented by a secondary set of standby units that are prepared to take over in the event that the primary set of units fails. The efficiency of such systems, on the other hand, is contingent upon their capacity to swiftly identify and correct any errors that may occur.

The dependability analysis of a repairable that included incomplete fault coverage and in which the technician took multiple vacations between repairs was the primary focus of **Jain and Gupta (2013)**. Jain wrote about the transient analysis of a machining system with multiple standbys. Both the ideas of administration interference and need were integrated into the examination. **Sharma's (2015)** research focused on the reliability analysis of a repairable system under N-policy, setup, and incomplete coverage. The repairman is activated so that he can perform system repair when the system has at least "N" failed components. A k-out-of-n method: G repairable framework with N-strategy and rehashed excursions, **Wu et al. (2015)** looked into the system's steady-state reliability behavior. According to research carried out by **Liu et al. (2016)**, various phase-type distributions were observed over the components' lifetimes, the amount of time it took the repairperson to repair the components, and the amount of vacation time the repairperson took.A k-out-of-n: G system with multiple vacations and redundant reliance was the subject of research conducted by **He et al. (2018)**. The N-policy was used to regulate the repair of a failing component inside the system.The reliability analysis of a system with poor fault coverage was the topic of discussion in **Kalaiarasi et al. (2018).** Taking into consideration a system consisting of two machines, one of which is operational and has a failure rate of μ, and the other of which is a cold standby spare that does not fail.**Krishnan (2020)** gave a rundown of the examination that has been directed on unwavering quality evaluations of k-out-of-n: G networks. Dependability assessment procedures, life expectancy conveyances of disappointment paces of parts, and frameworks that contain various parts are instances of strategies. A fault-tolerant system with vacation policies and working breakdowns was the subject of an investigation by **Kumar and Jain (2020)** into the reliability measures and queueing characteristics. In this framework, the reboot cycle was done if a bombed machine was not effectively fixed. In order to get the system back to working order, this was done. Shekhar (2020) conceived of a computing network as a Markovian warm standby repairable system with multiple working and vacation breaks. This system's warm standby nodes are susceptible to switching malfunctions and other catastrophes. The reliability characteristics of the computing network were identified by the authors using the transient-state probabilities model. The exploration that Yang and Wu (2021) led involved deciding the accessibility and reliability of a repairable framework that was encountering exchanging disappointment in a Markovian climate. The system contained warm standby components. To determine the system's mean time to failure (MTTF) and the explicit form of the reliability function, they make use of the Laplace transform approach so that they may construct the reliability model.A novel phase combination theory was proposed by **Wang et al. (2023)**, who investigated the unique structure of k-out-of-n PMSs. Several phases that satisfy particular conditions are merged into a single phase in accordance with the combination theory that has been proposed. This results in a decreased number of mission phases, which in turn significantly improves the effectiveness of the PMS reliability measurement.

II. Within the scope of this investigation, we investigate the reliability analysis of k-out-of-n systems that are equipped with cold standby, with a particular emphasis on the incorporation of excellent fault covering mechanisms. Identifying problems within the system is an essential step before beginning repair operations, and these processes play a significant part in this process. With the incorporation of perfect fault coverage, our objective is to provide a full understanding of the reliability dynamics of such systems. This understanding will take into consideration a variety of parameters, including statistical independence of states and insignificant repair durations. Our research seeks to contribute insights that might inform the optimization of system design and maintenance techniques in real-world applications, thereby strengthening dependability and resilience. These insights will be gleaned through rigorous analysis and simulation.

II. MATHEMATICAL FORMULATION

Within the scope of this work, we decided to investigate the dependability of a k-out-of-n system that utilized cold standby. All of the units are comparable in every way, and there is a facility that can be repaired. It is as good as new after the repair. Fault coverage is performed prior to the repair of any unit; if the coverage is successful, the unit is repaired; otherwise, it is not. Very little time is spent waiting for repairs to be completed. Failure of the system occurs when there is no longer any unit that can be replaced. In a fault coverage system that does not include any observing states, states are statistically independent and perfect fault coverage. There is a Markov chain for an n-unit system that has perfect fault coverage that we are considering here. The following is an example of the state transition diagram.

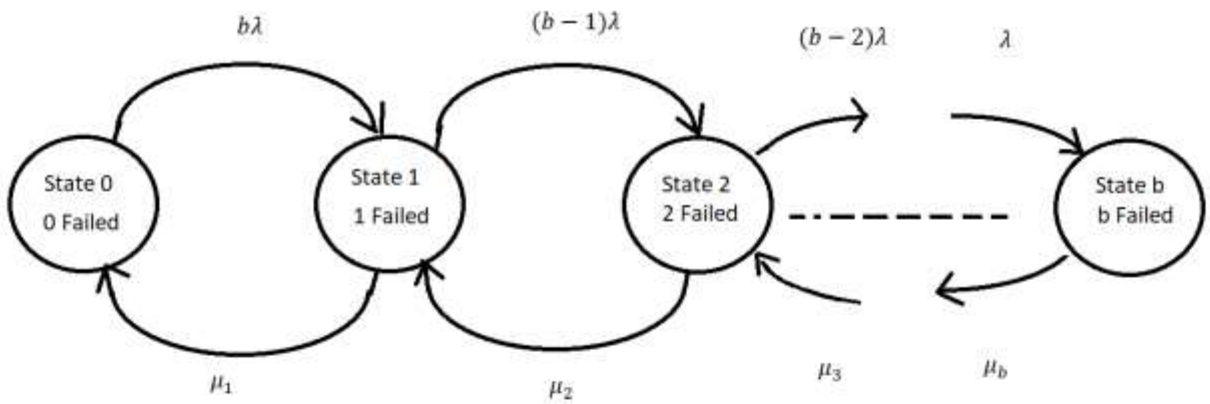


Figure 1: State transition rate diagram of proposed system

In order to provide an explanation for this system, a birth-death process is utilized, wherein the failure rate λ is constant and the repair rate μ_i is also constant.

This is where $i$stand in terms of the number of failures. When it comes to the uncommon circumstance of

$$\lambda_n = \lambda \tag{1}$$

$$\mu_i = \begin{cases} i\mu & for i = 0,1,2, \dots k-1 \\ k\mu & for i = i \leq k \end{cases} \tag{2}$$

We are able to determine the likelihood of i units being present in the system at the moment $(t + \Delta t)$, which is denoted by the expression $(i > 0)$.

$$p_i(t+\Delta t) = [1 - (\lambda + i\mu)\Delta t]p_i(t) + \lambda \Delta t p_{i-1}(t) + \mu_{i+1}p_{i+1}(t) + O(\Delta t) \tag{3}$$

This is the likelihood that there will be no unit $(i = 0)$ in the system at the moment $(t + \Delta t)$in question.

$$p_0(t+\Delta t) = (1 - \lambda \Delta t)p_0(t) + \mu_1 \Delta t p_1(t) + O(\Delta t) \tag{4}$$

Similar, the probability that at any time k units fails $(i \leq k)$

$$p_i(t+\Delta t) = [1 - (\lambda + k\mu)\Delta t]p_i(t) + \lambda \Delta t p_{i-1}(t) + \mu_{i+1}p_{i+1}(t) + O(\Delta t) \tag{5}$$

The following is the differential difference equation that will be used for the birth-death process for the system:

$$\frac{dp_i}{dt} = -(\lambda + i\mu)p_i(t) + \lambda p_{i-1}(t) + \mu_{i+1}p_{i+1}(t) \quad (6)$$

$$\frac{dp_0}{dt} = -\lambda p_0(t) + \mu_1 p_1(t) \quad (7)$$

$$\frac{dp_i}{dt} = -(\lambda + k\mu)p_i(t) + \lambda p_{i-1}(t) \quad (8)$$

The solution to the aforementioned equations in steady state is what we need now, so make sure it's available.

Keeping in mind that the k-unit might break at any moment, we can express these equations

$$p_i'(t) = -(\lambda + i\mu)p_i(t) + \lambda p_{i-1}(t) + \mu_{i+1}p_{i+1}(t)\ (i > 0) \quad (9)$$

$$p_0'(t) = -\lambda p_0(t) + \mu_1 p_1(t) \qquad (i = 0) \quad (10)$$

$$p_i'(t) = -(\lambda + k\mu)p_i(t) + \lambda p_{i-1}(t)(i \le k) \quad (11)$$

In the case of steady state

$P_i'(t) = 0$ and $P_0'(t) = 0$

III. SOLUTION OF THE PROBLEM

Therefore steady states form of the equations (9), (10) and (11) as follows:

$$-(\lambda + i\mu)p_i + \lambda p_{i-1} + \mu_{i+1}p_{i+1} = 0 \quad (12)$$

$$-\lambda p_0 + \mu_1 p_1 = 0 \quad (13)$$

$$-(\lambda + k\mu)p_i + \lambda p_{i-1} = 0 \quad (14)$$

On solving the equations (12),(13) and (14), we get $p_1 = \frac{\lambda}{\mu_1}p_0$

Putting $i = 1$ in equation (12), we get

$$p_2 = \frac{\lambda}{\mu_2}p_1 = \frac{\lambda^2}{\mu_1\mu_2}p_0$$

$$p_3 = \frac{\lambda}{\mu_3}p_2 = \frac{\lambda^3}{\mu_1\mu_2\mu_3}p_0$$

Similarly $p_i = \frac{\lambda}{\mu_i}p_{i-1} = \frac{\lambda^i}{\mu_1\mu_2\mu_3 \dots \mu_i}p_0$

The chance of having $n \le k$units in the system is now being considered.

$$p_i = \frac{k!}{(k-i)!}\left(\frac{\lambda}{\mu}\right)^i p_0 = \frac{k!}{(k-i)!}\rho^i p_0 \quad (15)$$

IV. PERFORMANCE MEASURES

- **Availability:** $A_S = 1 - \sum p_i = 1 - \frac{n!}{(k-1)}\rho^{n-k+1}$ (16)

$$A_M = 1 - \sum p_i = 1 - \frac{n!}{(k-1)(n-k+1)}\rho^{n-k+1} = 1 - \binom{n}{k-1}\rho^{n-k+1} \quad (17)$$

- **Mean time between failures (MTBF):** MTBF for a single and multiple repair facility at time $t = 0$ for the state $(n - k)$.

$$\begin{Bmatrix} MTBF(K-1,n) = \frac{1}{\lambda(k-1)} + MTBF(k,n) \\ MTBF(k,n) = \frac{1}{\lambda}\sum_{j=k}^{n}\frac{1}{j} \end{Bmatrix} \quad (18)$$

Then $MTBF_S = MTBF_S, approx = \frac{(k-1)!}{n!}\frac{\rho^{n-k}}{\lambda}$ (19)

$MTBF_M = MTBF_M, approx = \frac{\rho^{n-k}}{k\lambda\binom{n}{k}}$ (20)

$MTBF = \int_0^{\infty}[1 - P_{n-k+1}(t)]dt = \sum_{m\in\Omega} T_m$ (21)

$T_m = \int_0^{\infty} P_m(t)dt$ (22)

The MTBF, signified as (21), depends with the understanding that the framework is in state $(n - k)$at $t = 0$and from there on changes to state $(n - k + 1)$. When the Chapman-Kolmogorov equations are applied and the system is in steady-state, it is in state $(n - k + 1)$. Furthermore

$P_m(0) = \eta(m = n - k)$ (23)

$P_m(\infty) = \eta(m = n - k + 1)$ (24)

The system of equations is transformed

$[(n - m)\lambda + \mu_m]T_m - [(n - m - 1)\lambda + 0]T_{m-1} - (0 + \mu_{m+1})T_{m+1} = \eta, m = n - k, 0 \le m \le n - k$ (25)

$\mu_{m-1} = \mu_{n-k+1}$ (26)

The Chapman-Kolmogorov equation is integrated across the interval $0 \le t \le \infty$ to provide Equation (24). Therefore

$[(n - m)\lambda + 0]T_m - \mu_{m+1}T_{m+1} = 0$

$T_m = \frac{\mu_{m+1}T_{m+1}}{(n-m)\lambda};\ 0 \le m \le n - k$ (27)

$T_{n-k} = \frac{\mu_{n-k+1}T_{n-k+1}}{(n-n+k)\lambda} = \frac{1}{k\lambda}$ (28)

To get the precise MTBF, use Equation (15).

Then $MTBF_S = MTBF_S, approx = \sum_{m\in\Omega_s}\frac{\rho^m n!}{(n-m)!}$ (29)

$MTBF_M = MTBF_M, approx. \sum_{m\in\Omega_s}\binom{n}{m}\rho^m$ (30)

$A_M = (1+\rho)^{-n}\sum_{m\in\Omega_s}\binom{n}{m}\rho^m$ (31)

- **1-out-of-3: G System with multiple Facilities for Repair:**

The space for system state$\Omega_p = \{0,1,2,3\}$ where $\Omega_s = \{0,1,2\}$ and $\Omega_f = \{3\}$.

Applying equations (17) and (31) to the variable A_M

$A_M = 1 - \binom{n}{k-1}\rho^{n-k+1} \approx (1+\rho)^{-n}\sum_{m\in\Omega_s}\binom{n}{m}\rho^m$ (32)

$n = 3, k = 1$

Then

$A_M = 1 - \binom{3}{0}\rho^{3-1+1} \approx (1+\rho)^{-3}\left[\binom{3}{0}\rho^0 + \binom{3}{1}\rho^1 + \binom{3}{2}\rho^2\right]$

$A_M = 1 - \rho^3 \approx (1+\rho)^{-3}[1 + 3\rho + 3\rho^2]$ (33)

Applying the MTBF formulas (20) and (30) and $\lambda \ll \mu$

$$MTBF_M = MTBF_M, approx = \frac{\rho^{3-1}}{\lambda\binom{3}{1}} = \frac{\rho^2}{3\lambda}$$

$$MTBF_M = MTBF_M, approx. \sum_{m \in \Omega_s} \binom{n}{m} \rho^m = \frac{\rho^2}{3\lambda}\left[\sum_{m \in \Omega_s} \binom{n}{m} \rho^m\right]$$

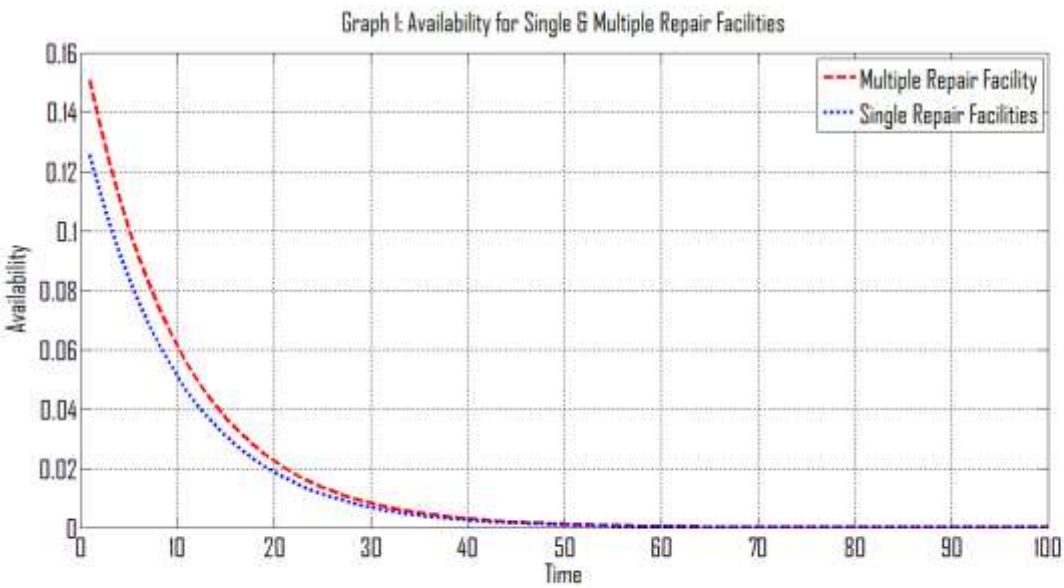
$$= \frac{\rho^2}{3\lambda}\left[\binom{3}{0}\rho^0 + \binom{3}{1}\rho^1 + \binom{3}{2}\rho^2\right] = \frac{\rho^2}{3\lambda}(1 + 3\rho + 3\rho^2) \approx \frac{\rho^2}{3\lambda} \quad (33)$$

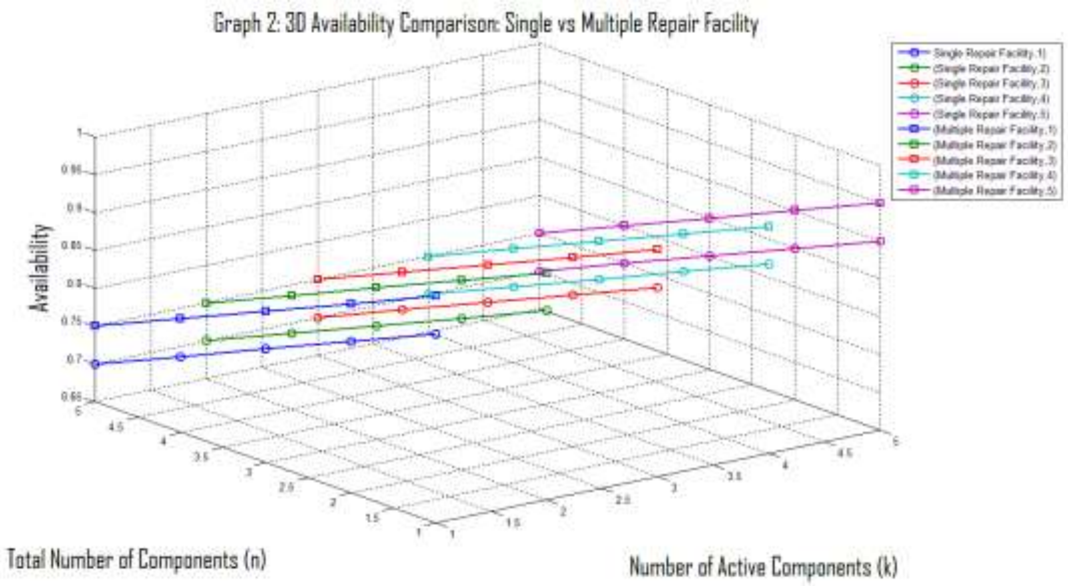
The purpose of this step is to determine the MTTR, or mean time to repair, as

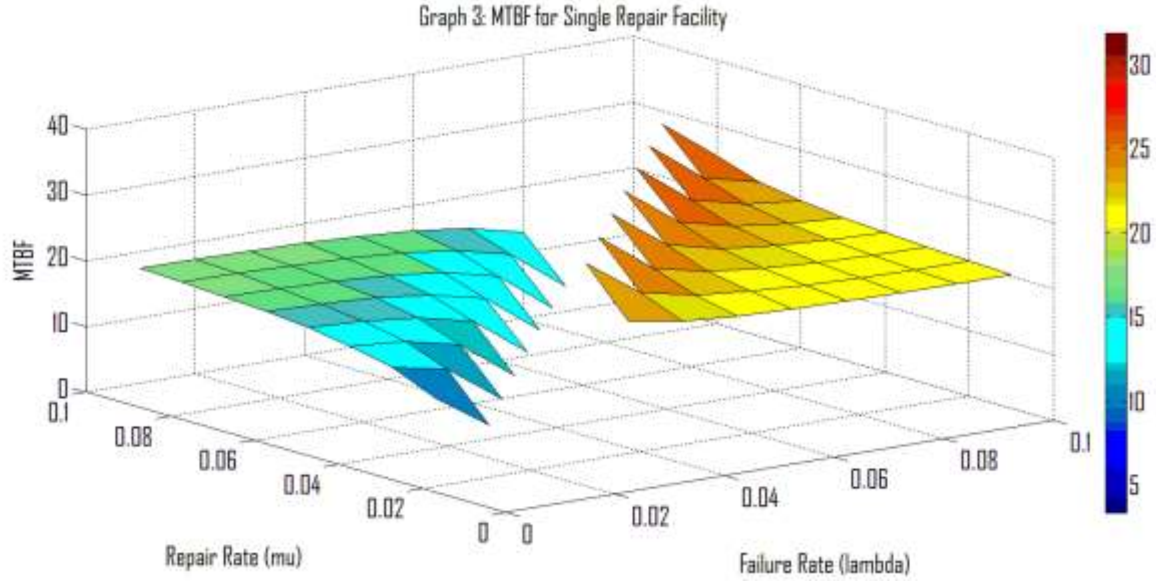
According to the MTTR definition

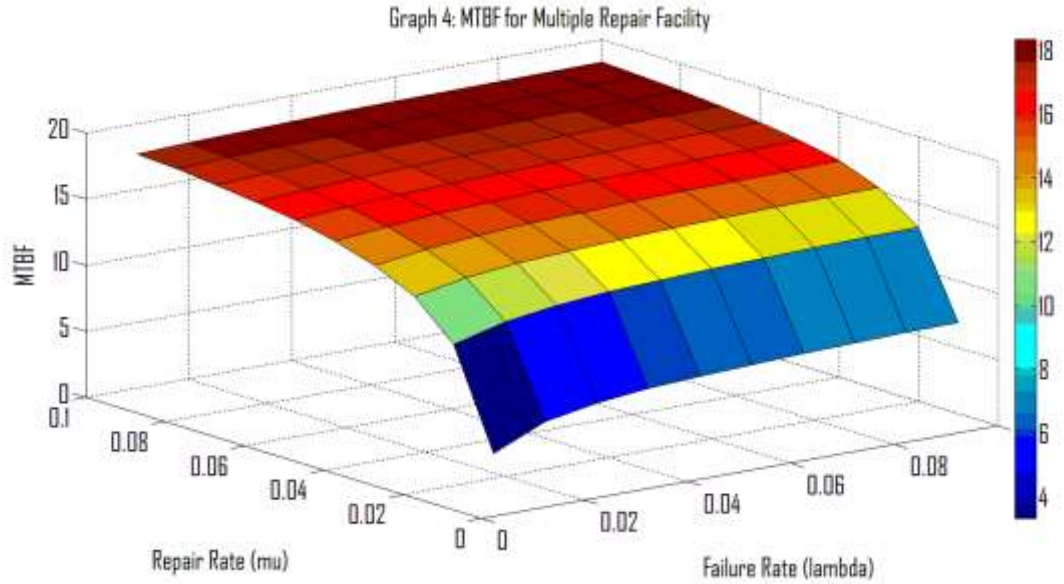
$$MTTR = \frac{MTBF_M(1-A_M)}{A_M} = \frac{\left(\frac{\rho^2}{3\lambda}\right)(1-1+\rho^3)}{(1-\rho^3)} = \frac{\rho^5}{3\lambda(1-\rho^3)} \approx \frac{\rho^5}{3\lambda} \ (\lambda \ll \mu) \quad (34)$$

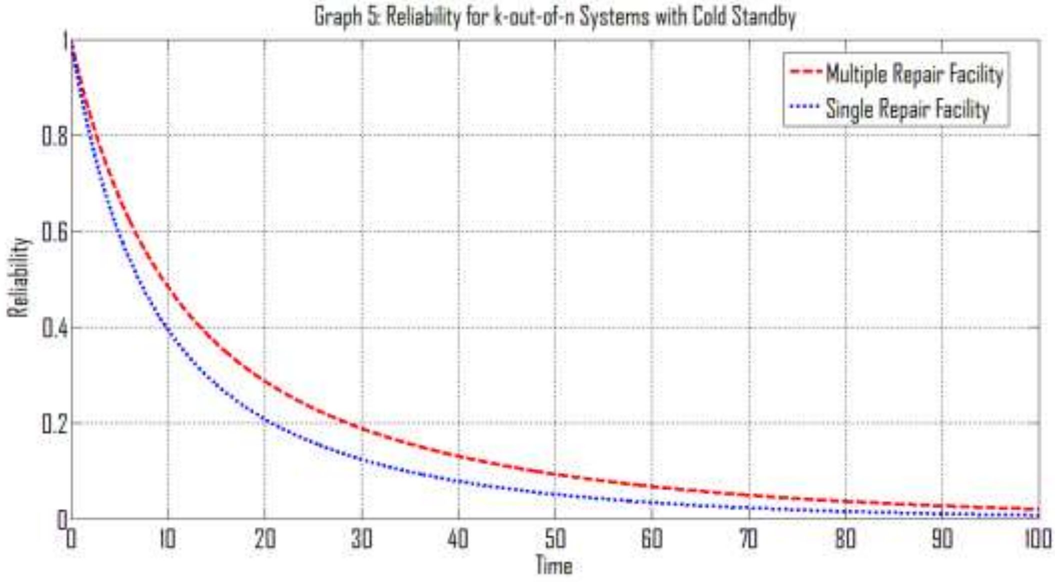
V. RESULTS AND DISCUSSION

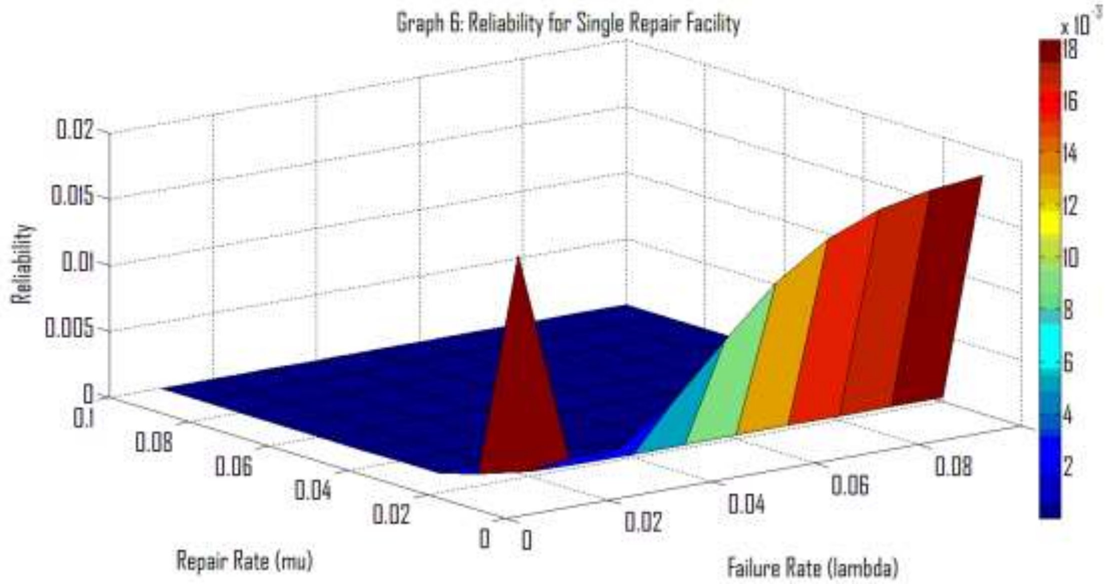


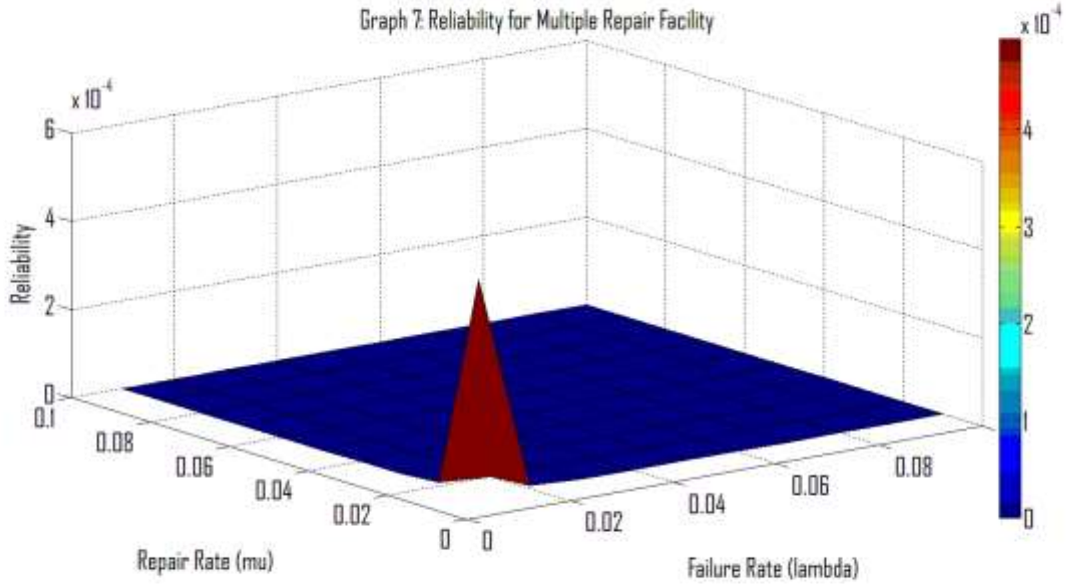



Graph 3: MTBF for Single Repair Facility
MTBF
40
30
20
10
0
0.1
0.08
0.06
0.04
0.02
0
Repair Rate (mu)
0
0.02
0.04
0.06
0.08
0.1
Failure Rate (lambda)
30
25
20
15
10
5


Graph 4: MTBF for Multiple Repair Facility
MTBF
20
15
10
5
0
0.1
0.08
0.06
0.04
0.02
0
Repair Rate (mu)
0
0.02
0.04
0.06
0.08
Failure Rate (lambda)
18
16
14
12
10
8
6
4







A fully operating system with all components working well is shown in Graph (1), where the availability curve starts at or near its highest value at the commencement of the operation. The availability curve drops when failures happen, showing that system availability drops because functional components are no longer available. Greater overall availability and resilience to failures of individual servers are indicated by the smaller and fewer dips in the availability graph of a facility with numerous servers as opposed to a facility with a single server.

The availability of a system indicates the likelihood that it is operational and performing as expected at any one moment. It is a reflection of the system's dependability and maintainability. Improved dependability is shown by a higher availability. When comparing two situations, it's important to note the difference between those in which there is a single server and those in which there are several servers. The availability of a system can be affected by the quantity of servers in a reliability analysis, particularly when redundancy or standby components are involved. In graph (2), a three-dimensional figure is shown to show the relationship between availability and changes in n, k, and the kind of facility (single server vs. several servers). We observed that the impact of various system configurations on availability and reliability by studying this plot. Maximizing availability while taking into account aspects like component count and redundancy offered by backup components or numerous servers is one way it aids in optimizing system architecture.

MTBF is an important reliability statistic that measures the average amount of time that has passed between individual failures of a system or component. MTBF values that are higher indicate that the system is more reliable and that it can continue to function without interruption for longer periods of time. It is used as an indicator of system dependability. Within the single and multiple server facility configurations, the three-dimensional surface plots (3) and (4) illustrate how MTBF of the system varies with changes in the failure rate and repair rate associated with the system. The surface of the figure illustrates the MTBF values that correspond to various combinations of failure and repair rates, while each of the axes of the plot represents one of these factors. The examination of these plots enables one to gain an understanding of the ways in which modifications to the rates of failure and repair have an effect on the overall reliability of the system. By way of illustration, a decrease in the failure rate or an increase in the repair rate often results in higher MTBF values. These numbers indicate that the system's reliability has improved and that there are longer intervals between failures. On the other hand, a lower MTBF number may be the result of a larger failure rate or a slower repair rate, which indicates a decreased reliability and more frequent downtimes.

A system's reliability is defined as the likelihood that, under specified conditions and for a given duration, it will carry out its intended function as intended without failure. An essential idea in reliability engineering, it shows how well a system may keep working over time. The usual way to describe dependability is as a probability value between zero and one, with zero representing complete failure and one representing perfect reliability.

The analysis of plot (5) facilitates the examination of dependability patterns in both single and many server facilities, enabling meaningful comparisons. The dependability curve of a single server facility demonstrates a progressive decrease over time, as the probability of component failures escalates with extended periods of operation. Conversely, in a facility with several servers, the reliability curve exhibits a progressive decrease or even reaches a stable state over time as a result of the redundancy offered by extra servers. This redundancy helps to reduce the impact of failures in individual components.

The three-dimensional surface plots (6) and (7) depict the relationship between the reliability of the system and variations in the failure rate and repair rate, encompassing both single and multiple server facilities. The plot's axis indicates each parameter, while the surface illustrates the dependability values associated with various combinations of failure and repair rates. These graphs aid in the determination of the most effective combinations of failure and repair rates to enhance the dependability of the system, taking into account the limitations of both single and multiple server facility configurations. This facilitates the process of making well-informed decisions pertaining to system design, maintenance strategies, and resource allocation, with the aim of maximizing reliability and minimizing periods of inactivity.

VI. CONCLUDING REMARKS

Finally, a major step forward in comprehending system dependability in real-world settings has been achieved by incorporating 100% fault coverage into the reliability analysis of k-out-of-n systems with cold standby. System designers and engineers seeking to optimize system performance will find vital insights from this work, which improves reliability estimates by accounting for perfect fault coverage. The results provide useful information for strengthening the stability and dependability of vital systems in various sectors, in addition to adding to the development of reliability engineering theory. In the conclusion, this study lays the groundwork for more reliability analysis work, which will encourage ongoing development and improvement in the field to guarantee the efficacy and dependability of complicated systems in ever-changing operational settings.

REFERENCES

1. He G., Wu W., Zhang Y. (2018): "Analysis of a multi-component system with failure dependency, N-policy and vacations", *Operations Research Perspectives*, 5: 191–198.
2. Jain M. (2013): "Transient analysis of machining systems with service interruption, mixed standbys and priority", *International Journal of Mathematics in Operational Research*, 5: 604–625.
3. Jain M., Gupta R. (2013): "Optimal replacement policy for a repairable system with multiple vacations and imperfect fault coverage", *Computers & Industrial Engineering*, 66: 710–719.
4. KalaiarasiS. ,.Venkatesan G. , Venkatesan E. , Nithyapriya N. (2018): " Analysis of imperfect fault coverage reliability of a system", *Journal of Emerging Technologies and Innovative Research*, 5(8): 1-6.
5. Krishnan R. (2020): "Reliability analysis of k-out-of-n: g system: a short review", *International Journal of Engineering and Applied Sciences (IJEAS)*, 7(2): 1-4.
6. Kumar P., Jain M. (2020): "Reliability analysis of a multi-component machining system with service interruption, imperfect coverage, and reboot", *Reliability Engineering & System Safety*, 202:106991.
7. Liu B, Cui L, Wen Y, Guo F. (2016): "A cold standby repairable system with the repairman having multiple vacations and operational, repair, and vacation times following phase-type distributions", *Communications in Statistics-Theory and Methods*, 45: 850–858.

8. Sharma R. (2015): "Reliability *Analysis for a Repairable System under N-policy and Imperfect Coverage*", *Proceedings of the International Multi Conference of Engineers and Computer Scientists*, IMECS, 2:1-4.
9. Shekhar C., Kumar A., Gupta A., Varshney S. (2020): "Warm-spare provisioning computing network with switching failure, common cause failure, vacation interruption, and synchronized reneging", *Reliability Engineering & System Safety*,199:106910.
10. Wang C., Xing L., Yu J., Guan Q., Yang C., Yu M. (2023): "Phase reduction for efficient reliability analysis of dynamic *k*-out-of-*n* phased mission systems",*Reliability Engineering and System Safety*, 237:109349.
11. Wu W., Tang Y., Yu M, Jiang Y. (2015): "Computation and profit analysis of a K-out-of-n: G repairable system under N-policy with multiple vacations and one replaceable repair facility", *RAIRO-Operations Research*, 49: 717–734.
13. Yang D.Y., Wu C.H. (2021): "Evaluation of the availability and reliability of a standby repairable system incorporating imperfect switchovers and working breakdowns", *Reliability Engineering and System Safety*, 207:107366.