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An EPQ Model with Imperfect Production Process incorporating Learning approach in Fuzzy Environment and Controllable Carbon Emission

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Abstract

Nowdays decision-makers confront multifaceted challenges stemming from uncertainties inherent in production processes and environmental regulations. This study introduces a novel hybrid approach that integrates an Economic Production Quantity (EPQ) model, learning dynamics, and a cap-and-trade policy framework, to tackle these challenges comprehensively. Emphasizing a fuzzy production environment characterized by imperfect production processes and reliability concerns, our approach incorporates learning dynamics to facilitate adaptive decision-making in response to evolving conditions. Moreover, by integrating a cap-and-trade policy mechanism, our framework offers a proactive strategy for managing controllable carbon emissions while maintaining economic viability. Through rigorous numerical simulations and sensitivity analyses, we empirically demonstrate the effectiveness of the proposed hybrid model in optimizing production decisions, advancing environmental sustainability goals, and enhancing resilience in complex manufacturing systems. This research significantly contributes to the development of decision support tools tailored to navigating uncertainties in production environments while simultaneously addressing sustainability objectives.

Keywords: EPQ Model, Imperfect Production Process, Learning effect, Fuzzy Environment, Controllable Carbon Emission

1. Introduction

Inventory modeling with controllable carbon emissions extends traditional inventory management approaches by integrating environmental sustainability objectives alongside traditional performance metrics. In response to growing concerns about climate change and carbon footprint, this approach aims to minimize inventory costs while also reducing carbon emissions throughout the supply chain. By incorporating controllable carbon emissions as an additional objective, decision-makers can optimize inventory policies to not only improve operational efficiency but also mitigate environmental impacts. Techniques such as multi-objective optimization algorithms and carbon emission accounting methods allow for the simultaneous consideration of multiple conflicting objectives, enabling the identification of trade-offs between cost, service levels, and carbon emissions. Through multi-objective inventory modeling with controllable carbon emissions, organizations can achieve a more sustainable and environmentally responsible inventory management strategy, aligning with corporate social responsibility initiatives and regulatory requirements while maintaining competitiveness in the market.

The Economic Production Quantity (EPQ) model serves as a fundamental framework for optimizing production and inventory decisions in manufacturing environments. Its inclusion in our study is essential due to its ability to minimize total production costs by determining the optimal production quantity that balances inventory holding costs and setup costs. By integrating the EPQ model into our hybrid approach, we aim to provide decision-makers with a robust methodology for managing production activities efficiently and economically.

In today's rapidly evolving industrial backdrop, learning dynamics play a crucial role in adapting to uncertainties and improving decision-making processes over time. Incorporating learning dynamics into our model allows for the integration of historical data and past experiences, enabling the system to continuously refine its strategies in response to changing conditions. This aspect is particularly relevant given the dynamic nature of production environments and the necessity for adaptive and resilient decision-making frameworks.

The inherent uncertainty and imprecision in production processes necessitate the consideration of fuzzy logic methodologies. Fuzzy environments acknowledge the vagueness and ambiguity present in real-world systems, providing a more realistic representation of manufacturing operations. By modeling the production environment as fuzzy, our study accounts for factors such as variability in demand, process parameters, and resource availability, ensuring the robustness and applicability of our proposed approach in complex and uncertain settings.

Environmental sustainability is an increasingly critical concern in modern manufacturing, driven by regulatory frameworks aimed at mitigating greenhouse gas emissions and promoting eco-friendly practices. The integration of a cap-and-trade policy mechanism in our model aligns with the growing emphasis on environmental responsibility within the industrial sector. By incorporating carbon pricing and emissions trading, we provide decision-makers with a proactive strategy for addressing environmental impacts while balancing economic considerations.

Real-world production processes are often characterized by imperfections, including machine breakdowns, quality variations, and material defects. Accounting for these imperfections is essential for accurately modeling production systems and optimizing decision-making strategies. By considering the imperfect nature of production processes in our study, we ensure the practical relevance and applicability of our proposed hybrid model to real-world manufacturing environments.

Reliability is a critical aspect of manufacturing operations, influencing product quality, customer satisfaction, and overall operational efficiency. Incorporating reliability considerations into our model allows for the assessment and optimization of production systems in terms of their dependability and performance under varying conditions. By addressing reliability concerns, our study contributes to enhancing the robustness and resilience of manufacturing processes, ultimately improving overall system performance and competitiveness.

2. Literature review

A comprehensive literature review synthesizes recent advancements and insights, aiming to clarify the current state of knowledge in the field. It provides valuable insights into emerging trends, challenges, and opportunities, guiding future research endeavours.

EPQ model

The EPQ model remains a focal point in production and inventory management research, with recent studies exploring its applications and extensions. For instance, research by Xiao et al. (2013) proposed a dynamic EPQ model considering deteriorating items and partial backordering, enhancing its relevance in real-world manufacturing scenarios. Moreover, Chen et al. (2016) extended the EPQ model to account for quality inspection and rework processes, addressing the imperfections often present in production environments. Additionally, the integration of sustainability considerations into

EPQ models has gained attention, as demonstrated by Zhang et al. (2018), who developed a green EPQ model considering carbon emissions and recycling activities, aligning with environmental sustainability objectives. Recent research has continued to explore the application and refinement of EPQ models in modern manufacturing contexts. For instance, Li et al. (2022) proposed a dynamic EPQ model considering stochastic demand and production disruptions, providing insights into mitigating the impact of uncertainties on production planning. Additionally, research by Wang et al. (2020) extended the EPQ model to incorporate carbon emissions and sustainability objectives, aligning production decisions with environmental considerations. Moreover, studies such as Tang et al. (2021) and Zhang et al. (2022) investigated multi-product EPQ models with batch production setups, enhancing the model's applicability in diverse production environments.

Learning

Recent research has focused on the integration of learning dynamics into decision-making processes to enhance adaptability and performance. For example, Zhang et al. (2016) investigated the application of reinforcement learning algorithms for optimizing production scheduling in dynamic environments, demonstrating significant improvements in system performance. Additionally, Li et al. (2012) proposed a novel learning-based approach for demand forecasting in manufacturing, leveraging machine learning techniques to capture complex demand patterns and improve forecasting accuracy. Recent literature has focused on leveraging learning dynamics to enhance decision-making processes in manufacturing. Additionally, studies such as Wang et al. (2020) and Liu et al. (2021) investigated learning-based approaches for predictive maintenance, enabling proactive maintenance strategies to improve equipment reliability and minimize downtime.

Fuzzy Environment:

Recent advancements in fuzzy logic have expanded its applicability in modeling uncertainties in manufacturing environments. For instance, Kumar et al. (2013) developed a fuzzy logic-based decision support system for production planning, incorporating fuzzy inference techniques to handle imprecise information and optimize production schedules. Moreover, research by Lee et al. (2017) explored the integration of fuzzy logic and genetic algorithms for optimizing production parameters, demonstrating improvements in production efficiency and resource utilization. Recent advancements in fuzzy logic have expanded its application in modeling uncertainties in manufacturing environments. For example, research by Wang et al. (2019) developed a fuzzy logic-based decision support system for production planning, incorporating fuzzy inference techniques to handle imprecise information and optimize production schedules. Moreover, studies such as Liu et al. (2020) and Cai et al. (2021) investigated the integration of fuzzy logic and optimization algorithms for production scheduling, demonstrating improvements in production efficiency and resource utilization.

Cap-and-Trade Policy:

Recent literature has extensively explored the implementation and impact of cap-and-trade policies on environmental sustainability and industrial practices. For instance, research by Sabzevaret al. (2017) provided a comprehensive overview of cap-and-trade systems, highlighting their effectiveness in reducing emissions and fostering innovation. Moreover, studies such as Li et al. (2015) and Wei et al. (2018) investigated the economic and environmental implications of cap-and-trade policies in various contexts, emphasizing the importance of regulatory frameworks in balancing environmental protection and economic growth. Recent literature has extensively explored the implementation and impact of cap-and-trade policies on environmental sustainability and industrial practices. For instance, research by Goulder and Schein (2019) provided empirical evidence on the effectiveness of cap-and-trade systems in reducing carbon emissions while minimizing adverse effects on economic growth. Moreover, studies such as Zhang et al. (2020) and Wang et al. (2023) investigated the design and optimization of cap-and-trade mechanisms in specific industries, highlighting the importance of tailored policy frameworks to address sector-specific challenges and opportunities.

Imperfect Production Process:

In recent years, research has focused on addressing imperfections in production processes to enhance operational efficiency and product quality. For example, studies by Wang et al. (2012) and Zhou et al. (2017) investigated strategies for mitigating the impacts of imperfect production processes, including machine breakdowns and quality variations, through proactive maintenance and quality control measures. Additionally, research by Wu et al. (2019) proposed optimization models considering imperfect production processes and reliability constraints, aiming to improve production planning and scheduling decisions. In recent years, research has focused on addressing imperfections in production processes to enhance operational efficiency and product quality. For example, studies by Xu et al. (2019) and Chen et al. (2021) investigated strategies for mitigating the impacts of imperfect production processes through proactive quality control measures and adaptive production planning techniques. Moreover, research by Wang et al. (2022) proposed optimization models considering imperfect production processes and reliability constraints, aiming to improve production planning and scheduling decisions in complex manufacturing environments.

2.2 Assumptions and Notation

The following assumptions are considered to develop the model:

1. An EPQ model is developed here and the production process of the manufacturer is assumed to be non-reliable.

2. Therefore some defective products are produced during the production. These products are sold in the secondary market in a single lot.
3. Uncertainty is accounted for in appropriate manner. Hence the model is developed in a fuzzy environment.
4. Model is developed considering learning approach for the production process.
5. Carbon emission is considered and controlled by the cap and trade policy

Development of the model in crisp environment

Following notations have been used in our study:

P_s	Constant production rate of the first manufacturer
α	Ratio of the perfect items
D	Demand rate of the manufacturer
C_s	Selling price for the material sold in from secondary market
$\left(C_m + \frac{C'_m}{n^\eta}\right)$	Production cost of the first manufacturer
C_m	Selling price of the first manufacturer
h_s	Holding cost for the first manufacturer
h_{imp}	Holding cost of imperfect items
C_{me}	Carbon emission during production of items
h_{se}	Carbon emission during storage of perfect items
h_{impe}	Carbon emission during storage of imperfect items
δ	Carbon tax
Z	Carbon cap
G_m	Green technology
ϵ	Reduction in carbon emission
A_s	Set up cost for the first manufacturer
$Y(A_s, r)$	Total cost of interest and depreciation for production process per production cycle
t_s	Production time of the producer
T_s	Cycle length of the producer
$I_{imp}(t)$	stock of the imperfect items at time t
$I_s(t)$	First manufacturer's Inventory at time t

2.2 Model Description

In this model we have developed a model fora manufacturer to formulate of the profit function. The manufacturer produces the with a constant production rate αP_s and fulfilled the second manufacturer's demand with a rate of P_m units per unit time hence the inventory is increases due to the combine effect of production and demand to the time $t = t_s$. After that the first manufacturer's inventory decreases due to the demand only until the time $t = T_s$. From the produced items a certain amount is found to be defective which is sold in the secondary market in a single lot and the perfect items are sold in the main stream market. The inventory level of the manufacturer is shown in the Figure 1.

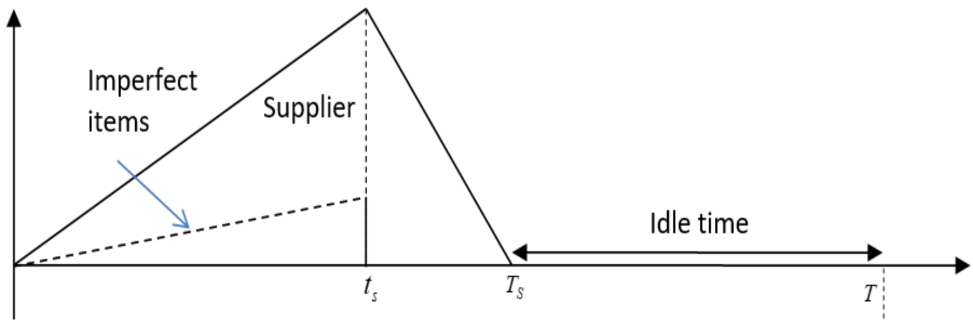

Fig. 1: Behaviour of inventory of the manufacturer

The inventory level of the first manufacturer is governed by the following differential equation

$$\frac{dI_s(t)}{dt} = \alpha P_s - \mathrm{D} \qquad I_s(0) = 0 \qquad 0 \le t \le t_s \qquad (1.1)$$

$$\frac{dI_s(t)}{dt} = -\mathrm{D} \qquad I_s(T_s) = 0 \qquad t_s \le t \le T_s \qquad (1.2)$$

Differential equation for the imperfect items is given by

$$\frac{dI_{imp}(t)}{dt} = (1-\alpha)P_s, \qquad 0 \le t \le t_s \qquad (2)$$

Solution of the differential equations (1) is

$$I_s(t) = \{\alpha P_s - \mathrm{D}\}t \qquad 0 \le t \le t_s \qquad (3.1)$$

$$I_s(t) = \mathrm{D}(T_s - t) \qquad t_s \le t \le T_s \qquad (3.2)$$

From the inequality $I_s(t_s)^- = I_s(t_s)^+$ we have $\alpha P_s t_s = \mathrm{D}T_s$

By which we get $T_s = \frac{\alpha P_s t_s}{\mathrm{D}}$ (4)

From the (2), we have

$I_{imp}(t) = (1-\alpha)P_s t, \quad 0 \le t \le t_s$ (5)

Total imperfect items $Q = (1-\alpha)P_s t_s$ (6)

Holding cost for the first manufacturer

$$= h_s \left\{ \int_0^{t_s} \{\alpha P_s - \mathrm{D}\} t dt + \int_{t_s}^{T_s} \mathrm{D}(T_s - t) dt \right\}$$

$$= h_s \left\{ \{\alpha P_s - \mathrm{D}\} \frac{t_s^2}{2} + \mathrm{D} \frac{(T_s - t_s)^2}{2} \right\}$$

$$= h_s \frac{\alpha t_s^2 P_s}{2\mathrm{D}} \{\alpha P_s - \mathrm{D}\}$$

Holding cost of imperfect items

$$= h_{imp} \int_0^{t_s} (1-\alpha) P_s t dt = h_{imp}(1-\alpha) P_s \frac{t_s^2}{2}$$

Production cost with learning effect $= \left(C_m + \frac{C'_m}{n^{\square}}\right) P_s t_s$

Carbon emission

$$= C_{me} P_s t_s + h_{se} \frac{\alpha t_s^2 P_s}{2\mathrm{D}} \{\alpha P_s - \mathrm{D}\} + h_{impe}(1-\alpha) P_s \frac{t_s^2}{2}$$

Carbon emission cost with cap and trade policy

$$= \delta \left\{ (1 - \epsilon(1 - \mathrm{Exp}[-\eta G_m])) \left(C_{me} P_s t_s + h_{se} \frac{\alpha t_s^2 P_s}{2\mathrm{D}} \{\alpha P_s - \mathrm{D}\} + h_{impe}(1-\alpha) P_s \frac{t_s^2}{2} \right) - Z \right\}$$

Accumulated revenue by the manufacturer from main stream market

$$= C_p \alpha P_s t_s$$

Accumulated revenue by the manufacturer from Secondary market

$$= C_s (1-\alpha) P_s t_s$$

Ordering cost is

$$= A_s$$

Normally, in a non-reliable environment the productivity effects and reduces due to production of the imperfect items. To account this depletion in the worth of product researchers feel a need to develop a cost which is called as interest and depreciation cost and calculated as

$$Y(A_s, r) = a A_s^{-b} r^c$$

$TP_s =$ Average profit for the first manufacturer

$$TP = \frac{1}{T}\Big[C_p \alpha P_s t_s + C_s(1-\alpha)P_s t_s - \left(C_m + \frac{C'_m}{n^\eta}\right)P_s t_s - h_s \frac{\alpha t_s^2 P_s}{2D}\{\alpha P_s - D\} - h_{imp}(1-\alpha)P_s \frac{t_s^2}{2} - \delta\left\{\left(1-\epsilon(1-\mathrm{Exp}[-\eta G_m])\right)\left(C_{me}P_s t_s + h_{se}\frac{\alpha t_s^2 P_s}{2D}\{\alpha P_s - D\} + h_{impe}(1-\alpha)P_s \frac{t_s^2}{2}\right) - Z\right\} - A_s - aA_s^{-b} r^c\Big] \tag{7}$$

Total average profit for the whole system is

$$TP = \frac{1}{T}[\text{Profit of first manufacturer + profit of second manufacturer + profit of retailer}]$$

The total profit (13) per unit time of the system, is a function of t_s and T_r, so, the necessary condition for having an optimum solution for the problem is

$$\frac{\partial TP}{\partial t_s} = 0$$

Development of the model in fuzzy environment

To develop the model in fuzzy environment, following parameters are considered in fuzzy nature and represented by non-negative fuzzy triangular numbers. Here we have considered the production rate demand rate return rate and the number of shipments as a fuzzy parameter.

$\widetilde{P_s} = [P_{s1}, P_{s0}, P_{s2}]$ Fuzzy selling price for the secondary market

$\widetilde{P_m} = [P_{m1}, P_{m0}, P_{m2}]$ Fuzzy production cost for the first manufacturer

The total profit of the supply chain in fuzzy sense can be derived from eq. (13) incorporating the fuzzy parameters instead of crisp parameters.

$$\widetilde{TP}(\widetilde{t_s}) = \frac{1}{\widetilde{t_s}}\left[C_p\alpha\widetilde{P_s}\widetilde{t_s} + C_s(1-\alpha)\widetilde{P_s}\widetilde{t_s} - \left(C_m + \frac{C'_m}{n^\eta}\right)\widetilde{P_s}\widetilde{t_s} - h_s\frac{\alpha\{\widetilde{t_s}\}^2\widetilde{P_s}}{2\widetilde{\mathrm{D}}}\{\alpha\widetilde{P_s} - \widetilde{\mathrm{D}}\}\right.$$
$$- h_{imp}(1-\alpha)\widetilde{P_s}\frac{\{\widetilde{t_s}\}^2}{2}$$
$$- \delta\left\{(1-\epsilon(1\right.$$
$$\left.- \mathrm{Exp}[-\eta G_m]))\left(C_{me}\widetilde{P_s}\widetilde{t_s} + h_{se}\frac{\alpha\{\widetilde{t_s}\}^2\widetilde{P_s}}{2\widetilde{\mathrm{D}}}\{\alpha\widetilde{P_s} - \widetilde{\mathrm{D}}\} + h_{impe}(1-\alpha)\widetilde{P_s}\frac{\{\widetilde{t_s}\}^2}{2}\right) - Z\right\}$$
$$\left.- A_s - aA_s^{-b}r^c\right]$$

Representing the total profit in fuzzy sense by triangular fuzzy number, we have

$$\widetilde{TP} = (TP_1, TP_0, TP_2)$$

$$\widetilde{t_s} = (t_{s1}, t_{s0}, t_{s2})$$

$$TP_1 = \frac{1}{t_{s1}}\left[C_p\alpha P_{s1}t_{s1} + C_s(1-\alpha)P_{s1}t_{s1} - \left(C_m + \frac{C'_m}{n^\eta}\right)P_{s1}t_{s1} - h_s\frac{\alpha t_{s1}^2 P_{s1}}{2D_1}\{\alpha P_{s1} - D_1\}\right.$$
$$- h_{imp}(1-\alpha)P_{s1}\frac{t_{s1}^2}{2}$$
$$- \delta\left\{(1-\epsilon(1-\mathrm{Exp}[-\eta G_m]))\left(C_{me}P_{s1}t_{s1} + h_{se}\frac{\alpha t_{s1}^2 P_{s1}}{2D_1}\{\alpha P_{s1} - D_1\}\right.\right.$$
$$\left.\left.+ h_{impe}(1-\alpha)P_{s1}\frac{t_{s1}^2}{2}\right) - Z\right\} - A_s - aA_s^{-b}r^c\Bigg]$$

$$TP_0 = \frac{1}{t_{s0}}\left[C_p\alpha P_{s0}t_{s0} + C_s(1-\alpha)P_{s0}t_{s0} - \left(C_m + \frac{C'_m}{n^\eta}\right)P_{s0}t_{s0} - h_s\frac{\alpha t_{s0}^2 P_{s0}}{2D_0}\{\alpha P_{s0} - D_0\}\right.$$
$$- h_{imp}(1-\alpha)P_{s0}\frac{t_{s0}^2}{2}$$
$$- \delta\left\{(1-\epsilon(1-\mathrm{Exp}[-\eta G_m]))\left(C_{me}P_{s0}t_{s0} + h_{se}\frac{\alpha t_{s0}^2 P_{s0}}{2D_0}\{\alpha P_{s0} - D_0\}\right.\right.$$
$$\left.\left.+ h_{impe}(1-\alpha)P_{s0}\frac{t_{s0}^2}{2}\right) - Z\right\} - A_s - aA_s^{-b}r^c\Bigg]$$

$$TP_2 = \frac{1}{t_{s2}}\Bigg[C_p\alpha P_{s2}t_{s2} + C_s(1-\alpha)P_{s2}t_{s2} - \left(C_m + \frac{C'_m}{n^\eta}\right)P_{s2}t_{s2} - h_s\frac{\alpha t_{s2}^2 P_{s2}}{2D_2}\{\alpha P_{s2} - D_2\}$$
$$- h_{imp}(1-\alpha)P_{s2}\frac{t_{s2}^2}{2}$$
$$- \delta\Bigg\{\left(1-\epsilon(1-\mathrm{Exp}[-\eta G_m])\right)\Bigg(C_{me}P_{s2}t_{s2} + h_{se}\frac{\alpha t_{s2}^2 P_{s2}}{2D_2}\{\alpha P_{s2} - D_2\}$$
$$+ h_{impe}(1-\alpha)P_{s2}\frac{t_{s2}^2}{2}\Bigg) - Z\Bigg\} - A_s - aA_s^{-b}r^c\Bigg]$$

Total profit in fuzzy sense is represented by the fuzzy number$\widetilde{TP} = (TP_1, TP_0, TP_2)$where $TP_1 < TP_0 < TP_2$. The membership function $\mu_{TP}(x)$ of$\widetilde{TP}$ is defined as follows:

$$\mu_{\widetilde{TP}}(x) = \begin{cases} \dfrac{(x-TP_1)}{(TP_0-TP_1)} & TP_1 \le x \le TP_0, \\ \dfrac{(TP_2-x)}{(TP_2-TP_0)} & TP_0 \le x \le TP_2, \\ 0 & otherwise. \end{cases}$$

Now, to find the optimal values of the decision variables, defuzzification of total profit is performed by centroid method. Using this method, equivalent crisp total cost expression is as follows.

$$F(\widetilde{TP}) = \frac{\int_{-\infty}^{\infty} x\mu_{\widetilde{TP}}(x)dx}{\int_{-\infty}^{\infty} \mu_{\widetilde{TP}}(x)dx} = \frac{1}{3}(TP_1 + TP_0 + TP_2) \ (15)$$

Numerical analysis for crisp model

We consider the following numerical example to illustrate the above solution procedure.

Example 1: We have considered the values of parameters based on the previous studies, but a rescannable estimation has been done.

$$P_s = 800, \alpha = 0.85, D = 600, C_p = 56, C_s = 25, C_m = 20, C'_m = 250, n = 25, \eta = 0.5, h_s = 2.5, h_{imp} = 1.5, C_{me} = 0.75, h_{se} = 0.5, h_{impe} = 0.25, \delta = 0.5, Z = 1200, G_m = 650, \epsilon = 1.2, A_s = 1500, r = 2.5, a = 100, b = 0.5, c = 0.75$$

Then, the optimal result for integrated/collaborating system is $t_s = 6.44685$, $T_R = 0.117995$, $T =$ 13.8176 and the total profit of the chain is 4587.94

Sensitivity analysis

The behaviour of the different parameters on the different variable has been analysed an table 1 revels the effect of variation of parameter on the profit function.

***Table**. 1. Sensitivity analysis of the problem.*

Parameters		T_R	t_s	TP
P_s	700	0.6566	1.0177	7163.25
	750	0.6354	0.9849	7134.15
	800	0.6195	0.9602	7111.31
	850	0.6070	0.9408	7092.88
	900	0.5970	0.9253	7077.70
D	500	0.7281	1.1285	5623.18
	550	0.6687	1.0364	6363.82
	600	0.6195	0.9602	7111.31
	650	0.5782	0.8962	7864.92
	700	0.5433	0.8421	8624.03
δ	0.40	0.6195	0.9602	6956.31
	0.45	0.6195	0.9602	7033.81
	0.50	0.6195	0.9602	7111.31
	0.55	0.6195	0.9602	7188.81
	0.60	0.6195	0.9602	7266.31

C_p	52	0.6404	0.9926	6951.27
	54	0.6299	0.9764	7031.28
	56	0.6195	0.9602	7111.31
	58	0.6090	0.9439	7191.36
	60	0.5985	0.9276	7271.45
α	0.75	0.6290	0.9750	6733.01
	0.80	0.6248	0.9684	6935.23
	0.85	0.6195	0.9602	7111.31
	0.90	0.6135	0.9509	7265.94
	0.95	0.6070	0.9409	7402.78
C_m	16	0.5865	0.9090	7092.47
	18	0.6136	0.9511	7102.30
	20	0.6195	0.9602	7111.31
	22	0.6136	0.9510	7119.44
	24	0.6012	0.9318	7126.75

Observations from the table 1.

- It is observed that, as the production rate of first manufacturer is increasing, the optimal cycle time of the retailer and the optimal time for production both are decreasing which is reasonable due to the ample production to meet the demand. The average profit decreases with the increment in production rate because of the increment in the holding cost.
- Table 1 reveals that as the demand rate of the customer is increased, the optimal time for production is increased therefore the accumulated revenue from the market increased and hence the profit increased.

- It seems that the optimal cycle time is unaffected by the change in returned rate because both of the cycle length is independent of the retuned rate (equation 17 and 20), while the profit increases in a constant ratio with the increment in returned rate.
- It is observed that as the ratio of the perfect items is increased, imperfect items are rarely detected in the inventory, that is, the perfect items will be more in volume. Hence the optimal time for the production run decreases. As the volume of imperfect items decreases, the loss due to those items decreases and hence the profit increases.
- It is observed that the profit is positive sensitive to the change in the accumulated revenue from the secondary market which is reasonable while the optimal cycle time of retailer is negatively sensitive.
- As the number of cycles increases, the retailer's optimal cycle time increases first and then starts to decrease while the optimal time for production is increasing. Furthermore, as the number of cycles increases, the retailer has to store less inventory in his warehouse hence the holding cost to store the inventory decreases thus the profit increases.

Numerical analysis for fuzzy model

Example 2: We consider the values of fuzzified parameters as follows while the value of the remaining parameters has been taken as same as taken in the example 1:

$P_m = (180 \text{ units}, 200 \text{ units}, 250 \text{ units})$ per unit time, $P_s = (210 \text{ units}, 250 \text{ units}, 320 \text{ units})$ per unit time,
$R = (25 \text{ units}, 35 \text{ units}, 55 \text{ units})$ per unit time, $d_c = (50 \text{ units}, 100 \text{ units}, 200 \text{ units})$ per unit time,
$n = (10, 20, 25)$ shipments

After applying the solution procedure provided in the above section, we have we have derived the optimal results, the outcomes for the given integrated system are as follows

$t_s = 6.40172$ months , $T_R = 0.120057$ months, $T = 14.1517$ months and the total profit of the chain is $ 5470.76.

Table 2: Effect of uncertainty on the crisp model.

	T_R	t_s	T	TP
Crisp Model	0.117995	6.44685	13.8176	4587.94
Fuzzy Model	0.120057	6.40172	14.1517	5470.76
Increment	1.75%	-0.70%	2.42%	19.24%

Conclusion

The study done fabricated a model which provides an analytical approach to find the optimum replenishment policies. A closed form solution has been given. We have explored this model for tennis racket. It is usually found that after spending a useful life the product has some components or the product in proper condition. In case of racket, sometimes either the frame or the string of the racket is in reusable condition. Study reveals that the profit increases in a constant ratio with the increment in returned rate. So, the adoption of the reverse logistics could be a profitable deal for the practitioners, not only at the environmental point of view but economically too. Thus, it will be a suggestion for the sport companies that they should also try to open a reverse channel for the products to recollect their used materials from the sports academies or any other places from where they can collect the material in a mass volume. A fuzzified model is also provided here where the demand rate, production rate, return rate and number of shipments are taken as fuzzy parameters. On comparison of the effects of the uncertainty of all the parameters individually on the crisp model, it is concluded that the uncertainties in demand affects the crisp model much more than the uncertainties of other parameters. So, it is advisable to the practitioners to be more conscious toward the impreciseness in the demand parameter. This article can be further extended for the decaying items. Permissible delay in payment in the vendor buyer integration is also a possible extension of the article.

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