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Radio Contra Harmonic Mean D-Distance Number of Some Bistar Related Graphs

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Abstract : This study has been undertaken to investigate the least upper bound of graphs with respect to the condition radio contra harmonic mean labeling and D-Distance of the graph by formulating constrains mathematically. Some Bistar related graphs are encountered here. These kinds of concepts is employed in X-rays, crystallography, coding theory, computing etc.

***IndexTerms* – D-Distance, D-diameter,**

I. Introduction

Graph labeling was introduced by Alexandra Rosa in 1967 [1]. For standard terminologies we follow Harary [5]. Radio labeling was introduced by Chartrand G, Erwin D, Zhang P and Harary F [3]. Concept of D-distance was developed by Reddy Babu D [8]. Radio contra harmonic mean number of Graphs was introduced by Ashika TS and Asha S [2]. Dr. Asha S, Ashika T S introduced the concept radio contra harmonic mean Dd-distance number [4]. John Bosco, K. Vishnu Priya, BS introduced the concept Radio D-Distance in harmonic mean labeling of some basic graphs [6]. In this Sequence, we introduced radio contra harmonic mean D-distance labeling and radio contra harmonic mean D-distance number of graphs. In this work we examined the bahaviour of some bistar related graphs according to the labeling introduced.

II. PRELIMINARIES

Definition 2.1. [5]The graph $K_{1,n}$ is a *star* for $n \geq 1$.

Definition 2.2. [5]The *Bistar* $B_{m,n}$ is the graph obtained by joining the two central vertices of $K_{1,m}$ and $K_{1,n}$.

Definition 2.3. [7]The *splitting graph* $S'(G)$ of a graph G is constructed by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

III. METHODOLOGY

The radio contra harmonic mean D-distance labeling of a connected graph G is an injective function $f: V(G) \to \mathbb{Z}^+$ such that for any two distinct vertices u, v

$$d^D(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq 1 + diam^D(G) \forall\, u, v \in V(G) \quad \ldots\ldots\ldots\ldots\ldots\ (1)$$

or $$d^D(u,v) + \left\lfloor \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rfloor \geq 1 + diam^D(G)\ \forall\, u, v \in V(G) \quad \ldots\ldots\ldots\ldots\ldots\ (2)$$

where $d^D(u,v)$ denote D-distance or D-length between u and v of G and $diam^D(G)$ denotes the of D-diameter of G, then G is a radio contra harmonic mean D-distance graph.

Further, if u and v are vertices of connected graph G, [5] the D-length of a connected $u - v$ path s is defined as $d^D(u,v) = min\{l^D(s)\}$, $l^D(s) = l(s) + deg(v) + deg\,(u) + \Sigma\, deg(w)$, where the sum runs over all intermediate vertices w in s of G and $diam^D(G) = \max\{d^D(u,v)\}$. The radio contra harmonic mean D-Distance number of the function f is denoted as $rchm^D n(f)$ is the highest positive integer assigned to any vertex $v \in V(G)$ under the mapping f and radio contra harmonic mean D-Distance number of the graph G is denoted as $rchm^D n(G)$ is the smallest span of $rchm^D n(f)$ taken across every radio contra harmonic mean labeling of G. Obviously $rchm^D n(G) \geq |V(G)|$. If $rchm^D n(G) = |V(G)|$ then G is called radio contra harmonic mean D-Distance graceful graph. In this paper we studied the behaviour of some bistar related graphs.

IV. RESULTS AND DISCUSSION

Theorem 4.1. For every bistar graph $B_{m,n}$, $rchm^D n(B_{m,n}) = \begin{cases} 2m+n+4 \ if \ m=1, \ n \geq 1 \\ 2m+n+2 \ if \ m \leq n, m,n \geq 2 \end{cases}$

Proof: Let $V(B_{m,n}) = \{u_i, u, v, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $X(B_{m,n}) = \{u_i u, uv, vv_j : 1 \leq i \leq m, 1 \leq j \leq n\}$.

Then $diam^D(B_{m,n}) = m+n+7$ for $m,n \geq 1$.

Therefore inequality (1) reduces to

$$d^D(u,v) + \left\lceil \frac{(f(u))^2 + (f(v))^2}{f(u)+f(v)} \right\rceil \geq m+n+8 \ldots\ldots\ldots\ldots\ldots (3)$$

Now to show that $B_{m,n}$ admits radio D-distance in contra harmonic mean labeling for $m,n \geq 2$.

Case (i) If $m = 1, n \geq 1$

Allocate the vertices as follows by defining the map $f: V(B_{m,n}) \to \mathbb{Z}^+$ such that:

$$f(u_1) = m+3$$
$$f(v_i) = m+i+3, 1 \leq i \leq n$$
$$f(v) = 2m+n+3$$
$$f(u) = 2m+n+4$$

Subcase (i) for the pair of vertices (u_1, v_i), $d^D(u_1, v_i) = n+8$ for $1 \leq i \leq n$,

$$\left\lceil \frac{(m+3)^2 + (m+i+3)^2}{2m+i+6} \right\rceil \geq m$$

Subcase (ii) for the pair of vertices (u_1, v), $d^D(u_1, v) = n+6$,

$$\left\lceil \frac{(m+3)^2 + (2m+n+3)^2}{3m+n+6} \right\rceil \geq m+2$$

Subcase (iii) for the pair of vertices (u_1, u), $d^D(u_1, u) = 4$,

$$\left\lceil \frac{(m+3)^2 + (2m+n+4)^2}{3m+n+7} \right\rceil \geq m+n+4$$

Subcase (iv) for the pair of vertices (v_i, v), $d^D(v_i, v) = n+3$ for $1 \leq i \leq n$,

$$\left\lceil \frac{(m+i+3)^2 + (2m+n+3)^2}{3m+n+i+6} \right\rceil \geq m+5$$

Subcase (v) for the pair of vertices (v_i, u), $d^D(v_i, u) = n+6$ for $1 \leq i \leq n$,

$$\left\lceil \frac{(m+i+3)^2 + (2m+n+4)^2}{3m+n+i+7} \right\rceil \geq m+2$$

Subcase (vi) for the pair of vertices (v, u), $d^D(v, u) = n+4$,

$$\left\lceil \frac{(2m+n+3)^2 + (2m+n+4)^2}{4m+2n+7} \right\rceil \geq m+4$$

Subcase (vii) for the pair of vertices (v_i, v_j), $d^D(v_i, v_j) = n+5$ for $1 \leq i,j \leq n, i \neq j$,

$$\left\lceil \frac{(m+i+3)^2 + (m+j+3)^2}{2m+i+j+6} \right\rceil \geq m+3$$

It is clear that f is an injective function and every distinct pair of vertices satisfies the inequality (3). Thus $B_{m,n}$ admits radio contra harmonic mean D-distance labeling and the largest number assigned is $2m+n+4$ to the vertex u.

Case (ii) If $m \leq n$ where $m,n \geq 2$

Allocate the vertices as follows by defining the map $f: V(B_{m,n}) \to \mathbb{Z}^+$ such that:

$$f(u_1) = m+1$$
$$f(v_j) = m+j+1, 1 \leq j \leq n$$
$$f(u_i) = m+n+i, 2 \leq i \leq m$$
$$f(v) = 2m+n+1$$
$$f(u) = 2m+n+2$$

Subcase (i) a) for the pair of vertices (u_1, u), $d^D(u_1, u) = m+3$,

$$\left\lceil \frac{(m+1)^2 + (2m+n+2)^2}{3m+n+3} \right\rceil \geq n+5$$

b) for the pair of vertices (u_i, u), $d^D(u_i, u) = m+3$ for $2 \leq i \leq m$,

$$\left\lceil \frac{(m+n+i)^2 + (2m+n+2)^2}{3m+2n+2+i} \right\rceil \geq n+5$$

Subcase (ii) a) for the pair of vertices (u_1, v), $d^D(u_1, v) = m+n+5$,

$$\left\lceil \frac{(m+1)^2 + (2m+n+1)^2}{3m+n+2} \right\rceil \geq 3$$

b) for the pair of vertices (u_i, v), $d^D(u_i, v) = m + n + 5$ for $2 \le i \le m$,

$$\left\lceil \frac{(m+n+i)^2 + (2m+n+1)^2}{3m+2n+1+i} \right\rceil \ge 3$$

Subcase (iii) a) for the pair of vertices (u_1, v_j), $d^D(u_1, v_j) = m + n + 7$ for $1 \le j \le n$,

$$\left\lceil \frac{(m+1)^2 + (m+j+1)^2}{2m+j+2} \right\rceil \ge 1$$

b) for the pair of vertices (u_i, v_j), $d^D(u_i, v_j) = m + n + 7$ for $2 \le i \le m, 1 \le j \le n$,

$$\left\lceil \frac{(m+n+i)^2 + (m+j+1)^2}{2m+n+i+j+1} \right\rceil \ge 1$$

Subcase (iv) for the pair of vertices (u, v), $d^D(u, v) = m + n + 3$,

$$\left\lceil \frac{(2m+n+2)^2 + (2m+n+1)^2}{4m+2n+3} \right\rceil \ge 5$$

Subcase (v) for the pair of vertices (u, v_j), $d^D(u, v_j) = m + n + 5$ for $1 \le j \le n$,

$$\left\lceil \frac{(2m+n+2)^2 + (m+j+1)^2}{3m+n+j+3} \right\rceil \ge 3$$

Subcase (vi) for the pair of vertices (v, v_j), $d^D(v, v_j) = n + 3$ for $1 \le j \le n$,

$$\left\lceil \frac{(2m+n+1)^2 + (m+j+1)^2}{3m+n+j+2} \right\rceil \ge m+5$$

Subcase (vii) a) for the pair of vertices (u_1, u_j), $d^D(u_1, u_j) = m + 5$ for $2 \le j \le m$,

$$\left\lceil \frac{(m+1)^2 + (m+n+j)^2}{2m+n+j+1} \right\rceil \ge n+3$$

b) for the pair of vertices (u_i, u_j), $d^D(u_i, u_j) = m + 5$ for $2 \le i, j \le m$, $i \ne j$,

$$\left\lceil \frac{(m+n+i)^2 + (m+n+j)^2}{2m+2n+i+j} \right\rceil \ge n+3$$

Subcase (viii) for the pair of vertices (v_i, v_j), $d^D(v_i, v_j) = n + 5$ for $1 \le i, j \le n$, $i \ne j$,

$$\left\lceil \frac{(m+i+1)^2 + (m+j+1)^2}{2m+i+j+2} \right\rceil \ge m+3$$

Thus f is an injective function and every distinct pair of vertices satisfies the inequality (3). Thus $B_{m,n}$ admits radio contra harmonic mean D-distance labeling and the largest number assigned is $2m + n + 2$ to the vertex u.

Hence $rchm^D n(B_{m,n}) = \begin{cases} 2m+n+4 \ \ if \ m = 1, \ n \ge 1 \\ 2m+n+2 \ if \ m \le n, m, n \ge 2 \end{cases}$

Theorem 4.2. $rchm^D n\left(S'(B_{n,n})\right) = \begin{cases} 6n+6 \ \ if \ n = 1 \\ 6n+4 \ if \ n \ge 2 \end{cases}$

Proof: Let $V\left(S'(B_{n,n})\right) = \{u_i, u, v, v_j, u_i', u', v', v_j' : 1 \le i, j \le n\}$ and

$X\left(S'(B_{n,n})\right) = \{u_i'u, uu_i, u_iu', u'v, uv', uv, v_j'v, vv_j, v_jv' : 1 \le i, j \le n\}$.

Then D-distance diameter of $\left(S'(B_{n,n})\right)$ is $4n + 9$.

Therefore inequality (1) reduces to

$$d^D(u, v) + \left\lceil \frac{\left(f(u)\right)^2 + \left(f(v)\right)^2}{f(u) + f(v)} \right\rceil \ge 4n + 10 \ldots\ldots\ldots\ldots\ldots (7)$$

Case (i) if $n = 1$

Allocate the vertices by defining the map $f: V(B_{n,n}) \to \mathbb{Z}^+$ as follows

$$f(u_1') = 2n + 3$$
$$f(v_1') = 2n + 4$$
$$f(u') = 3n + 4$$
$$f(v') = 3n + 5$$
$$f(u) = 4n + 5$$
$$f(v) = 4n + 6$$
$$f(v_i) = 6n + 5$$
$$f(u_i) = 6n + 6$$

Subcase (i) for the pair of vertices (u_1, u), $d^D(u_1, u) = 2n + 5$,

$$\left\lceil \frac{(6n+6)^2 + (4n+5)^2}{10n+11} \right\rceil \ge 2n+5$$

Subcase (ii) for the pair of vertices (u_1, v), $d^D(u_1, v) = 3n + 7$,

$$\left\lceil \frac{(6n+6)^2 + (4n+6)^2}{10n+12} \right\rceil \geq n+3$$

Subcase (iii) for the pair of vertices (u_1, v_1), $d^D(u_1, v_1) = 3n + 10$,

$$\left\lceil \frac{(6n+6)^2 + (6n+5)^2}{12n+11} \right\rceil \geq n$$

Subcase (iv) for the pair of vertices (u, v), $d^D(u, v) = 4n + 5$,

$$\left\lceil \frac{(4n+5)^2 + (4n+6)^2}{8n+11} \right\rceil \geq 5$$

Subcase (v) for the pair of vertices (u, v_1), $d^D(u, v_1) = 3n + 8$,

$$\left\lceil \frac{(4n+5)^2 + (6n+5)^2}{10n+10} \right\rceil \geq n+2$$

Subcase (vi) for the pair of vertices (v, v_1), $d^D(v, v_1) = 2n + 5$,

$$\left\lceil \frac{(4n+6)^2 + (6n+5)^2}{10n+11} \right\rceil \geq 2n+5$$

Subcase (vii) for the pair of vertices (u_1', u'), $d^D(u_1', u') = 3n + 10$,

$$\left\lceil \frac{(2n+3)^2 + (3n+4)^2}{5n+7} \right\rceil \geq n$$

Subcase (viii) for the pair of vertices (u_1', v'), $d^D(u_1', v') = 3n + 7$,

$$\left\lceil \frac{(2n+3)^2 + (3n+5)^2}{5n+8} \right\rceil \geq n+3$$

Subcase (ix) for the pair of vertices (u_1', v_1'), $d^D(u_1', v_1') = 4n + 9$,

$$\left\lceil \frac{(2n+3)^2 + (2n+4)^2}{4n+7} \right\rceil \geq 1$$

Subcase (x) for the pair of vertices (u', v'), $d^D(u', v') = 4n + 9$.

$$\left\lceil \frac{(3n+4)^2 + (3n+5)^2}{6n+9} \right\rceil \geq 1$$

Subcase (xi) for the pair of vertices (u', v_1'), $d^D(u', v_1') = 3n + 6$,

$$\left\lceil \frac{(3n+4)^2 + (2n+4)^2}{5n+8} \right\rceil \geq n+4$$

Subcase (xii) for the pair of vertices (v', v_1'), $d^D(v', v_1') = 3n + 9$,

$$\left\lceil \frac{(3n+5)^2 + (2n+4)^2}{5n+9} \right\rceil \geq n+1$$

Subcase (xiii) for the pair of vertices (u_1, u_1'), $d^D(u_1, u_1') = n + 7$,

$$\left\lceil \frac{(6n+6)^2 + (2n+3)^2}{8n+9} \right\rceil \geq 3n+3$$

Subcase (xiv) for the pair of vertices (u_1, u'), $d^D(u_1, u') = n + 4$,

$$\left\lceil \frac{(6n+6)^2 + (3n+4)^2}{9n+10} \right\rceil \geq 3n+6$$

Subcase (xv) for the pair of vertices (u_1, v'), $d^D(u_1, v') = 3n + 7$,

$$\left\lceil \frac{(6n+6)^2 + (3n+5)^2}{9n+11} \right\rceil \geq n+3$$

Subcase (xvi) for the pair of vertices (u_1, v_1'), $d^D(u_1, v_1') = 3n + 9$,

$$\left\lceil \frac{(6n+6)^2 + (2n+4)^2}{8n+10} \right\rceil \geq n+1$$

Subcase (xvii) for the pair of vertices (u, u_1'), $d^D(u, u_1') = 2n + 4$,

$$\left\lceil \frac{(4n+5)^2 + (3n+4)^2}{7n+9} \right\rceil \geq 2n+6$$

Subcase (xviii) for the pair of vertices (u, u'), $d^D(u, u') = 3n + 7$,

$$\left\lceil \frac{(4n+5)^2 + (3n+4)^2}{7n+9} \right\rceil \geq n+3$$

Subcase (xix) for the pair of vertices (u, v'), $d^D(u, v') = 3n + 4$,

$$\left\lceil \frac{(4n+5)^2 + (3n+5)^2}{7n+10} \right\rceil \geq n+6$$

Subcase (xx) for the pair of vertices (u, v_1'), $d^D(u, v_1') = 4n + 7$,

$$\left\lceil\frac{(4n+5)^2+(2n+4)^2}{6n+9}\right\rceil \geq 3$$

Subcase (xxi) for the pair of vertices (v, u_1'), $d^D(v, u_1') = 4n+7$,

$$\left\lceil\frac{(4n+6)^2+(2n+3)^2}{6n+9}\right\rceil \geq 3$$

Subcase (xxii) for the pair of vertices (v, u'), $d^D(v, u') = 3n+4$,

$$\left\lceil\frac{(4n+6)^2+(3n+4)^2}{7n+10}\right\rceil \geq n+6$$

Subcase (xxiii) for the pair of vertices (v, v'), $d^D(v, v') = 3n+7$,

$$\left\lceil\frac{(4n+6)^2+(3n+5)^2}{7n+11}\right\rceil \geq n+3$$

Subcase (xxiv) for the pair of vertices (v, v_1'), $d^D(v, v_1') = 2n+4$,

$$\left\lceil\frac{(4n+6)^2+(2n+4)^2}{6n+10}\right\rceil \geq 2n+6$$

Subcase (xxv) for the pair of vertices (u_1', v_1), $d^D(u_1', v_1) = 3n+9$,

$$\left\lceil\frac{(2n+3)^2+(6n+5)^2}{8n+8}\right\rceil \geq n+1$$

Subcase (xxvi) for the pair of vertices (u', v_1), $d^D(u', v_1) = 3n+7$,

$$\left\lceil\frac{(3n+4)^2+(6n+5)^2}{9n+9}\right\rceil \geq n+3$$

Subcase (xxvii) for the pair of vertices (v_1, v'), $d^D(v_1, v') = n+4$,

$$\left\lceil\frac{(6n+5)^2+(3n+5)^2}{9n+10}\right\rceil \geq 3n+6$$

Subcase (xxviii) for the pair of vertices (v_1, v_1'), $d^D(v_1, v_1') = n+7$,

$$\left\lceil\frac{(6n+5)^2+(6n+6)^2}{12n+11}\right\rceil \geq 3n+3$$

Clearly f is an injective function and every distinct pair of vertices satisfies the inequality (7). Thus $B_{m,n}$ admits radio contra harmonic mean D-distance labeling and the largest number assigned is $6n+6$ to the vertex u_1.

Case (ii) if $n \geq 2$

Allocate the vertices by defining the map $f: V(B_{n,n}) \rightarrow \mathbb{Z}^+$ as follows

$$f(u_1') = 2n+1$$
$$f(v_1') = 2n+2$$
$$f(u') = 2n+3$$
$$f(v') = 2n+4$$
$$f(v_j') = 2n+3+j, 2 \leq j \leq n$$
$$f(u_i') = 3n+2+i, 2 \leq i \leq n$$
$$f(u) = 4n+3$$
$$f(v) = 4n+4$$
$$f(v_j) = 4n+4+j, 1 \leq j \leq n$$
$$f(u_i) = 5n+4+i, 1 \leq i \leq n$$

Subcase (i) for the pair of vertices (u_i, u), $d^D(u_i, u) = 2n+5$ for $1 \leq i \leq n$,

$$\left\lceil\frac{(5n+4+i)^2+(4n+3)^2}{9n+7+i}\right\rceil \geq 2n+5$$

Subcase (ii) for the pair of vertices (u_i, v), $d^D(u_i, v) = 3n+7$ for $1 \leq i \leq n$,

$$\left\lceil\frac{(5n+4+i)^2+(4n+4)^2}{9n+8+i}\right\rceil \geq n+3$$

Subcase (iii) for the pair of vertices (u_i, v_j), $d^D(u_i, v_j) = 3n+10$ for $1 \leq i, j \leq n$,

$$\left\lceil\frac{(5n+4+i)^2+(4n+4+j)^2}{9n+i+j+8}\right\rceil \geq n$$

Subcase (iv) for the pair of vertices (u, v), $d^D(u, v) = 4n+5$,

$$\left\lceil\frac{(4n+3)^2+(4n+4)^2}{8n+7}\right\rceil \geq 5$$

Subcase (v) for the pair of vertices (u, v_j), $d^D(u, v_j) = 3n+8$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(4n+3)^2+(4n+4+j)^2}{8n+j+7} \right\rceil \geq n+2$$

Subcase (vi) for the pair of vertices (v, v_j), $d^D(v, v_j) = 2n+5$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(4n+4)^2+(4n+4+j)^2}{8n+j+8} \right\rceil \geq 2n+5$$

Subcase (vii) for the pair of vertices (u_i, u_j), $d^D(u_i, u_j) = n+7$ for $1 \leq i, j \leq n$, $i \neq j$,

$$\left\lceil \frac{(5n+4+i)^2+(5n+4+j)^2}{10n+i+j+8} \right\rceil \geq 3n+3$$

Subcase (viii) for the pair of vertices (v_i, v_j), $d^D(v_i, v_j) = n+7$ for $1 \leq i, j \leq n$, $i \neq j$.

$$\left\lceil \frac{(4n+4+i)^2+(4n+4+j)^2}{8n+i+j+8} \right\rceil \geq 3n+3$$

Subcase (ix) a) for the pair of vertices (u_1', u'), $d^D(u_1', u') = 3n+10$,

$$\left\lceil \frac{(2n+1)^2+(2n+3)^2}{4n+4} \right\rceil \geq n$$

b) for the pair of vertices (u_i', u'), $d^D(u_i', u') = 3n+10$ for $2 \leq i \leq n$,

$$\left\lceil \frac{(3n+2+i)^2+(2n+3)^2}{5n+5+i} \right\rceil \geq n$$

Subcase (x) a) for the pair of vertices (u_1', v'), $d^D(u_1', v') = 3n+7$,

$$\left\lceil \frac{(2n+1)^2+(2n+4)^2}{4n+5} \right\rceil \geq n+3$$

b) for the pair of vertices (u_i', v'), $d^D(u_i', v') = 3n+7$ for $2 \leq i \leq n$,

$$\left\lceil \frac{(3n+2+i)^2+(2n+4)^2}{5n+6+i} \right\rceil \geq n+3$$

Subcase (xi) a) for the pair of vertices (u_1', v_j'), $d^D(u_1', v_j') = 4n+9$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(2n+1)^2+(2n+3+j)^2}{4n+j+4} \right\rceil \geq 1$$

b) for the pair of vertices (u_i', v_1'), $d^D(u_i', v_1') = 4n+9$ for $2 \leq i \leq n$,

$$\left\lceil \frac{(3n+2+i)^2+(2n+2)^2}{5n+i+4} \right\rceil \geq 1$$

c) for the pair of vertices (u_1', v_1'), $d^D(u_1', v_1') = 4n+9$,

$$\left\lceil \frac{(2n+1)^2+(2n+2)^2}{4n+3} \right\rceil \geq 1$$

d) for the pair of vertices (u_i', v_j'), $d^D(u_i', v_j') = 4n+9$ for $2 \leq i, j \leq n$,

$$\left\lceil \frac{(3n+2+i)^2+(2n+3+j)^2}{5n+i+j+5} \right\rceil \geq 1$$

Subcase (xii) for the pair of vertices (u', v'), $d^D(u', v') = 4n+9$.

$$\left\lceil \frac{(2n+3)^2+(2n+4)^2}{4n+7} \right\rceil \geq 1$$

Subcase (xiii) a) for the pair of vertices (u', v_1'), $d^D(u', v_1') = 3n+6$,

$$\left\lceil \frac{(2n+3)^2+(2n+2)^2}{4n+5} \right\rceil \geq n+4$$

b) for the pair of vertices (u', v_j'), $d^D(u', v_j') = 3n+6$ for $2 \leq j \leq n$,

$$\left\lceil \frac{(2n+3)^2+(2n+3+j)^2}{4n+j+6} \right\rceil \geq n+4$$

Subcase (xiv) a) for the pair of vertices (v', v_1'), $d^D(v', v_1') = 3n+9$,

$$\left\lceil \frac{(2n+4)^2+(2n+2)^2}{4n+6} \right\rceil \geq n+1$$

b) for the pair of vertices (v', v_j'), $d^D(v', v_j') = 3n+9$ for $2 \leq j \leq n$,

$$\left\lceil \frac{(2n+4)^2+(2n+3+j)^2}{4n+j+7} \right\rceil \geq n+1$$

Subcase (xv) a) for the pair of vertices (u_1', u_j'), $d^D(u_1', u_j') = 2n+6$ for $2 \leq j \leq n$,

$$\left\lceil \frac{(2n+1)^2+(3n+2+j)^2}{5n+3+j} \right\rceil \geq 2n+4$$

b) for the pair of vertices (u_i', u_j'), $d^D(u_i', u_j') = 2n + 6$ for $2 \le i, j \le n$, $i \ne j$,

$$\left\lceil \frac{(3n+2+i)^2 + (3n+2+j)^2}{6n+4+i+j} \right\rceil \ge 2n+4$$

Subcase (xvi) a) for the pair of vertices (v_1', v_j'), $d^D(v_1', v_j') = 2n + 6$ for $2 \le j \le n$,

$$\left\lceil \frac{(2n+2)^2 + (2n+3+j)^2}{4n+j+5} \right\rceil \ge 2n+4$$

b) for the pair of vertices (v_i', v_j'), $d^D(v_i', v_j') = 2n + 6$ for $2 \le i, j \le n$, $i \ne j$,

$$\left\lceil \frac{(2n+3+i)^2 + (2n+3+j)^2}{4n+i+j+6} \right\rceil \ge 2n+4$$

Subcase (xvii) a) for the pair of vertices (u_i, u_1'), $d^D(u_i, u_1') = n + 7$, for $1 \le i \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (2n+1)^2}{7n+5+i} \right\rceil \ge 3n+3$$

b) for the pair of vertices (u_i, u_j'), $d^D(u_i, u_j') = n + 7$ for $1 \le i \le n, 2 \le j \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (3n+2+j)^2}{8n+6+i+j} \right\rceil \ge 3n+3$$

Subcase (xviii) for the pair of vertices (u_i, u'), $d^D(u_i, u') = n + 4$ for $1 \le i \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (2n+3)^2}{7n+7+i} \right\rceil \ge 3n+6$$

Subcase (xix) for the pair of vertices (u_i, v'), $d^D(u_i, v') = 3n + 7$ for $1 \le i \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (2n+4)^2}{7n+8+i} \right\rceil \ge n+3$$

Subcase (xx) a) for the pair of vertices (u_i, v_1'), $d^D(u_i, v_1') = 3n + 9$ for $1 \le i \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (2n+2)^2}{7n+6+i} \right\rceil \ge n+1$$

b) for the pair of vertices (u_i, v_j'), $d^D(u_i, v_j') = 3n + 9$ for $1 \le i \le n, 2 \le j \le n$,

$$\left\lceil \frac{(5n+4+i)^2 + (2n+3+j)^2}{7n+7+i+j} \right\rceil \ge n+1$$

Subcase (xxi) a) for the pair of vertices (u, u_1'), $d^D(u, u_1') = 2n + 4$,

$$\left\lceil \frac{(4n+3)^2 + (2n+1)^2}{6n+4} \right\rceil \ge 2n+6$$

b) for the pair of vertices (u, u_i'), $d^D(u, u_i') = 2n + 4$ for $2 \le i \le n$,

$$\left\lceil \frac{(4n+3)^2 + (3n+2+i)^2}{7n+5+i} \right\rceil \ge 2n+6$$

Subcase (xxii) for the pair of vertices (u, u'), $d^D(u, u') = 3n + 7$,

$$\left\lceil \frac{(4n+3)^2 + (2n+3)^2}{6n+6} \right\rceil \ge n+3$$

Subcase (xxiii) for the pair of vertices (u, v'), $d^D(u, v') = 3n + 4$,

$$\left\lceil \frac{(4n+3)^2 + (2n+4)^2}{6n+7} \right\rceil \ge n+6$$

Subcase (xxiv) a) for the pair of vertices (u, v_1'), $d^D(u, v_1') = 4n + 7$,

$$\left\lceil \frac{(4n+3)^2 + (2n+2)^2}{6n+5} \right\rceil \ge 3$$

b) for the pair of vertices (u, v_j'), $d^D(u, v_j') = 4n + 7$ for $2 \le j \le n$,

$$\left\lceil \frac{(4n+4)^2 + (2n+3+j)^2}{6n+6+j} \right\rceil \ge 3$$

Subcase (xxv) a) for the pair of vertices (v, u_1'), $d^D(v, u_1') = 4n + 7$,

$$\left\lceil \frac{(4n+4)^2 + (2n+1)^2}{6n+5} \right\rceil \ge 3$$

b) for the pair of vertices (v, u_i'), $d^D(v, u_i') = 4n + 7$ for $2 \le i \le n$,

$$\left\lceil \frac{(4n+4)^2 + (3n+2+i)^2}{7n+6+i} \right\rceil \ge 3$$

Subcase (xxvi) for the pair of vertices (v, u'), $d^D(v, u') = 3n + 4$,

$$\left\lceil \frac{(4n+4)^2 + (2n+3)^2}{6n+7} \right\rceil \ge n+6$$

Subcase (xxvii) for the pair of vertices (v, v'), $d^D(v, v') = 3n + 7$,

$$\left\lceil \frac{(4n+4)^2 + (2n+4)^2}{6n+8} \right\rceil \geq n+3$$

Subcase (xxviii) a) for the pair of vertices (v, v_1'), $d^D(v, v_1') = 2n + 4$,

$$\left\lceil \frac{(4n+4)^2 + (2n+2)^2}{6n+6} \right\rceil \geq 2n+6$$

b) for the pair of vertices (v, v_j'), $d^D(v, v_j') = 2n + 4$ for $2 \leq j \leq n$,

$$\left\lceil \frac{(4n+4)^2 + (2n+3+j)^2}{6n+7+j} \right\rceil \geq 2n+6$$

Subcase (xxix) a) for the pair of vertices (u_1', v_j), $d^D(u_1', v_j) = 3n + 9$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(2n+1)^2 + (4n+4+j)^2}{6n+5+j} \right\rceil \geq n+1$$

b) for the pair of vertices (u_i', v_j), $d^D(u_i', v_j) = 3n + 9$ for $2 \leq i \leq n, 1 \leq j \leq n$,

$$\left\lceil \frac{(3n+2+i)^2 + (4n+4+j)^2}{7n+6+i+j} \right\rceil \geq n+1$$

Subcase (xxx) for the pair of vertices (u', v_j), $d^D(u', v_j) = 3n + 7$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(2n+3)^2 + (4n+4+j)^2}{6n+7+j} \right\rceil \geq n+3$$

Subcase (xxxi) for the pair of vertices (v_j, v'), $d^D(v_j, v') = n + 4$ for $1 \leq j \leq n$,

$$\left\lceil \frac{(4n+4+j)^2 + (2n+4)^2}{6n+8+j} \right\rceil \geq 3n+6$$

Subcase (xxxii) a) for the pair of vertices (v_i, v_1'), $d^D(v_i, v_1') = n + 7$ for $1 \leq i \leq n$,

$$\left\lceil \frac{(4n+4+i)^2 + (2n+2)^2}{6n+6+i} \right\rceil \geq 3n+3$$

b) for the pair of vertices (v_i, v_j'), $d^D(v_i, v_j') = n + 7$ for $1 \leq i \leq n, 2 \leq j \leq n$,

$$\left\lceil \frac{(4n+4+i)^2 + (2n+3+j)^2}{6n+7+i+j} \right\rceil \geq 3n+3$$

Clearly f is an injective function and every distinct pair of vertices satisfies the inequality (7). Thus $B_{m,n}$ admits radio contra harmonic mean D-distance labeling and the largest number assigned is $6n + 4$ to the vertex u_n.

Hence $rchm^D n\left(S'(B_{n,n})\right) = \begin{cases} 6n+6 & if\ n = 1 \\ 6n+4 & if\ n \geq 2 \end{cases}$.

OPEN PROBLEM

Find $rchm^D n\left(S'(B_{m,n})\right)$ for $m \neq n$.

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