JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

GENERALIZATIONS OF TOPOLOGICAL SPACES

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Abstract –In this paper, the notation of spg-interior and spg-closure are introduced and studied in topological spaces. It is proved that the complment of spg-interior of A is the closure of the complement of A and some properties of the new concepts have been studied.

Keywords – Spg closed sets,Spg open sets, Spg-closure, Spg-interior.

Mathematics subject classification (2010): 54A05.

1 .INTRODUCTION

N.Levine[1] introduced the concept of generalized sets of a topological space in 1970.Dunham[2] defined generalized closure operator in 1982.Mashhour,Abd El-Monsef and Deeb[3] introduced the concept of pre-closed sets in 1982.Gnanambal[4] introduced and studied the concept of gpr closed sets in topological space. Gnanambal and Balachandran[5], introduced and studied the concept ofgpr- interior and gpr-closure operator in topological space in 1999. Recently In the year 1999 N.Nagaveni [6] introduced and studied semi weakly generalized closed sets (briefly-spg)and semi weakly generalized open [7] sets in topological spaces.

2. Preliminaries. A subset A of a topological space (X,τ) is called

(i) Generalized closed set(briefly g-closed) if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is open in X.

(ii) Pre-openset if A $\subseteq$int(cl(A)) and pre-closed set if cl(int(A)) $\subseteq$ A.

(iii) Generalized pre regular closed set(briefly gpr-closed) if pcl(A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.

(iv) A subset A of topological space (X,τ) is called a pre generalized pre regular ωeakly closed set(briefly pgprω-closed set) if pCl(A)$\subseteq$U whenever A$\subseteq$U and U is rgα open in (X,τ).

(v) A subset A in (X,τ) is called semi weakly generalized open set in X if A^c is semi weakly generalized closed set in X.

Throughout this paper space $(X, \tau)$and(Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c, P-Cl(A) and P-int(A) denote the Closure of A, Interior of A, Compliment of A,pre closure of A and pre-interior of A in X respectively.

3.spg-closure and spg- Interior

In this section the notation of spg-closure and spg-Interior is defined and some of its basic properties are studied.

Definition 3.1 : For a subset A of (X,τ), spg-closure of A is denoted by spg-cl(A) and defined as spg-cl(A)= $\cap$\{G:A$\subseteq$G, G is spg–closed in (X,τ)\} or $\cap$\{G:A$\subseteq$G, Gϵspg-C(X) \}

Theorem3.2 If A and B are subsets of space (X,τ) then

(i) spg-cl (X) = X ,spg-cl(ϕ)=ϕ

(ii)A $\subseteq$spg-cl(A)

(iii)If B is any spg-closed set containing A, then spg-cl(A)$\subseteq$ B.

(iv)If A$\subseteq$B then spg-cl(A)$\subseteq$spg-cl(B)

(v)spg-cl(A)=spg-cl(spg-cl(A))

(vi) spg-cl(A$\cup$B) =spg-cl(A)$\cup$spg-cl(B).

Proof:

(i) By definition of spg-closure, X is only spg-closed set containing X. Therefore spg-cl(X) = Intersection of all the spg-closed set containing X = ∩{X} =X therefore spg-cl (X) = X and again by definition of spg-closure.

spg-cl(ϕ) = Intersection of all spg-closed sets containing ϕ.

= ϕ∩any spg-closed set containing ϕ= ϕ. Therefore spg-cl(ϕ)=ϕ.

(ii) By definition ofspg-closure of A, it is obvious that A ⊆spg-cl(A).

(iii)Let B be any spg-closed set containing A. Since spg-cl(A) is the intersection of all spg-closed set containing A, spg-cl(A) is contained in every spg-closed set containing A. Hence in particular spg-cl(A)⊆B

(iv)Let A and B be subsets of (X,τ) such that A⊆B by definition spg-closure ,Spg-cl(B)=∩{F : B ⊆F ϵspg-C(X)}. If B ⊆F ϵspg-C(X), then spg-cl(B) ⊆ F. since A ⊆B, A⊆B ⊆ Fϵ spg-C(X), we have spg-cl(A)⊆F, spg-cl(A)⊆∩{F : B⊆F ϵspg-C(X)} = spg-cl(B).Therefore spg-cl(A) ⊆spg-cl(B).

(v)Let A be any subset of X by definition of spg-closure ,spg-cl(A)= ∩{F : A ⊆F ϵspg-C(X)}.
If A ⊆ Fϵspg-C(X) then spg-cl(A) ⊆ F, since F is spg-closed set containing spg-cl(A) by (iii) spg-cl(spg-cl(A))⊆F; Hence spg-cl(spg-cl(A))=∩{F:A⊆Fϵspg-C(X)}=spg-cl(A).
Therefore;spg-cl(spg-cl(A))=spg-cl(A).

(Vi) Let A and B be subsets of X, Clearly A⊆A∪B , B⊆A∪B from (iv) spg-cl(A)⊆spg-cl(A∪B),spg-cl(B)⊆spg-cl(A∪B);hence, spg-cl(A)∪ spg-cl(B)⊆spg-cl(A∪B)……..(1)
Now we have to prove that spg-cl(A∪B) ⊆spg-cl(A)∪ spg-cl(B).
Suppose x∉spg-cl(A)∪ spg-cl(B) then ∃spg-closed set A_1 and B_1 with A⊆A_1 , B⊆B_1 and x∉A_1∪B_1.We have A∪B⊆A_1∪B_1 and A_1∪B_1 is the spg-closed set(we know that union of two spg closed subsets of X is spg closed set in X) such that x∉A_1∪B_1.Thus x∉spg-cl(A∪B) hence
spg-cl(A∪B)⊆spg-cl(A)∪ spg-cl(B)------(2).From (1) and (2) we have spg-cl(A∪B)=spg-cl(A)∪spg-cl(B).

Theorem 3.3 If A ⊆X is spg-closed set then spg-cl(A) =A

Proof : Let A be spg-closed subset of X. we know that A ⊆spg-cl(A) –(1).
Also A⊆A and A is spg-closed set by theorem3.2 (iii) spg-cl(A)⊆A –(2) . Hence spg-cl(A) =A

Example3.4 Let X={a,b,c,d}, τ={X,ϕ,{a},{c,d},{a,c,d}}.A={a,c}

spg-cl(A)={a,c}=A then A is not spg-closed set

Theorem 3.5 If Aand B are subsets of space X then spg-cl(A∩B)⊆spg-cl(A)∩spg-cl(B)

Proof: Let A and B be subsets of X, Clearly A∩B⊆A, A∩B⊆B by theorem3.2(iv) spg-cl(A∩B)⊆spg-cl(A),Spg-cl(A∩B)⊆spg-cl(B);hence spg-cl(A∩B) ⊆spg-cl(A)∩spg-cl(B).

Remark 3.6:In-general;spg-cl(A)∩spg-cl(B)⊈spg-cl(A∩B) as seen from the following example.

Example3.7 Consider X={a,b,c,d}, τ ={X,ϕ,{a},{b},{a,b},{a,b,c}}, A={b,c} , B={c,d},A∩B={c},spg-cl(A)={b,c,d},spg-cl(B)={c,d},spg-cl(A∩B)={c} and spg-cl(A)∩spg-cl(B)={c,d} therefore spg-cl(A)∩spg-cl(B) ⊈spg-cl(A∩B).

Theorem3.8: For an x ϵX, xϵspg-cl(A) if and if A∩V≠ ϕ for every spg-open set V containing x.

Proof: Let xϵspg-cl(A).To prove A∩V ≠ ϕ for every spg-open set V containing x by contradiction.Suppose ∃spg-open set V containing x s.t A∩V = ϕ.Then A ⊆X-V , X-V is spg-closed set , spg-cl(A)⊆ X-V. This shows that x∉spg-cl(A) which is contradiction . Hence A∩V ≠ ϕ for every spg-open set V containing x.

Conversely:

Let A∩V ≠ ϕ for every spg-open set V containing x. To prove xϵspg-cl(A). we prove the result by contradiction. Suppose x∉spg-cl(A) then there exist a spg-closed subset Fcontaining A s.t x ∉ F. Then xϵX-F is spg-open.Also (X-F)∩A=ϕ which is contradiction. Hence xϵspgcl(A).

Theorem 3.9 If Ais subset of space X then

(i) spg-cl(A) ⊆ cl(A)

(ii) spg-cl(A) ⊆pcl(A)

Proof: (i) Let A besubset of space X by definition of Closure Cl(A)=⊆∩{F : A ⊆F ϵ C(X)}.If A⊆FϵC(X) then A⊆F ϵspg-C(X) because every closed set is spg-closed that is spg-cl(A)⊆F therefore spg-cl(A) ⊆∩{F : A⊆F ϵ C(X)} = cl(A). Hence spg-cl(A) ⊆ cl(A).

(ii) Let A be subset of space X by definition of p-closure pcl(A)= ⊆∩{F : A ⊆F ϵ p-C(X)}.If A⊆ F ϵ p-C(X) then A⊆F ϵspg-C(X) because every p-closed set is spg-closed that is spg-cl(A)⊆F .therefore spg-cl(A) ⊆∩{F : A⊆F ϵ p-C(X)} = pcl(A). Hence spg-cl(A) ⊆ p-cl(A)

Remark 3.10 Containment relation in the above theorem 3.9 may be proper as seen from following example**.**

Example3.11 Let X={a,b,c,d} ,τ={X,ϕ,{a},{b},{a,b},{a,b,c}},A={a} cl(A)={a,c,d} .
spg-cl(A)={a,d}, pcl(A)={a,c,d} It follows that spg-cl(A)⊂cl(A) and spg-cl(A) ⊂ pcl(A)

Theorem 3.12 : If A is subset of space X then gpr-cl(A)⊆spg-cl(A) where gpr-cl(A)= ∩{F : A ⊆F ϵ GPR-C(X)}

Proof: Let A be a subset of X by definition of spg-closure, spg-cl(A)=∩{F : A ⊆F ϵspg-C(X)} If A⊆F ϵspg-C(X) then A⊆F ϵ GPR-C(X), because every spg-closed is gpr-closed i.egpr-cl(A) ⊆F therefore gpr-cl(A) ⊆∩{F : A ⊆F ϵspg-C(X)}= spg-cl(A). Hence gpr-cl(A) ⊆spg-cl(A).

Theorem 3.13: spg-closure is a Kuratowski closure operator on a space X.

Proof: Let A and B be the subsets space X.

(i) spg-cl(X)= X , spg-cl(ϕ)=ϕ
(ii) A ⊆spg-cl(A)
(iii) spg-cl(A)= spg-cl (spg-cl(A))
(iv)spg-cl(A∪B) = Spg-cl(A)∪spg-cl(B) by theorem3.2
Hence spg-closure is a Kuratowski closure operator on a space X.

Definition3.14: For a subset A of (X,τ) , spg-interior of A is denoted by spg-int(A) and defined as spg-int(A)= ∪{G:G⊆A and G is spg-open in X} or ∪{G: G⊆A and G ϵspg-O(X)}
i.e spg-int(A) is the union of all spg-open set contained in A.

Theorem 3.15: Let A and B be subset of space X then

(i)spg-int(X) = X ,spg-int(ϕ)=ϕ

(ii)spg-int(A)⊆ A

(iii)If B is any spg-open set contained in A, then B⊆spg-int(A)

(iv)IfA⊆B then spg-int(A)⊆spg-int(B)

(v)spg-int(A)= spg-int(spg-int(A))

(vi)spg-int(A∩B)=spg-int(A)∩spg-int(B)

Proof :(i) and (ii) by definition of spg-interior of A, it is obvious.

(iii) Let B be any spg-open set s.t B⊆ A. Let x ϵB , B is an spg-open set contained in A, x is an spg-interior of A i.e. x ϵspg-int(A). Hence B ⊆spg-int(A).

(iv) ,(v), (vi) similar proof as theorem 3.2 and definition of spg-interior.

Theorem 3.16 If a subset A of X is spg-open then spg-int(A) = A

Proof: Let A be spg-open subset of X. We know that spg-int(A) ⊆ A –(1)

Also A is spg-open set contained in A from Theorem 3.15(iii)A ⊆spg-int(A) --(2).

Hence From(1) and(2) spg-int(A) = A

Theorem 3.17: If A and B are subsets of space X then spg-int(A)∪spg-int(B)⊆spg-int(A∪B)

Proof: We know that A ⊆A∪B and B⊆A∪B ,We have Theorem 3.15(iv)spg-int(A)⊆spg-int(A∪B) and spg-int(B)⊆spg-int(A∪B). This implies that spg-int(A)∪spg-int(B)⊆A∪B

Remark 3.18: The converse of the above theorem need not be true as seen from the following example.

Example3.19: Let X={a,b,c,d} and τ ={X,ϕ,{a},{b},{a,b},{a,b,c}},A={b,c}, B={a,d},A ∪ B={a,b,c,d},spg-int(A)={b,c},spg-int(B)={a},spg-int(A ∪ B)=X,spg-int(A) ∪ spg-int(B)= {a,b,c}; therefore spg-int(A ∪ B) ⊈ spg-int(A)∪ spg-int(B).

Theorem 3.20 If A is a subset of X then (i) int(A) ⊆spg-int(A)(ii) p-int(A) ⊆spg-int(A).

Proof:(i) Let A be a subset of a space X. Let x ϵint(A) => x ϵ ∪{G : G is open , G ⊆ A}=>∃ an open set G s.t.x ϵ G ⊆ A =>∃ an spg-open set G s.t.x ϵ G ⊆ A, as every open set is an spg-open set in X=> x ϵ ∪{G : G is spg-open set in X}=> x ϵspg-int(A). Thus x ϵint(A) => x ϵspg-int(A).Hence int(A)⊆spg-int(A).

(ii) Let A be a subset of a space X. Let x ϵ p − int(A) => x ϵ ∪{G : G is p-open , G ⊆ A}=>∃ an p-open set G s.t.x ϵ G ⊆ A =>∃ an spg-open set G s.t.x ϵ G ⊆ A, as every p-open set is an spg-open set in X=> x ϵ ∪{G : G is spg-open set in X}=> x ϵspg-int(A). Thus x ϵ p-int(A)
=>x ϵspg-int(A). Hence p-int(A)⊆spg-int(A).

Remark 3.21: Containment relation in the above theorem may be proper as seen from the followingexample

Example 3.22 Let X={a,b,c,d}
τ={X,ϕ,{a},{b},{a,b},{a,b,c}};A={b,c}, Int(A)={b},p-int(A)={b},
Spg-int(A)={b,c}therefore int(A)⊆spg-int(A) and p-int(A)⊆spg-int(A)

Theorem 3.23: If A issubset of X, then spg-int(A)⊆gpr-int(A), where gpr-int(A) is given bygpr-int(A) = ∪{G ⊆ X : G is gpr-open , G ⊆ A}

Proof: Let A be a subset of a space X.
Let x ϵspg −int(A) => x ϵ ∪{G : G is spg-open , G ⊆ A}=>∃ an spg-open set G s.t.x ϵ G ⊆ A, as every spg-open set is an gpr-open set in X=> x ϵ ∪{G : G is gpr-open , G ⊆ A }=> xϵgpr-int(A). Thus xϵspg-int(A) => xϵgpr-int(A)Hence spg-int(A)⊆gpr-int(A).

Theorem 3.24: For any subset A of X

(i) X – spg-int(A) = spg-cl(X – A)
(ii) spg-int(A) = X – spg-cl(X – A)
(iii) spg-cl(A) = X –spg-int(X – A)
(iv)X – spg-cl(A) = spg-int(X – A)

Proof:xϵX-spg-int(A) then x is not inspg-int(A)i.e. everyspg-open set G containing x s.t. G⊈A . This implies every spg open set G containing x intersects (X–A) i.e. $G \cap (X - A) \neq \phi$. Then by theorem 3.8xϵ spg-cl(X-A)therefore X-spg-int(A)⊆spg-cl (X-A)–(1) and Let, x ∈spg-cl(X-A), then every x∈spg open set ,G containing x intersects X – A i.e. $G \cap (X - A) \neq \phi$, i.e. everyspg-open G containing x s.t. $G \subseteq A$. Then by definition 3.14, x is not inspg-int(A), i.e. xϵX-spg-int(A) and so – spg-cl (X-A) $\subseteq$ X- spg-int(A)-(2)

Thus X – spg-int(A) = spg-cl(X – A) .

(ii)Follows by taking complements in (i).

(iii) Follows by replacing A by X-A in (i)

(iv) Follows by taking complements in(iii).

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