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K Power 3 Heronian Mean Labeling of Graphs

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Abstract : A function f is called k – Power 3 Heronian Mean Labeling of a graph $G = (V, E)$ with p vertices and q edges if $f : V(G) \rightarrow \{k, k+1, \ldots\ldots, k+q\}$ be an injective function and the induced edge labeling $f(e = uv)$ be defined by $f(e) = \left\lceil \sqrt[3]{\frac{f(a)^3 + (f(a)f(b))^{\frac{3}{2}} + f(b)^3}{3}} \right\rceil$ *or* $\left\lfloor \sqrt[3]{\frac{f(a)^3 + (f(a)f(b))^{\frac{3}{2}} + f(b)^3}{3}} \right\rfloor$ with distinct edge labels. In this paper we have proved the k – Power 3 Heronian Mean labeling behaviour of Path, Twig Graph, Triangular ladder L_n , $L_n \odot k_1$. Also we prove that K_n is not k – Power 3 Heronian Mean graph.

Keywords: Graph, Power 3 Heronian Mean Graph, k- Power 3 Heronian Mean Graph.

AMS Classification: 05C78

I. INTRODUCTION

By a graph $G = ((V(G), E(G))$ with p vertices and q edges we Mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. The term not defined here are used in the sense of Harary [1]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. A useful survey on graph labeling by J.A. Gallian (2016) can be found in [2]. If the domain of the mapping is the set of vertices (or edges) then the labeling is called a vertex labeling (or an edge labeling). Somasundaram and Ponraj have introduced the notion of Mean labeling of graphs. R. Ponraj and D. Ramya introduced Super Mean labeling of graph. S. Somasundaram and S.S. Sandhya introduced the concept of Harmonic Mean labeling and studied their behavior. In this paper, we introduce the concept of k-Power 3 Heronian Mean labeling and we investigate k-Power 3 Heronian Mean labeling of some graphs.

Definition: 1.1

A graph G with p vertices and q edges is called a Power 3 Heronian Mean graph,if it is possible to label the vertices $v \in V$ with distinct labels $f(x)$ from $1, 2, \ldots\ldots, q+1$ in such a way that in each edge $e = uv$ is labeled with $f(e) =$

$\left\lceil \sqrt[3]{\frac{f(a)^3+(f(a)f(b))^{\frac{3}{2}}+f(b)^3}{3}} \right\rceil$ or $\left\lfloor \sqrt[3]{\frac{f(a)^3+(f(a)f(b))^{\frac{3}{2}}+f(b)^3}{3}} \right\rfloor$. Then , the edge labels are distinct. In this case f is called Power 3 Heronian Mean labeling of G.In this case, f is a Power 3 Heronian Mean labeling of G and G is called a Power 3 Heronian Mean Graph.

Definition: 1.2

A function f is called k – Power 3 Heronian Mean Labeling of a graph $G = (V, E)$ with p vertices and q edges if $f : V(G) \rightarrow \{k, k + 1, \ldots\ldots, k + q\}$ be an injective function and the induced edge labeling $f(e = uv)$ be defined by $f(e) = \left\lceil \frac{f(u)^3+f(v)^3}{2} \right\rceil^{\frac{1}{3}}$ or $\left\lfloor \frac{f(u)^3+fv)^3}{2} \right\rfloor^{\frac{1}{3}}$ with distinct edge labels.

Definition 1.3

A Twig graph is a tree obtained from a path by attaching exactly two pendent edges to each internal vertex of the path.

Definition 1.4

A Triangular ladder $TL_n, n \geq 2$ is a graph obtained from a ladder L_n by adding the edges $u_i v_{i+1}$ for $1 \leq i \leq n-1$, where u_i and v_i ; $1 \leq i \leq n$, are the vertices of L_n such that $u_1 u_2 \ldots \ldots u_n$ and $v_1 v_2 \ldots \ldots v_n$ are two paths of length n in L_n.

Definition 1.5

If G has order n, the Corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i^{th} vertex of G with an edge to every vertex in the i^{th} copy of H.

II Main Results

Theorem 2.1

Path P_n is a k-Power 3 Heronian Mean graph for all k and $n \geq 2$.

Proof:

Let $V(P_n) = \{v_i ; 1 \leq i \leq n\}$ and

$E(P_n) = \{ e_i = (v_i v_{i+1}); 1 \leq i \leq n - 1\}$

Define a function $f: V(P_n) \rightarrow \{k, k + 1, k + 2, \ldots, k + q\}$ by

$f(v_i) = k + i - 1 ; 1 \leq i \leq n.$

Then the induced edge labels are $f(e_i) = k + i - 1; 1 \leq i \leq n - 1$.

The above defined function f provides k- Power 3 Heronian Mean labeling of the graph. Hence P_n is a k-Power 3 Heronian Mean graph.

Example 2.2 500 - Power 3 Mean labeling of P_7 is given below.

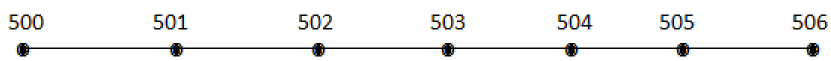

Figure : 1

Theorem 2.3

Twig graph is k-Power 3 Heronian Mean graph for all k and $n \geq 3$

Proof:

Let G be a Twig Graph.Let $V(G) = \{v_i ; 0 \leq i \leq n - 1, u_i w_i; 1 \leq i \leq n - 2\}$ and

$E(G) = \{u_i v_i,\ v_i w_i\ ;\ 1 \le i \le n-2,\ v_i v_{i+1}\ ;\ 0 \le i \le n-2\}$. The ordinary labeling is

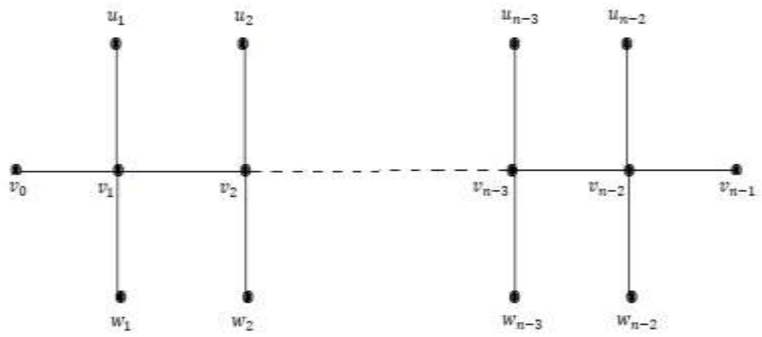

Figure : 2

Define a function $f : V(G) \to \{k, k+2, k+2, \ldots, k+q\}$ by $f(v_0) = k$

$f(v_i) = k + 3i - 2, \quad for\ 1 \le i \le n-1$

$f(w_i) = k + 3i - 1, \quad for\ 1 \le i \le n-2$

$f(u_i) = k + 3i, \quad for\ 1 \le i \le n-2$

Then the induced edge labels are

$f(v_i v_{i+1}) = k + 3i, \quad for\ 0 \le i \le n-2$

$f(v_i u_i) = k + 3i - 1, \quad for\ 1 \le i \le n-2$

$f(v_i w_i) = k + 3i - 2, \quad for\ 1 \le i \le n-2$

The above defined function f provides k - Power 3 Heronian Mean labeling of the graph. Hence Twig is a k-Power 3 Heronian Mean graph.

Example 2.4 100 - Power 3 Heronian Mean labeling of Twig Graph is shown below.

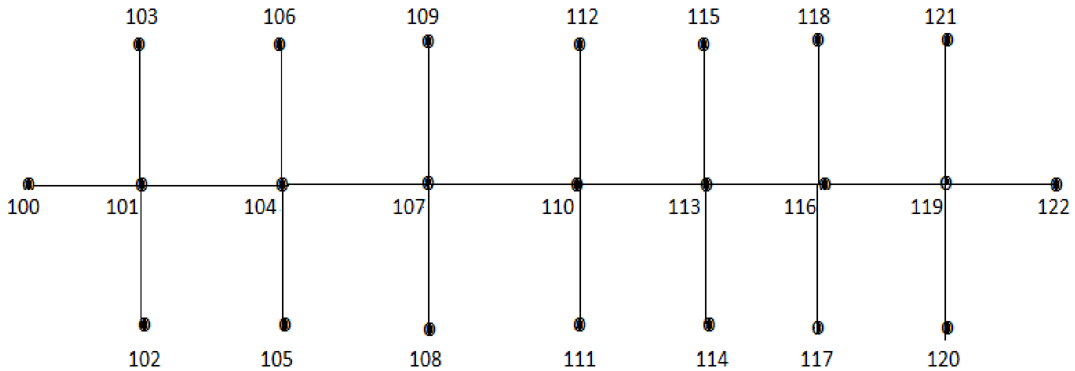

Figure : 3

Theorem 2.5

The Triangular ladder TL_n is k-Power 3 Heronian Mean graph for all k and $n \ge 2$.

Proof:

Let $V(TL_n) = \{u_i, v_i\ ;\ 1 \le i \le n\}$ and

$E(TL_n)\ \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1}\ ;\ 1 \le i \le n-1,\ u_i v_i\ ;\ 1 \le i \le n\}$. The ordinary labeling is

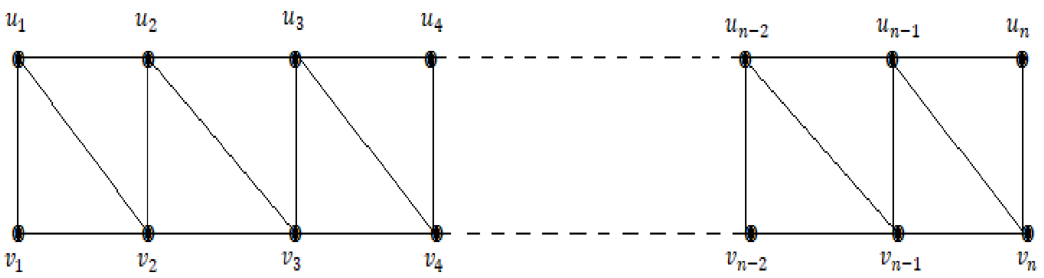

Figure : 4

First we label the vertices as follows

Define a function $f : V(TL_n) \rightarrow \{k, k + 1, k + 2, \ldots k + q\}$ by

$$f(u_i) = k + 4i - 3, \; for\ 1 \leq i \leq n$$

$$f(v_1) = k\ f(v_i) = k + 4i - 5, \; for\ 2 \leq i \leq n.$$

Then the induced edge labels are

$$f(u_i u_{i+1}) = k + 4i - 1, \; for\ 1 \leq i \leq n - 1$$

$$f(v_i v_{i+1}) = k + 4i - 3, \; for\ 1 \leq i \leq n - 1$$

$$f(u_i v_i) = k + 4i - 4, \; for\ 1 \leq i \leq n$$

$$f(u_i v_{i+1}) = k + 4i - 2, \; for\ 1 \leq i \leq n - 1$$

The above defined function f provides k-Power 3 Heronian Mean labeling of the graph.

Hence TL_n is a k-Power 3 Heronian Mean graph.

Example 2.6 700-Power 3 Heronian Mean labeling of TL_7 is shown below

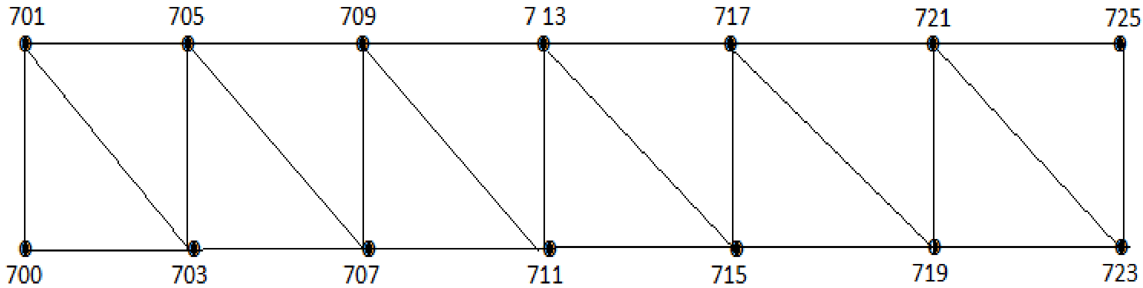

Figure : 5

Theorem 2.7

$L_n \odot k_1$ is k-Power 3 Heronian Mean graph for all k and $n \geq 2$.

Proof:

Let $V(L_n \odot k_1) = \{u_i, v_i, w_i, x_i,; \; 1 \leq i \leq n\}$ and

$$E(L_n \odot k_1) = \{u_i v_i, u_i w_i, v_i x_i; 1 \leq i \leq n, u_i u_{i+1}, v_i v_{i+1}, 1 \leq i \leq n - 1\}$$

The ordinary labeling is

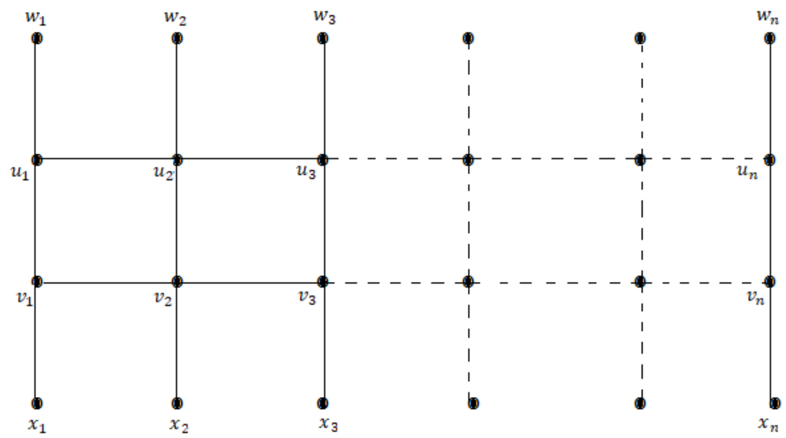

Figure : 6

First we label the vertices as follows

Define a function $f : V(L_n \odot k_1) \rightarrow \{k, k+1, k+2, \ldots k+q\}$ by

$$f(u_i) = k + 5i - 3, \quad for\ 1 \leq i \leq n$$
$$f(v_i) = k + 5i - 4, \quad for\ 1 \leq i \leq n$$
$$f(w_i) = k + 5i - 2, \quad for\ 1 \leq i \leq n$$
$$f(x_i) = k + 5i - 5, \quad for\ 1 \leq i \leq n$$

Then the induced edge labels are

$$f(u_i u_{i+1}) = k + 5i - 1, \quad for\ 1 \leq i \leq n - 1$$
$$f(v_i v_{i+1}) = k + 5i - 2, \quad for\ 1 \leq i \leq n - 1$$
$$f(u_i v_i) = k + 5i - 4, \quad for\ 1 \leq i \leq n$$
$$f(u_i w_i) = k + 5i - 3, \quad for\ 1 \leq i \leq n$$
$$f(v_i x_i) = k + 5i, \quad for\ 1 \leq i \leq n$$

The above defined function f provides k-Power 3 Heronian Mean labeling of the graph. Hence $L_n \odot k_1$ is a k-Power 3 Heronian Mean graph.

Example 2.8: 800 - Power 3 Heronian Mean labeling of $L_6 \odot k_1$ is shown below

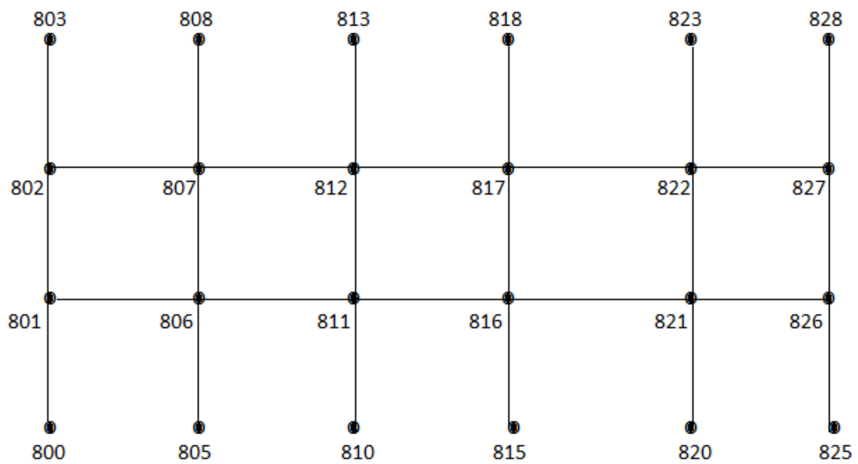

Figure : 7

Conclusion: Here we discuss K Power 3 Heronian mean labeling in the context of graph elements. The results are demonstrated by means of sufficient illustrations which provide better understanding.

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