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ONE-EDGE MAGIC LABELING FOR SOME CLASSES OF GRAPHS

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Abstract:

The primary objective of this research is to present a novel graph labeling technique, termed 1-Edge Magic Labeling, and to determine the conditions under which particular graphs admit this labeling.

Keywords: Graph Labeling, Edge Labeling, 0-Edge Magic Labeling

Introduction:

We denote graphs as G(p, q), with p vertices and q edges, and represent vertex and edge sets as V(G) and E(G), respectively. Labeling refers to a one-to-one mapping of graph elements to numbers, specifically integers. We categorize labeling into:

- Vertex labeling (vertices)
- Edge labeling (edges)
- Total labeling (vertices and edges)

This concept, introduced by Rosa, has gained significant attention in graph theory due to its mathematical challenges and diverse applications:

- X-ray crystallography
- Coding theory
- Cryptography
- Astronomy
- Circuit design
- Communication networks

Recent decades have seen extensive research (over 1300 papers) on:

- Magic labeling
- Anti-magic labeling
- Prime labeling
- Graceful labeling
- k-Graceful labeling
- Odd labeling
- Even and odd mean labeling

- Strongly labeling

Building on J. Jayapriya and K. Thirusangu's 0-Edge Magic Labeling (2012), we introduce 1-Edge Magic Labeling for specific graphs:

- Path graphs (P_n)
- Cycle graphs (C_n)
- Corona graphs ($G \odot K_1$)
- Double stars (Smn)

This paper explores the existence of 1-Edge Magic Labeling in these graph structures.

1-Edge magic Labelling: A 1-edge magic labeling of a graph G is a labeling of the edges of G with positive integers such that the sum of the labels of the edges incident to each vertex is the same.

Vertex-Weight: The vertex-weight of a vertex v in a graph G under an edge labeling is the sum of the labels of all edge's incident with v.

Vertex-Magic Labeling:A vertex-magic labeling is an edge labeling or total labeling where all vertices in G have the same weight, denoted ask. This constant k is called the magic constant.

Vertex-Antimagic Labeling:A vertex-antimagic labeling is an edge labeling or total labeling where all vertices in G have different weights.

Edge-Weight: The edge-weight of an edge e under a vertex labeling is the sum of the labels of the vertex's incident with e. Under a total labeling, the edge-weight is the sum of the labels of the vertex's incident with e and the label of e itself.

Edge-Magic Vertex Labeling: An edge-magic vertex labeling is a vertex labeling where all edges in G have the same edge-weight, denoted as k. This constant k is called the magic constant.

Edge-Magic Total Labeling: An edge-magic total labeling is a total labeling where all edges in G have the same edge-weight, denoted as k. This constant k is called the magic constant.

Edge-Antimagic Vertex Labeling: An edge-antimagic vertex labeling is a vertex labeling where all edges in G have distinct edge-weights.

Edge-Antimagic Total Labeling: An edge-antimagic total labeling is a total labeling where all edges in G have distinct edge-weights.

(1,0) Edge-Magic Graph:A (p, q) graph G is said to be (1,0) edge-magic with a common edge count k if there exists a bijection $f: V(G) \rightarrow \{1, 2, ..., p\}$ such that for all edges $e = uv \in E(G)$, $f(u) + f(v) = k$.

(1,0) Edge-Anti magic Graph:A (p, q) graph G is said to be (1,0) edge-antimagic if for all edges $e = uv \in E(G)$, $f(u) + f(v)$ are distinct.

(0,1) Vertex-Magic Graph:A (p, q) graph G is said to be (0,1) vertex-magic with a common vertex count k if there exists a bijection $f: E(G) \rightarrow \{1, 2, ..., q\}$ such that for each vertex $u \in V(G)$, the sum of the labels of the edges incident on u is equal to k.

(0,1) Vertex-Anti magic Graph:A (p, q) graph G is said to be (0,1) vertex-antimagic if for each vertex $u \in V(G)$, the sum of the labels of the edges incident on u is distinct.

(1,1) Edge-Magic Graph:A (p, q) graph G is (1,1) edge-magic with common edge count k if:

There exists a bijection $f: V(G) \cup E(G) \rightarrow \{1, 2, ..., p + q\}$

For all edges e = uv∈ E(G), f(u) + f(v) + f(e) = k

(1,1) Edge-Anti magic Graph:A (p, q) graph G is (1,1) edge-antimagic if:

There exists a bijection f: V(G) ∪ E(G) → {1, 2, ..., p + q}

For all edges e = uv∈ E(G), f(u) + f(v) + f(e) are distinct.

(0-Edge Magic Labeling): Given a graph G = (V, E), where:V = {vi | 1 ≤ i ≤ n},E = {vi vi+1 | 1 ≤ i ≤ n-1},A 0-edge magic labeling is a labeling f: V → {-1, 1} and f*: E → {0} such that:For all edges uv∈ E, f*(uv) = f(u) + f(v) = 0.

G+ = GΘK₁:G+ is a graph obtained by joining exactly one pendant edge (K_1) to every vertex of a graph G.

Results:

The concept of 0-Edge Magic Labeling motivate us to define the following new definition of 1-Edge Magic Labeling.

1-Edge Magic Labeling: Let G=(V, E) be a graph where V= { vi, 1≤i≤n } and E = {vi v i+1,1≤i≤n} Let f : V →

{-1, 2} and f*: E → {1} such that for all uvεE, f*(uv)= f(u) + f(v) =1 then the labeling is said to be 1-Edge Magic Labeling. Using this new definition, we prove some results as follows:

Path Graphs (P_n)

A path graph is a graph that consists of a single path. Path graphs admit 1-edge magic labeling.

Theorem 1:P_n admits 1-Edge Magic Labeling for all n.

Proof: Let G =(V, E) be a graph where V= {v_i, 1≤i≤n} and E = {v_i v_{i+1} ,1≤i≤n}. Let f: V → {-1, 2} Such that f(v_i) =-1 if i is odd. F (v_{i+1}) =2 if i is even 1≤i≤n f* (v_i, v_{i+1}) = -1+2=1 if i is odd. f* (v_i, v_{i+1}) =2-1=1 if i is even.

Hence p_n admits 1-Edge Magic Labeling.

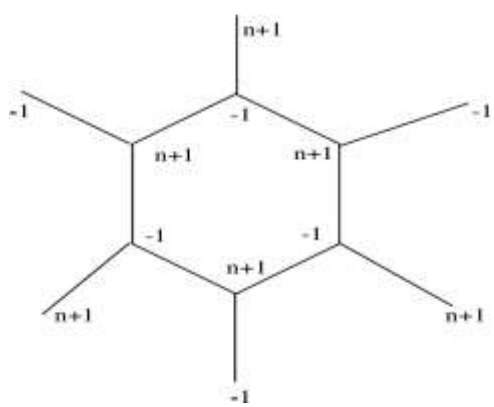
Theorem 2: Cn admits 1-Edge Magic Labeling when n is even.

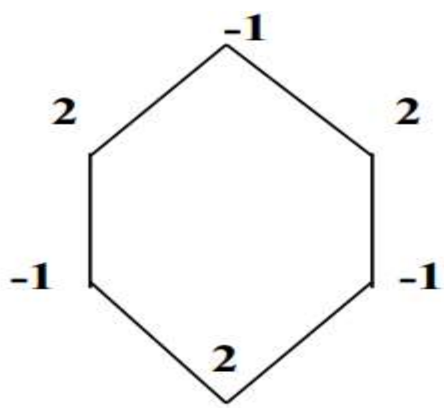
Proof: Let G=(V, E) be a graph where V={v_i: 1 ≤ i ≤ n} and E={v_i. v_{i+1}, 1 ≤ i ≤ n}.

Let f: V→{-1,2}such that f(v_i)= -1 if i is odd f(v_i)= 2 if i is even 1≤i≤n f * (v_i. v_{i+1}) = -1+2=1

if i is odd. f* (v_i. v_{i+1}) =2-1=1 if i is even,

Hence Cn admits 1-Edge Magic Labeling for every even n.





1- Edge Magic labeling for c_6

Theorem 3:If G admits 1- Edge Magic Labeling then GʘK1 admits 1- Edge Magic Labeling.

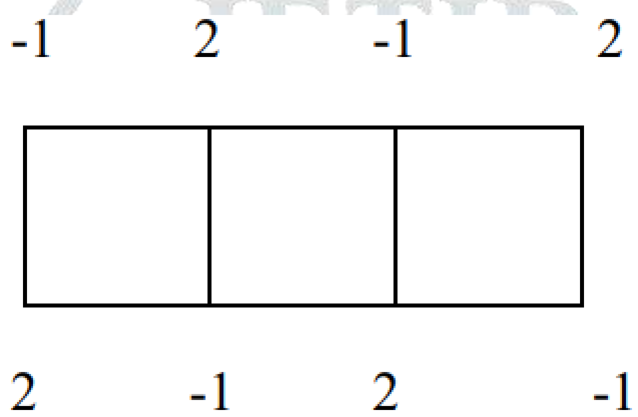
Proof: Let G=(V, E) be a graph where V={v_i: $1 \leq i \leq n$} and E={$v_i . v_{i+1}$, $1 \leq i \leq n$}.

Let f: V→{-1,2} and f*:E→{1} such that for all $v_i . v_{i+1}$ ϵ E f* ($v_i . v_{i+1}$) = f (v_i)+ f (v_{i+1}) = 1 Let G$ʘK_1$ = (V,E) U {vj: $1 \leq j \leq n$} U {$v_i . v_j$: $1 \leq i, j \leq n$}, then

let g: V→{-1, 2} and g*: E→{1}, then for all $v_i . v_{i+1}$ ϵ E.

Let v_j = -1 if v_{Ii} =2 or v j = 2 if vi = -1 then g*($v_i v_j$) = g*(v_i)+ g*(v_j)=1

Hence the theorem.



1-Edge Magic Labeling for $P_4ʘK_1$.

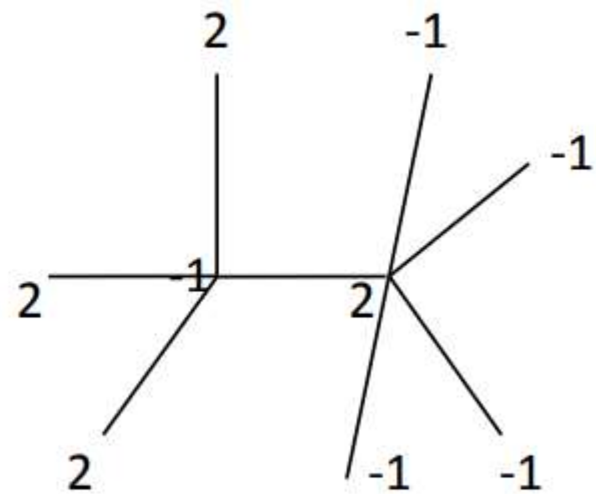
Theorem 4: Let G =Sm, n be a double star graph then G admits 1- Edge Magic Labeling.

Proof: Let G = (V, E) be a double star graph denoted by Sm,n and v_1 and v_2 are two vertices in Sm,n which are not pendent.

Let u_i's are m pendent vertices to v_i and u_j' are n pendent vertices to v_2.

Let f: V→{-1, 2} such that f (v_i)= -1 and f (v_2)=2 and f(u_i) = 2 for $1 \leq i \leq m$ and f(u_j)= -1 for $1 \leq j \leq n$ f * ($v_1 u_i$) = -1 + 2 = 1 if $1 \leq i \leq m$ Also f * (v_2 u_j') = 2 - 1 = 1 if $1 \leq j \leq n$.

Hence the proof.



1-Edge Magic Labeling for S4,5.

CONCLUSION

This study examines a particular class of graphs that exhibit 1-Edge Magic Labeling. Additional investigation is required to identify specific conditions under which graphs possess this labeling property.

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