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STUDY ON STRONG (N, $p_n)_k$ SUMMABILITY OF FOURIER SERIES

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1. Abstract

In the present note we deals with a theorem on strong (N, $p_n)_k$, $k \geq 1$, summability of a Fourier series. It is important to note that the summability (N, $p_n)_k$ is a generalization of both summability (N, $p_n)_k$ and (R, logn, 1). In the sequel our result improve several know results, for instance a wall known result due to Luan [2] on the summability of the Fourier series and sequence by Logarithmic mean and a recent result of Tripathi and Tripathi [3] on strong (N, p_n) summability of a Fourier series.

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2. Introduction:

Let $\{p_n\}$ be a sequence of non-zero constants with R_n as its non-vanishing n^{th} partial sum. Let $\{s_n\}$ be the sequence of partial sums of infinite series $\sum$ an. Following Hardy and Littlewood [1], we define that if

[1.1] $$\sum p_m|s_m - s|^k = o(R_n) \text{ as } n \to \infty,$$

Then the series $\sum a_n$ or sequence $\{s_n\}$ of its partial sum is said to be strongly summable by (N, $p_n)_k$ method or simply summable [N, $p_n]_k$ to the fixed finite sum s.

If k = 1 then summability (N, $p_n)_k$ reduces to the summability (N, p_n) . Also it is known that in the special case when $p_n = 1/n+1$ and k = 1, (N, $p_n)_k$ is equivalent to the summability (R, logn, 1).

Let F(t) be a 2π - periodic and Lebesque integrable function of t in the interval ($-\pi, \pi$). Then the Fourier series of the function f(t) is given by

[1.2] $$f(t) \approx a_0/2 + \sum (a_n \cos nt + b_n \sin nt)$$

Let us write, at the point t = x,

$$\phi(t) = f(x+t) + f(x-t) - 2f(x)$$

and let τ denote the integral part of $1/t$.

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* Adders for Communication

We prove the following theorem:

3 Theorem : Let $\{P_n\}$ be a non negative, monotonic non – increasing sequence of constant such that its non – vanishing n^{th} partial sum $R_n \to \infty$ as $n \to \infty$ and $\{\in_n\}$ be a suitable sequence of constants $\pm$ 1, such that

[3.1] $\sum |\Delta(m\, p_m \in_m)| + n\, p_n \in_n \quad = \quad o(1) \quad \text{as} \quad n \to \infty$,

if [3.2] $\int (|\phi(u)|/u)\, du \quad = \quad o(R_\tau) \quad \text{as} \quad n \to \infty$

then [3.3] $\sum p_m\, |\sigma_m(x) - f(x)|^k \quad = \quad o(R_n) \quad \text{as} \quad n \to \infty$

where $\sigma_n(x)$ is the m^{th} partial sum of the series [1.2].

4. For the proof of the our theorem, the following lemmas are required.

Lemma 1 (Tripathi and Tripathi [3])

$$|\sum \sin mt \,/\, m| = O(1) \qquad \text{for all n and t.}$$

Lemma 2 (Tripathi and Tripathi [3])

$$|\sum \sin mt \,/\, mt| = O(n) \qquad \text{for all } 0 < t \le 1/n .$$

Lemma 3 (Tripathi and Tripathi [3])

(i) $|\sum \in_m p_m \sin mt| = O(1)$ for all n and t.

(i) $|\sum (\in_m p_m \sin mt)/t| = O(n)$ for all $0 < t \le 1/n$.

5 Proof of the Theorem : Let $\sigma_n(x)$ be the n^{th} partial sum of the series [1.2] at the point t = x. Then we have, (Zygmund , (2.52) page 24)

[5.1] $\sigma_n(x) - f(x) \quad = \quad 1/\pi \int \phi(t)\,(\sin nt / t)\, dt + o(1)$

Therefore

$$\sum p_m\, |\sigma_m(x) - f(x)|^k = |\sum p_m \in_m \{\sigma_m(x) - f(x)\}^k|$$

$$= |\sum p_m \in_m \{1/\pi \int \phi(t)\,(\sin mt / t)\, dt + o(1)\}^k|$$

$$= |\sum (p_m \in_m)^{1-k}\{1/\pi \int \phi(t) \sum p_m \in_m (\sin mt / t)\, dt$$

$$+ o(\sum p_m \in_m)\}^k|$$

$$\sum p_m\, |\sigma_m(x) - f(x)|^k = |\sum (p_m \in_m)^{1-k}(1/\pi^k)\{(\int \phi(t) \sum p_m \in_m (\sin mt / t)\, dt$$

$$+ \int \phi(t) \sum p_m \in_m (\sin mt / t)\, dt) + o(\sum p_m \in_m)\}^k|$$

[5.2] $\leq \sum (p_m \in_m)^{1-k}\{ \int |\phi(t)| |\sum p_m \in_m (\sin mt / t)| dt$

$+ \int | \phi(t)/t | | \sum p_m \in_m \sin mt | dt) + o(\sum p_m \in_m)\}^k$

where $\in_n = \pm 1$, so as to make $\in_n \{ \sigma_n(x) - f(x)\} k \geq 0$, for n = 1, 2, 3 ,……….

Now, using particular integration, it can be easily established that if [3.2] holds then,

[5.3] $\int |\phi(t)| du = o(t R_\tau)$ as $t \to 0$.

Using lemma 3 in equation [4.2], we have

[5.4] $\sum p_m |\sigma_m(x) - f(x)|^k = O\{o(R_n)^{1-k}((O(n) \int |\phi(t)| dt + O(1)\int | \phi(t)/t | dt$

$+ o(R_n))^k \}$

using [3.2] and [5.3] in equation [5.4], we have

$$\sum p_m |\sigma_m(x) - f(x)|^k = O\{o(R_n)^{1-k}((O(n)\, o(R_n/n) + O(1)\, o(R_n) + o(R_n))^k \}$$

$$= O\{o(R_n)^{1-k}(o(R_n) + o(R_n) + o(R_n))^k \}$$

$$= O\{o(R_n)^{1-k}\, o(R_n))^k \}$$

$$= O\{o(R_n) \}$$

$$= o(R_n) \qquad \text{as } n \to \infty$$

This complete the proof of theorem .

It may be mentioned that for k = 1, the main theorem of Tripathi and Tripathi [3] becomes a particular case of our theorem which follows under the same conditions as used by them.

Reference:

1 Hardy, G. H. and Littlewood, J. E. : The strong summability of Fourier series : Fund. Math. 25(1935), 162 – 189.

2 Luan, L. S. ; Summability of Fourier series and related series and sequence by logarithmic mean : Nanta Math., 6(1973),

3 Tripathi, L. M. and Tripathi, M. : On strong (N, p_n) summability of Fourier series. Bull. Cal. Math. Soc., 82 (1990), 404 – 408.