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APPLICATION OF EMAD-SARA TRANSFORM FOR SOLVING MATHEMATICAL MODELS OCCURRING IN HEALTH SCIENCE AND BIOTECHNOLOGY

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ABSTRACT

Lot of mathematical models including differential equations play important role in healthcare and biotechnology. One of them is Malthus model. This model was developed by Thomas Malthus, in his essay on world population growth and resource supply. Another interesting equation is Advection diffusion equation and Predator prey model. We use a integral transform called as Emad-Sara transform to obtain the solutions of these models which are important in biotechnology and health sciences.

Key Words: Mathematical Models in health care; Emad-Sara transform; Integral transform; Malthus model; Predator -Prey model; Logistic model; System of differential equations.

1. INTRODUCTION: Mathematical models are of much importance in the optimization of the performance of the biotechnological process. Here we discuss growth law (Malthus equation), logistic model and Predator Prey model. Population is always modeled by growth law or differential equations, for the population u = u(t) of insects in the tube at a time t. For that Malthus model is used, which is

$$\frac{du(t)}{dt} = ru(t).$$

Meaning of this law is growth rate is proportional to current population and the proportionality constant r is intrinsic growth rate. By using variable separable method we can obtain its solution as $u(t) = u(0)e^{rt}$. From this equation we can conclude that the graph of population is increasing exponentially. This model is reasonable in early stage.

But as the resources like food, space and other factors are limited; there is competition for these resources the growth of population does not follow the exponential equation. So the Malthus model is replaced by logistic model.

$$\frac{du(t)}{dt} = ru(t)\left(1 - \frac{u(t)}{K}\right)$$

Where K is carrying capacity, which means that as the population grows and approaches to K then the growth approaches to zero and there is limit to the growth. Serdal Pamuk and Nagihan Saylu [37] used Laplace transform method for logistic growth in a population and predator models. The interaction between predator and prey commonly occurs among the bacterial species and protozoa. Hence the predator prey model is important model.

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms. Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Newton's law of Cooling is solved by using Kushare transform [3], Emad- Falih transform [22], Soham transform [23], HY integral transform[25] In April 2022 D. P. Patil, et al [4] used Kushare transform, HY integral transform[24], Emad Sara transform [28], Alenzi transform [29], Emad Falih transform[30], Kharrat Toma transform [39], KKAT transform[43], ARA transform[47], Ranging integral transform[48] and KKA transform [51] for solving the problems on population growth and decay. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9]. D .P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] solved the system of differential equations using Kushare transform. D. P. Patil [13] used Aboodh and Mahgoub transforms to solve boundary value problems of the system of ordinary differential equations. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems. Laplace, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al obtained solution of Volterra Integral equations of first kind by using, Anuj transform [17], Kushare transform [34]. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. [18] Emad Sara transform[19] and Emad-Falih transform [26] are used for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil et al [27] introduced double kushare transform. Nikam, Patil et al [31] used, Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, et al [36] developed generalized double ranging integral transform. Shinde, et al [37] used Kushare transform is used for solving Volterra Integro-Differential equations of first kind. Patil et al [38] used new general integral transform [49] and Kushare transform [52] to solve Abel's integral equations. Patil et al [40] used Kushare transform for evaluating integrals containing Bessel's functions. Thakare and Patil [41] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [42]. Models in health sciences and biotechnology are solved by using Soham transform [44] and Kushare transform [50]. © 2024 JETIR May 2024, Volume 11, Issue 5 www.jetir.org (ISSN-2349-5162) JETIR2405D34 Journal of Emerging Technologies and Innovative Research (JETIR) www.jetir.org n226 Kushare transform and NE transform is used in mechanics [45], [46]. Dighe et al [54] studies recent developments in integral transforms. Pawar and Patil [55] used NE transform to solve Volterra Integral equations of second kind.

This paper is organized as follows: Introduction is in first section. Second Section is devoted to the useful results and formulae which we are using to solve models. Logistic growth model which is important model in health care sciences is solved in third section. Fourth section is for Predator Prey Model. Applications and results are in fifth section and conclusion is in sixth section.

2. USEFUL RESULTS AND FORMULAE:

In this section we include some required definitions, some useful formulae and theorems based on Emad-Sara transform

In this section we state some preliminary concepts required ie defination , formulae and theorems from Emad-Sara Transform
A.Defination of Emad-Sara Integral Transform :

A={f(t): ∋ M,τ_1 τ_2> 0,$|f(t)|$<$Me^{\tau j \frac{|t|}{\tau}}$,if t ϵ $(-1)^j$X[0,□)}

ES[f(t)] = T(α)=$\frac{1}{\alpha^2}\int_0^{?} e^{-at} f(t)$dt

Where t ≥ 0, $\tau_1 \le \alpha \le \tau_2$ and α is a variable that is used as a factor to the variable in the function f.

B.Emad-Sara Transform of some functions.

In this section we state formula of some elementary function.

Sr.No	Function	Emad-Sara Transform
1	1	$\frac{1}{\alpha^3} = T(\alpha)$
2	t^n	$\frac{n!}{\alpha^n + 3}$
3	e^{at}	$\frac{1}{\alpha^2(\alpha - a)}$
4	$\sin(at)$	$\frac{a}{\alpha^2(\alpha^2 + a^2)}$
5	$\cos(at)$	$\frac{1}{\alpha(\alpha^2 + a^2)}$

Table 1: Emad-Sara Transform of some functions.

C.Emad-Sara Transform of derivative :

If T(α) is Emad-Sara transform of function $f(t)$ then

1. $ES[f'(t)] = \frac{-f(0)}{\alpha^2} + \alpha T(\alpha)$.

2. $ES[f''(t)] = \frac{-f'(0)}{\alpha^2} + \alpha\, ES[f'(t)]$.

3. $ES[f^{(n)}(t)] = \frac{-f^{(n-1)}(0)}{\alpha^2} + \alpha\, ES[f^{(n-1)}(t)]$

1 EMAD-SARA TRANSFORM FOR LOGISTIC GROWTH MODEL

Consider the Logistic growth model equation

$$\frac{du}{dt} = u - f(u), u(0) = u_0 \quad \ldots\ldots\ldots\ldots\ldots (3.1)$$

Here f is nonlinear function of u. Suppose that solution u of equation (3.1) is of the infinite power series as follows,

$$u = u(t) = \sum_{n=0}^{\infty} a_n t^n \quad \ldots\ldots\ldots\ldots (3.2)$$

Further (3.2) also satisfies the conditions for the existence of Soham transform.

Applying Emad-Sara transform on the both sides of the equation (3.1) we get

$$s\left(\frac{du}{dt}\right) = ES(u) - ESf(u)$$

$$\frac{u(0)}{\alpha^2} + \alpha T(\alpha) = T(\alpha) - F(\alpha) \ldots\ldots\ldots\ldots (3.3)$$

Where ,$T(\alpha) = ES(u(t))$ and $f(\alpha) = ES(f(u))$ are the Emad -Sara transform of the function u(t) and f(u) respectively.

Rearranging the terms in equations 3.3 we get,

$$\alpha T(\alpha) - T(\alpha) = -\frac{u(0)}{\alpha^2} - F(\alpha)$$

$$(\alpha - 1)T(\alpha) = -\frac{u(0)}{\alpha^2} - F(\alpha)$$

$$T(\alpha) = -\frac{u_0}{\alpha^2(\alpha - 1)} - \frac{F(\alpha)}{(\alpha - 1)} \ldots\ldots (3.4)$$

If we suppose $(u) = u^2$ then

$$f(u) = \left(\sum_0^\infty a_n t^n\right)^2$$

$$f(u) = (a_0 + a_1 t + a_2 t^2 + \cdots + a_n t^n)^2$$

$$f(u) = a_0^2 + 2a_0 a_1 t + (a_0 a_2 + a_0^2)t^2 + (2a_0 a_3 + 2a_1 a_2)t^3 + \cdots \dots \dots \dots (3.5)$$

Taking Emad-Sara transform on both side of equation 3.3,

$$f(\alpha) = \frac{a_0^2}{\propto^3} + \frac{2a_0 a_1}{\alpha^4} + (2a_0 a_2 + a_0^2)\frac{2!}{\alpha^5} + (2a_0 a_3 + 2a_1 a_3)\frac{3!}{\alpha^6} + \cdots \dots ..$$

$$\frac{f(\alpha)}{(\alpha-1)} = \frac{a_0^2}{\alpha^3(\alpha-1)} + \frac{2a_0 a_1}{\alpha^4(\alpha-1)} + (2a_0 a_1 + a_1^2)\frac{2!}{\alpha^5(\alpha-1)} + (2a_0 a_3 + 2a_1 a_3)\frac{3!}{\alpha^6(\alpha-1)} + \cdots$$

Applying the method of partial fractions to the terms in R.H.S of the above equation,

$$\therefore \frac{f(\alpha)}{(\alpha-1)} = \frac{1}{\alpha^2(\alpha-1)}[a_0^2 + 2a_0 a_1 + 4a_0 a_2 + 2a_1^2 + 12a_0 a_3 + 12a_1 a_3 \dots] - [a_0^2 + 2a_0 a_1 + 4a_0 a_2 + 2a_1^2 + 12a_0 a_3 + 12a_1 a_3 \dots .] - t[2a_0 a_1 + 4a_0 a_2 + 2a_1^2 + 12a_0 a_3 + 12a_1 a_3 \dots .] - \frac{t^2}{2}[4a_0 a_2 + 2a_1^2 + 12a_0 a_3 + 12a_1 a_3 \dots .] - \frac{t^3}{6}[12a_0 a_3 + 12a_1 a_3 \dots ..] - \cdots \dots \dots$$

Applying inverse Emad-Sara transform to both sides of the above equation and rearranging the term we get,

$$u(t) = u_0\left(1 + t + \frac{t^2}{2} + \frac{t^3}{6} + \cdots\right) - \left[a_0^2 t + \left(\frac{a_0^2}{2} + a_0 a_1\right)t^2 + \left(\frac{a_0^2}{6} + \frac{a_0 a_1}{3} + \frac{2a_0 a_2}{3} + \frac{a_1^2}{3}\right)t^3\right] + \cdots \dots \dots$$

$$\therefore u(t) = u_0 + (u_0 - a_0^2)t + \left(\frac{u_0}{2} - \frac{a_0^2}{2} - a_0 a_1\right)t^2 + \left(\frac{u_0}{6} - \frac{a_0^2}{6} - \frac{a_0 a_1}{3} - \frac{2a_0 a_2}{3} + \frac{a_1^2}{3}\right)t^3$$

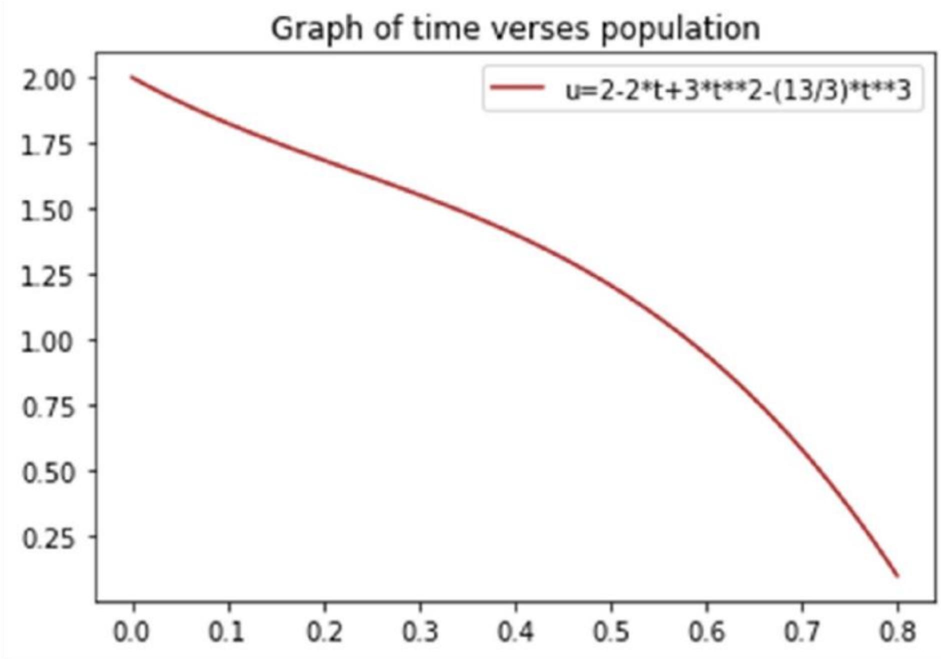
If we consider $u_0 = 2$ and compare with 3.2 we obtain

$$a_0 = 2, a_1 = -2, a_2 = 3, a_3 = \frac{-13}{3} \dots ..$$

Therefore ,

$$u(t) = 2 - 2t + 3t^2 + \frac{-13}{3}t^3 \dots ..$$

It is required solution and the graph of this solution is



This figure 1 is graph showing that how population of a species changes when a hazard function is acting in life of the species.

From graph we can conclude that, if there is more competition in life and more hazards then the population decreases. Here f(u) that is hazard function is taken as square of the population that means more hazard, so population of the insect reaches to zero in less than one unit interval of time.

4. Emad-Sara transform for predator prey model:

In this section we use Emad-Sara transform for predator Prey model.

The interaction between two species and their effect on each other is called as predator prey relationship. In this one species is feeding on the other species. An organism that eats or hunts other organism as food is called as predator and an organism that is killed by other organism for food is called as prey. Fox and rabbit, lion and zebra are examples of predator and prey. This concept of predator prey is not only applicable for animals but it is applicable for plants also. Grasshopper and leaf is an example of this.

Consider the system of differential equations governing predator prey model,

$$\frac{du}{dt} = u - f(u, v) \text{.....(4.1)}$$

$$\frac{du}{dt} = \beta[g(u, v) - v] \text{......(4.2)}$$

with initial conditions $u(0) = u_0$ and $v(0) = v_0$, f and g are nonlinear functions of u and v. β is a positive constant. Let u and v. be the solutions of this system, which are infinite series of the form

$$u(t) = \sum_{n=0}^{\infty} a_n t^n$$
$$v(t) = \sum_{n=0}^{\infty} b_n t^n$$

And they both also satisfy the required conditions for existence of Emad-Sara transform.

Applying Emad -Sara transforms to both sides of the equations 4.1 and 4.2

$$ES\left(\frac{du}{dt}\right) = ES(u) - ES[f(u,v)]$$

$$ES\left(\frac{dv}{dt}\right) = \beta[ES[g(u,v)] - ES(v)]$$

Using the Emad -Sara transform of derivative theorem ,

$$\frac{-1}{\alpha^2}\big(U(0)\big) + \alpha U(\alpha) = U(\alpha) - F(\alpha)$$

$$\frac{-1}{\alpha^2}\big(V(0)\big) + \alpha V(\alpha) = \beta G(\alpha) - \beta V(\alpha)$$

Where $P(u) = U(\alpha), P(v) = V(\alpha)$, $ES\big[f\big(u(t), v(t)\big)\big] = F(\alpha)\ and$

$$ES\big[g\big(u(t), v(t)\big)\big] = G(\alpha)$$

Rearranging the terms and simplifying we get ,

$$U(\alpha) = u_0 \frac{1}{\alpha^2(\alpha - 1)} - \frac{F(\alpha)}{(\alpha - 1)}$$

And

$$V(\alpha) = v_0 \frac{1}{\alpha^2(\alpha + \beta)} + \frac{\beta G(\alpha)}{(\alpha + \beta)}$$

Applying inverse Emad -Sara transform

$\mathrm{u}(t) = u_0 e^t - ES^{-1}\left(\frac{F(\alpha)}{(\alpha-1)}\right)$…..(4.3)

$v(t) = v_0 e^{-\beta t} + \beta ES^{-1}\left(\frac{G(\alpha)}{(\alpha+\beta)}\right)$……(4.4)

These equations (4.3) and (4.4) represent the solution of the system of equations (4.1) and (4.2)

5. Applications and results: In this section we use results in above section to solve some systems of differential equations arising in biotechnology and health of sciences.

EXAMPLE: 1. Consider the system of differential equations governing predator and prey model.

$$\frac{du}{dt} = u - uv \ldots\ldots\ldots (5.1)$$

$$\frac{dv}{dt} = uv - v \ldots\ldots\ldots (5.2)$$

With the initial conditions $u(0) = 1.3, v(0) = 0.6$

Suppose $u = \sum_0^\infty a_n t^n$, $v = \sum_0^\infty b_n t^n$

Be solution of the system of equations(5.1)and (5.2).

$$\therefore uv = a_0 b_o + (a_0 b_1 + a_1 b_0)t + (a_0 b_2 + a_1 b_1 + a_2 b_0)t^2 + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)t^3 + \cdots.$$

Applying Emad-Sara transform,

$$P(u,v) = a_0 b_o ES(1) + (a_0 b_1 + a_1 b_0) ES(t) + (a_0 b_2 + a_1 b_1 + a_2 b_0) ES(t^2) + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0) ES(t^3) + \cdots.$$

$$\therefore P(u,v) = a_0 b_o \left(\frac{1}{\alpha^3}\right) + (a_0 b_1 + a_1 b_0)\left(\frac{1!}{\alpha^4}\right) + (a_0 b_2 + a_1 b_1 + a_2 b_0)\left(\frac{2!}{\alpha^5}\right) + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)\left(\frac{3!}{\alpha^6}\right) + \cdots.$$

$$P(u,v) = a_0 b_o \left(\frac{1}{\alpha^3}\right) + (a_0 b_1 + a_1 b_0)\left(\frac{1}{\alpha^4}\right) + (a_0 b_2 + a_1 b_1 + a_2 b_0)\left(\frac{2}{\alpha^5}\right) + (a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)\left(\frac{6}{\alpha^6}\right) + \cdots.$$

Suppose, $P(u,v) = F(\alpha) = G(\alpha)$

By previous section we have, $P(v) = u_0 \frac{1}{\alpha^2(\alpha-1)} - \frac{F(\alpha)}{(\alpha-1)}$

$$\therefore P(v) = 1.3 \frac{1}{\alpha^2(\alpha-1)} - \left[\frac{a_0 b_0}{\alpha^3(\alpha-1)} + \frac{(a_0 b_1 + a_1 b_0)}{\alpha^4(\alpha-1)} + 2\left(\frac{(a_0 b_2 + a_1 b_1 + a_2 b_0)}{\alpha^5(\alpha-1)}\right) + 6\left(\frac{(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)}{\alpha^6(\alpha-1)}\right) + \cdots.\right]$$

Rearranging the terms

$$P(v) = 1.3 \frac{1}{\alpha^2(\alpha-1)} - \left[\frac{a_0 b_0 \alpha^3}{\alpha^3(\alpha-1)\alpha^3} + \frac{(a_0 b_1 + a_1 b_0)\alpha^4}{\alpha^4(\alpha-1)\alpha^4} + 2\left(\frac{(a_0 b_2 + a_1 b_1 + a_2 b_0)\alpha^5}{\alpha^5(\alpha-1)\alpha^5}\right) + 6\left(\frac{(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)\alpha^6}{\alpha^6(\alpha-1)\alpha^6}\right) + \cdots.\right]$$

Applying partial and rearranging terms

$$P(v) = 1.3 \frac{1}{\alpha^2(\alpha-1)} - \left[a_0 b_0 \left(\frac{1}{\alpha^2(\alpha-1)} - 1\right) + (a_0 b_1 + a_1 b_0)\left(-1 - \frac{1}{\alpha^3} + \frac{1}{\alpha^2(\alpha-1)}\right) + 2(a_0 b_2 + a_1 b_1 + a_2 b_0)\left(-1 - \frac{1}{\alpha^3} - \frac{1}{\alpha^4} + \frac{1}{\alpha^2(\alpha-1)}\right) + 6(a_0 b_3 + a_1 b_2 + a_2 b_1 + a_3 b_0)\left(-1 - \frac{1}{\alpha^3} - \frac{1}{\alpha^4} - \frac{1}{\alpha^5} + \frac{1}{\alpha^2(\alpha-1)}\right) \ldots\right]$$

Applying inverse Emad -Sara transform ,

$$u(t) = 1.3 + (1.3 - a_0 b_0)t + (1.3 - a_0 b_0 - a_0 b_1 - a_1 b_0)\frac{t^2}{2} + (1.3 - a_0 b_0 - a_0 b_1 - a_1 b_0 - a_0 b_2 + a_1 b_1 + a_2 b_0)\frac{t^3}{6} + \cdots \ldots (5.3)$$

Similarly we can obtain,

$v(t) = 0.6 + (0.6 - a_0b_0)t + (0.6 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \cdots \ldots (5.4)$

From equation (5.3)

$$\sum_{n=0}^{\infty} a_n t^n = 1.3 + (1.3 - a_0b_0)t + (1.3 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \cdots$$

Hence, $a_0 = 1.3$, $a_1 = (1.3 - a_0b_0)$, $a_2 = (1.3 - a_0b_0 - a_0b_1 - a_1b_0)$, $a_3 = (1.3 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)$, … … … …

From equation(5.4)

$$\sum_{n=0}^{\infty} b_n t^n = 0.6 + (0.6 - a_0b_0)t + (0.6 - a_0b_0 - a_0b_1 - a_1b_0)\frac{t^2}{2} + (0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)\frac{t^3}{6} + \cdots$$

Hence $b_0 = 0.6$, $b_1 = (a_0b_0 - 0.6)$, $a_2 = (0.6 - a_0b_0 - a_0b_1 - a_1b_0)$, $b_3 = (0.6 - a_0b_0 - a_0b_1 - a_1b_0 - a_0b_2 + a_1b_1 + a_2b_0)$, … … … …

Obtaining the values of $a_0, a_1, a_2, a_3, \ldots\ldots\ldots, b_0, b_1, b_2, b_3 \ldots\ldots\ldots\ldots,$

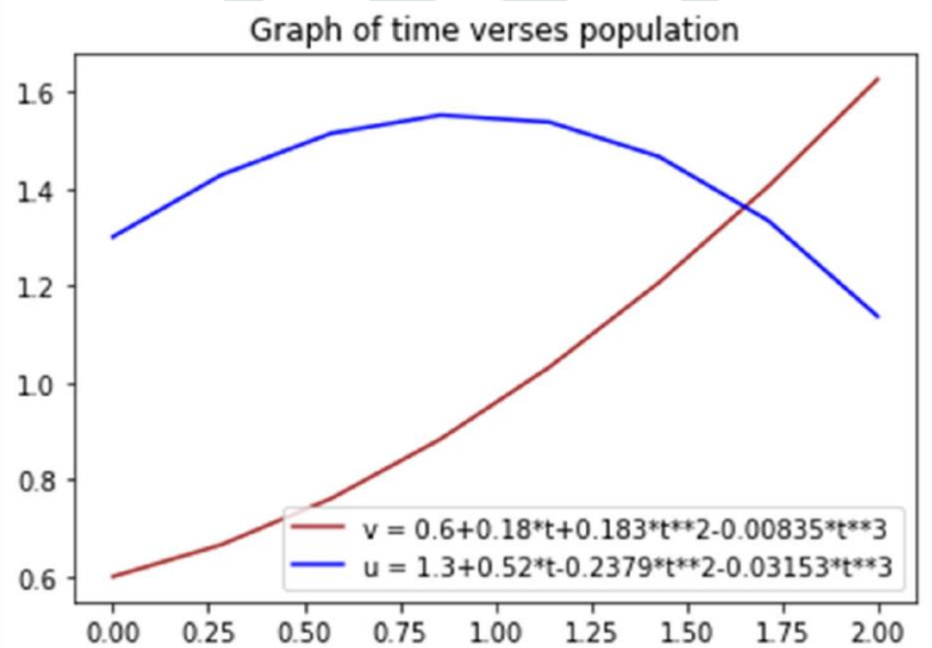
We get required solution of the system of equations

$$u(t) = 1.3 + 0.52t - 0.2379t^2 - 0.03153t^3 + \cdots \ldots\ldots$$

And

$$v(t) = 0.6 + 0.18t + 0.183t^2 - 0.00835t^3 + \cdots \ldots\ldots$$

Graph of solution of the system 5.1 and 5.2 with given initial conditions is,



This figure 2 is graph showing effect of predators on preys.

From this graph we can conclude that the number of predators and prey is maintained (conserved).in some limit. That means if number of preys increases then the number of predators will also increases due to increase in food supply. Increase in the predators consumes more food. It results reduction in food supply means number of preys reduces. A time comes when the number of predators and prey becomes equal. Then increase in predator results decrease in prey. Hence there is shortage of food for predators. Thus the chain is continued and number of predators and prey always remains in some specific limit.

6 CONCLUSION: By using Emad-Sara transform we can easily solve the mathematical models in biochemistry, health sciences and environmental sciences, containing ordinary differential equations.

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