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On A Group Concerning Spin Of The Electron

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Abstract:- In this paper we obtained a non-abelian group of sixteen order which is related to the spin of electron. The matricial representation of the elements of the group are given. The main intention of this paper is to stress the fact that the origin of the complex numbers as operators of the special type on R^2 should be kept in minds by mathematical community.

Key Words:- Non-abelian group, Spin of electron, operators

Introduction: Any group comes out as an extension of the trivial group consisting of identity only. For example if $G_0 = \{1\}$ then a multiplicative group we obtained $G_1 = \{1,-1\}$ by adjoining -1 to G_0, $G_2 = \{1,-1,i,-i\}$ by adjoining i. Similarly we extend the smaller groups to groups of higher orders. In this paper we have obtained a group G of sixteen order by adjoining the components of spinner angular momentum of electron or any particle of half spin. The matricial representation of this group in the Hilbert space R^4 over the field R has been given.

Observations and Results: We have assumed that the elements of our group G are linear operators on R^4 and their matricial representation are as follows:

$$(1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; \quad (-1) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$(i) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} ; \quad (-i) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$(\sigma_1) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} ; \quad (-\sigma_1) = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

$$(i\sigma_1) = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} ; \quad (-i\sigma_1) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$$

$$(\sigma_2)=\begin{bmatrix}0&0&0&1\\0&0&-1&0\\0&-1&0&0\\1&0&0&0\end{bmatrix};\quad (-\sigma_2)=\begin{bmatrix}0&0&0&-1\\0&0&1&0\\0&1&0&0\\-1&0&0&0\end{bmatrix}$$

$$(\imath\,\sigma_2)=\begin{bmatrix}0&0&1&0\\0&0&0&1\\-1&0&0&0\\0&-1&0&0\end{bmatrix};\quad (-\imath\sigma_2)=\begin{bmatrix}0&0&-1&0\\0&0&0&-1\\1&0&0&0\\0&1&0&0\end{bmatrix}$$

$$(\sigma_3)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix};\quad (-\sigma_3)=\begin{bmatrix}-1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$(\imath\,\sigma_3)=\begin{bmatrix}0&-1&0&0\\1&0&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix};\quad (-\imath\sigma_3)=\begin{bmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&-1\\0&0&1&0\end{bmatrix}$$

The set G of these sixteen matrices forms a non- abelian group which consists of the quantities useful in the studies of spin of the electrons. These matrices having the real coefficients are expected to be important for computational calculations. All of them are orthogonal matrices and representing operators on R^4. We are motivated from the book [2] of Professor P.A.M. Dirac. To represent the spin angular momentum, he has used ί as coefficients that is he has represented the operators concerned on C^2 over the field C. we feel that for computational point of view ί is troublesome quantity so its use should be avoided. To remove ί, we have used the concept given in Copson [1] that ί is in fact a special type operator on R^2 whose matrix representation is

$$\imath=\begin{bmatrix}0&-1\\1&0\end{bmatrix}$$

We may verified that square of this matrix is $\left[\begin{bmatrix}-1&0\\0&-1\end{bmatrix}\right]$ ie -1. Thus we get the set of complex numbers as sub space of dimension two spanned by the matrices $\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and $\begin{bmatrix}0&-1\\1&0\end{bmatrix}$

We write, $x+iy = x\begin{bmatrix}1&0\\0&1\end{bmatrix}+y\begin{bmatrix}0&-1\\1&0\end{bmatrix}=\begin{bmatrix}x&-y\\y&x\end{bmatrix}$

Thus complex numbers are special type of linear operators obtained on R^2 whereas the general linear operator on R^2 has the matrix of the form $\left[\begin{bmatrix}a_{11}&a_{12}\\a_{21}&a_{22}\end{bmatrix}\right]$

These general type of 2x2 matrices for linear space of dimension 4 over the filed are, the standard base being the set of matrices

$$\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&1\\0&0\end{bmatrix},\begin{bmatrix}0&0\\1&0\end{bmatrix}\text{ and }\begin{bmatrix}0&0\\0&1\end{bmatrix}$$

N = { 1, -1, ί - ί } is centre of the group G, the quotient group G/N = { N, $N\sigma_1$, $N\sigma_2$, $N\sigma_3$ }

Explicitly we may write -

N = { 1, -1, ί - ί }

$N\sigma_1$ = { σ_1, -σ_1, ίσ_1 - ίσ_1 }

$N\sigma_2$ = { σ_2, -σ_2, ίσ_2 - ίσ_2 }

$N\sigma_3$ = { σ_3, -σ_3, ίσ_3 - ίσ_3 }

We have to keep in our mind that σ_1, σ_2, σ_3 are quantities which anti- commute with each other and ${\sigma_1}^2 = {\sigma_2}^2 = {\sigma_3}^2$ = 1. Also we find that

$\sigma_1\sigma_2 = i\,\sigma_3$, $\sigma_2\,\sigma_3 = i\,\sigma_1$, $\sigma_3\sigma_1 = i\,\sigma_2$ and $\sigma_1\sigma_2\,\sigma_3 = i$

Now we give the group composition table below:

	1	-1	i	$-i$	σ_1	$-\sigma_1$	$i\sigma_1$	$-i\sigma_1$	σ_2	$-\sigma_2$	$i\sigma_2$	$-i\sigma_2$	σ_3	$-\sigma_3$	$i\sigma_3$	$-i\sigma_3$
1	1	-1	i	$-i$	σ_1	$-\sigma_1$	$i\sigma_1$	$-i\sigma_1$	σ_2	$-\sigma_2$	$i\sigma_2$	$-i\sigma_2$	σ_3	$-\sigma_3$	$i\sigma_3$	$-i\sigma_3$
-1	-1	1	$-i$	i	$-\sigma_1$	σ_1	$-i\sigma_1$	$i\sigma_1$	$-\sigma_2$	σ_2	$-i\sigma_2$	$i\sigma_2$	$-\sigma_3$	σ_3	$-i\sigma_3$	$i\sigma_3$
i	i	$-i$	-1	1	$i\sigma_1$	$-i\sigma_1$	$-\sigma_1$	σ_1	$i\sigma_2$	$-i\sigma_2$	$-\sigma_2$	σ_2	$-i\sigma_3$	$i\sigma_3$	$-\sigma_3$	σ_3
$-i$	$-i$	-1	$-i$	$-i$	$-i\sigma_1$	$i\sigma_1$	σ_1	$-\sigma_1$	$-i\sigma_2$	$i\sigma_2$	σ_2	$-\sigma_2$	$i\sigma_3$	$-i\sigma_3$	σ_3	$-\sigma_3$
σ_1	σ_1	$-\sigma_1$	$i\sigma_1$	$-i\sigma_1$	1	-1	i	$-i$	$i\sigma_3$	$-i\sigma_3$	$-\sigma_3$	σ_3	$-i\sigma_2$	$i\sigma_2$	σ_2	$-\sigma_2$
$-\sigma_1$	$-\sigma_1$	σ_1	$-i\sigma_1$	$i\sigma_1$	-1	1	$-i$	i	$-i\sigma_3$	$i\sigma_3$	σ_3	$-\sigma_3$	$i\sigma_2$	$-i\sigma_2$	$-\sigma_2$	σ_2
$i\sigma_1$	$i\sigma_1$	$-i\sigma_1$	$-\sigma_1$	σ_1	i	$-i$	-1	1	$-\sigma_3$	σ_3	$-i\sigma_3$	$i\sigma_3$	σ_2	$-\sigma_2$	$i\sigma_2$	$-i\sigma_2$
$-i\sigma_1$	$-i\sigma_1$	$i\sigma_1$	σ_1	$-\sigma_1$	$-i$	i	1	-1	σ_3	$-\sigma_3$	$i\sigma_3$	$-i\sigma_3$	$-\sigma_2$	σ_2	$-i\sigma_2$	$i\sigma_2$
σ_2	σ_2	$-\sigma_2$	$i\sigma_2$	$-i\sigma_2$	$-i\sigma_3$	$i\sigma_3$	σ_3	$-\sigma_3$	1	-1	i	$-i$	$i\sigma_1$	$-i\sigma_1$	$-\sigma_1$	σ_1
$-\sigma_2$	$-\sigma_2$	σ_2	$-i\sigma_2$	$i\sigma_2$	$i\sigma_3$	$-i\sigma_3$	$-\sigma_3$	σ_3	-1	1	$-i$	i	$-i\sigma_1$	$i\sigma_1$	σ_1	$-\sigma_1$
$i\sigma_2$	$i\sigma_2$	$-i\sigma_2$	$-\sigma_2$	σ_2	σ_3	$-\sigma_3$	$i\sigma_3$	$-i\sigma_3$	i	$-i$	-1	1	$-\sigma_1$	σ_1	$-i\sigma_1$	$i\sigma_1$
$-i\sigma_2$	$-i\sigma_2$	$i\sigma_2$	σ_2	$-\sigma_2$	$-\sigma_3$	σ_3	$-i\sigma_3$	$i\sigma_3$	$-i$	i	1	-1	σ_1	$-\sigma_1$	$i\sigma_1$	$-i\sigma_1$
σ_3	σ_3	$-\sigma_3$	$i\sigma_3$	$-i\sigma_3$	$i\sigma_2$	$-i\sigma_2$	$-\sigma_2$	σ_2	$-i\sigma_1$	$i\sigma_1$	σ_1	$-\sigma_1$	1	-1	i	$-i$
$-\sigma_3$	$-\sigma_3$	σ_3	$-i\sigma_3$	$i\sigma_3$	$-i\sigma_2$	$i\sigma_2$	σ_2	$-\sigma_2$	$i\sigma_1$	$-i\sigma_1$	$-\sigma_1$	σ_1	-1	1	$-i$	i
$i\sigma_3$	$i\sigma_3$	$-i\sigma_3$	$-\sigma_3$	σ_3	$-\sigma_2$	σ_2	$-i\sigma_2$	$i\sigma_2$	σ_1	$-\sigma_1$	$i\sigma_1$	$-i\sigma_1$	i	$-i$	-1	1
$-i\sigma_3$	$-i\sigma_3$	$i\sigma_3$	σ_3	$-\sigma_3$	σ_2	$-\sigma_2$	$i\sigma_2$	$-i\sigma_2$	$-\sigma_1$	σ_1	$-i\sigma_1$	$i\sigma_1$	$-i$	i	1	-1

This composition table can be verified by using corresponding matrices. Any computer programme for matrices multiplication can be used.

Results: We may avoid the complex representation by the real representations and find the result using computational programs.

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References-

[1] Copson E.T.(1974) An Introduction to the “Theory of Functions Of A Complex Variable.” (pp 2 to 5)

[2] Dirac P.A.M.(1973) “The Principles Of Quantum Mechanics (pp149 to 152)