JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

ON CERTAIN CLASSES OF MULTIVALENT MEROMORPHIC FUNCTIONS

[1]P.Priyanka, [2]V.Rathika, [3]P.Tamilselvi ,[4]N.A.Revathi

[1]Head & Assistant Professor, [2]Assistant Professor, [3]Assistant Professor,[4]Assistant Professor
[1]Department of Mathematicsr,
[1]Adhiyaman arts & science college for women, Krishnagiri, India

ABSTRACT

We drive properties of the certain class of meromorphic function and merorphic multivalent function. We defined analytic function, pole, differential equation, meromorphic function open disk unit disk and convex. To derive several properties of a certain class of multivalent function in the open unit disk. One of our main theorems unifies and extends several earlier result in the theory of analytic functions. We introduce the class $\mathscr{B}(p,n,\mu,\alpha)$ of analytic and p-valent functions to obtain some sufficient conditions and some angular properties for functions belonging to this class. Two new classes of meromorphic multivalent functions. Such results as subordination properties, coefficient inequalities, convolution properties and integral representations are proved. Several sufficient conditions for meromorphic multivalent starlikeness and convexity are also derive

INTRODUCTION

Let (p,n) be the class of functions f(z)of the form:

$$f(z)=z^p+\sum_{k=p+n}^{\infty} a_k z^k,(p,n\in{:=}*1,2,3,\ldots.+),$$

Which are analytic in the open unit disk $u = *z{:}z\epsilon\ \mathbb{C} and\ |z| < 1+$.A function f(z) in $A(p,n)$ is said to be in the class $A(p,n,\alpha)$ if it satisfies the inequality:

$$\Re 2\,1+\frac{zf''(z)}{f'(z)}\,3<\alpha,(z\in U;\alpha>p)$$

.for example of various other classes of univalent and multivalentfunctions.

For p=1andn=1,thefollowngresultareknown. Let p denote the class of functions of the form

$$p(z)=1+\sum_{n=1}^{\infty} p_n z^n,$$

Which are analytic and convex in$\mathbb{U}$and satisfy the condition

$$\mathfrak{N}(p(z))>0 \qquad (z\epsilon\mathbb{U}).$$

A function $f(z)\in A(p,n)$ is said to be in the class $s^*(p,n,\alpha)$ of p-valenty starlike of order α in $\mathcal{U}$ if and only if it satisfies the inequality

$$Re\left\{\frac{zf'(z)}{f(z)}\right\}>\alpha, \qquad z\in\mathcal{U};0\leq\alpha<p).$$

On the other hand, a function $f(z)\in A(p,n)$ is said to be in the class $\mathcal{K}(p,n,\alpha)$ of p-valenty convex of order α in $\mathcal{U}$ if and only if it satisfies the inequality

$$Re\left\{1+\frac{zf''(z)}{f(z)}\right\}>\alpha,(z\epsilon\mathcal{U};\leq\alpha<p$$

A function $f(z)\epsilon A(p,n)$ is the said to be in the class $C(p,n,\alpha)$ of p-valenty close-to convex of of order α in $\mathcal{U}$ if and only if it satisfies the inequality

$$Re\left(\frac{f'(z)}{z^{p-1}}\right)>\alpha,(z\in\mathcal{U};0\leq\alpha<P).$$

In particular, we write $s^*(1,1,0)=:s^*, K(1,1,0)=:K$ and $C(1,1,0)=:C$, where s^*, K and C are the usual subclasses of A consisting of functions which are starlike, convex and close-to- convex, respectively.

To investigate various properties of the following classes of analytic and p-valent functions defined as follows.

PRELIMINARIES:

In the several complex analysis, a meromorphic function on an open subset D of the complex plane is a function that is holomorphic on all of D except for a set of isolated points , which are poles of the function. Every meromorphic function on D can be expressed as the ratio between two holomorphic function defined on D, any pole must coincide with a zero of the denominators.

MEROMORPHIC FUNCTIONS

Pole Expansion

If a function f[z] only has isolated singularities, it is described as meromorphic.

For simplicity suppose that these singularities are simple pole at z_n where the index lists the poles in order of increasing distance from the origin.

Theorem

Mittag– Leffler theorem:

Suppose that the function f[z] is analytic everywhere except for isolated simple poles, is analytic at the origin, and that there exists a sequence of circles $\{c_k:|z|=R_k, k=1,n\}$ where each c_k encloses k poles within radius R_k.

Furthermore, assume that on these circles $|f|$ is bounded as $R_n\to\infty$. The function can then be expanded in the form

$$f[z]=f[0]+\sum_{k=1}^{\infty}\left(\frac{b_n}{z-z_n}+\frac{b_k}{z_n}\right)$$

Where$b_n$istheresidueforpolez_n.

UnlikeLaurentexpansionsforwhichthechoiceof $z_0$canbesomewhatarbitrary,thepole expansion for meromorphic functions depends only upon intrinsic properties of the function itself.

Although the present version places significant restrictions on the function, generalizations can often be made fairly easily.

For example, if f[z] has pole at the origin, one can apply the theorem to the closelyrelated function g[z] $= f[z + z_0]$ where $z_0$is any convenient point where f[z] is analytic.

Similarly.if$M_n \propto R_n^{m-1}$forlargeR_n.

Onecanemployan expansion of theform

$$f[z]=\sum_{k=1}^{m} f^{(k)}[0]\frac{z^k}{k!}+\sum_{n=1}^{\infty}\frac{b_n}{z-z_n}\left(\frac{z}{z_n}\right)^{m+1}$$

Polesofhigherordercan beaccommodatedalso,butweforegodetailedanalysisher.

Pole expansion appear in many branches of physics. If f[z] response of a dynamical system to some driving force, the poles generally represent resonances or normal modes of vibration while the residues represent resonances or normal modes. pole expansions can also be used to sum infinite series.

PROPERTIESOFMEROMORPHICMULTIVALENTFUNCTION

Let$A[a,n]$bethesubclassofanalyticfunction$g(w)$ in $\mathbb{U}=\{w\epsilon\mathbb{C}:|w|<1\}$ofthe following form: $g(w)=a=+a_n w^n+a_{n+1}w^{n+1}+\cdots(a\in\mathbb{C};w\epsilon\mathbb{U})$.

Furthermore,let$m_p$denotedthe classof allanalyticfunction$g(w)$of thefollowing form:

$$g(w)=w^{-p}+\sum_{k=1}^{\infty} a_k w^k (p\epsilon=\{1,2.3,..\}),$$

Whicharemeromorphicp-valentinthepunctureddisc$\mathbb{U}^*=\mathbb{U}\{0\}$.If$g_1(w)andg_2(w)$are analyticin$\mathbb{U}$,wesaythat$g_1(w)$issubordinateto$g_2(w)org_2(w)$issuperordinateto $g_1(w)$,writtenas$g_1(w)<g_2(w)$,iftheexistsananalyticfunctionv(w)in$\mathbb{U}$withv(0)=0and $|v(w)|<1(w\epsilon\mathbb{U})$suchthat

$$g_1(w)=g_2(v(w))(w\epsilon\mathbb{U}).$$

In particular,if$g_2(w)$is a univalentfunctionin$\mathbb{U}$,wehave thefollowingequalence

$$g_1(w)<g_2(w)\Leftrightarrow g_1(0)=g_2(0)and g_1(\mathbb{U})\subset g_2(\mathbb{U}).$$

Manysubclassesofmeromorphicallymultivalentfunctionshavebeenintroduced and investigatedbyseveralearlierauthors.Now,weintroduceacertainclass meromorphic multivalent functions by using the principle of subordination.

REFERENCES

1. Djiok. J; "Classes of meromorphic function defined by the hadamard product", International Journal of mathematics and mathematical sciences, volume 2010, Article ID 302583, 11pages.
2. Kulkarni.S.RandJoshi.S.S.;"On a subclass of meromorphic univalent functions with positive coefficients" , The journal of the Indian Academy of Mathematics, Vol 24, no. 1, pp.197-205,2002.
3. SilvermanH; "Univalent functions with varying arguments", Houston journal of Mathematics, volume 7, No.2, pp.283-287 , 1981.
4. Bulboca,T. Differential subordination and super ordinations; Recent Results ; House of science Book Publication: Cluj-Napoca, Romania,2005.
5. Miller, SS.,Mocanu, P.T. Differential subordinations: Theory and Applications, series on Monographs andTexbooks in pureand AppliedMathematics;MarcelDekker. New york, NY, USA; Basel , Switzerland, 2000; volume 225.
6. Khan, S.; Hussain ,S.;Darus, M. Certain subclasses of meromorphic multivalent q-starlike and q-convex functions Math. Slovaca 2022, 72, 635-646. [crossRef]
7. Liu, J-L. A note on meromorphically multivalent functions with missing coefficients. Math. Slovaca 2015, 63,77-82[crossRef]
8. Seoudy, TM.; Aouf, M.K. A class of p-valent meromorphic functions defined by the Lui- srivastava operator. UKr.Math. J.2015, 66, 1383-1392.
9. Srivastava, H.M,; Patel, J. Applications of differential subordination to certain subclasses of meromorphically multivalent functions. J.Inequal. Pure Appl. Math. 2005,6,1-17.
10. Xu, Y-H; Yang, Q; Liu J.-L. Some properties of meromorphically multivalent functions. J.inequal. Appl.2012, 86, 1-6.
11. Miller, SS.;Mocanu, P.T. subordinates of differential superordinates. Complex var. 2003, 48, 815-826.
12. Hallenbeck, D.J.; Ruscheweyh, St. subordination by convex functions. Proc. Amer. Math. Soc.1975, 52, 191-195.
13. Whittaker, E.T,; Watson, G.N. A course on Modern Analysis: An introduction to the General Theory of infinite Processes and of Analytic functions; With an Account ofthe principal Transcendental functions, 4th ed.; Cambridge university press: Cambridge, Uk, 1927.
14. Shanmugam, T.N,; Ravichandran, V.; Sivasubramanian, s. Differential sandwichtheorems for subclasses of subclasses of analytic functions. Australian J.Math. Anal. Appl.2006, 3, 1-11.
15. Ma,W.;Minda,D.A unified treatment of some special classes of univalent functions. In proceeding of the conference on complex Analysis; International press Inc .: Cambridge ,MA,USA,1994;pp.157-169.
16. Kumar V.;shukla, S.L. Certain integralsfor classesof p-valent meromorphic function. Bull Austral .Math Soc. 1982,25, 85-97.
17. Bernardi,SD.convexandstarlike univalentfunctions.Trans.AM.Math.Soc.1969,135
18. Kumar V.; shukla, SL. Certain integrals for classes of p-valent meromorphic functions. Bull. Austral. Math. Soc 1982 ,25,85-97.