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New Neighbourhood Properties in Topological spaces

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ABSTRACT: In this paper a new class of sets called Semi Pre Generalized open sets (briefly Spg open sets) and Spg neighbourhoods are introduced and studied in topological spaces and some properties of new concepts have been studied.

Keywords: Spg closed sets, Spg open sets, Spg -neighbourhoods.

AMS Mathematics subject classification (2010): 54A05.

1. INTRODUCTION

Stone[1] introduced and studied Regular open sets then Regular semi open sets, Pre-open sets, gspr closed sets, gpr closed sets, gp closed sets,Rg closed sets, rgα-closed sets, π-g-closed sets, Spg closed sets are introduced and studied by Cameron[2],Mashhour, Abd El-Monsef and El-Deeb[3], Govindappa Navalagi, Chandrashakarappa & Gurushantanavar[4], Gnanambal[5], Maki, Umehara and Noiri[6],Palaniappan and Rao[7], Vadivel & Vairamamanickam[8], Dontchev and Noiri[9], Wali and Vivekananda Dembre[10] Respectively.

2. PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c ,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) Regular open set [1] if A = int(cl(A)) and a regular closed set if A = cl(int(A)).

(ii) Regular semi open set [2] if there is a regular open set U such that U ⊆A⊆ cl(U).

(iii)Pre-open set [3] if A ⊆ int(cl(A)) and pre-closed set if cl(int(A)) ⊆ A.

(iv)Generalized semi pre regular closed (briefly,gspr-closed) set [4] if spcl(A)⊆U whenever A⊆U and U is regular open in X.

(v)Generalized pre regular closed set(briefly,gpr-closed)[5] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

(vi)Generalized pre closed (briefly,gp-closed) set [6] if pcl(A)⊆U whenever A⊆U and U is open in X.

(vii) Regular generalized closed set(briefly,rg-closed)[7] if cl(A)⊆U whenever A⊆U and U is regular open in X.

(viii)Regular-generalized- α closed set [8]if α- cl(A)⊆UwheneverA⊆Uand U is regular α -open in X.

(ix) π-generalized closed set (briefly,πg-closed)[9] if cl (A)⊆ U whenever A⊆ U and U is π-open in X.

(x)pre generalized pre regular ωeakly closed set(briefly spg-closed)[10] if pCl(A)⊆U whenever A⊆U and U is rgα-open in (X, τ).

3. SEMI PRE GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES:

Definition 3.1: A subset A of topological space (X, τ) is called a semi pre generalized closed sets (briefly spg-closedset) if SPCl(A)⊆U whenever A ⊆ U and U is Semi-open in (X, τ).

(i) Every closed set is spg-closed set in X.

(ii) Every regular closed set is Spg-closed set in X.

(iii)Every Spg-closed set is gspr, gpr, gp,rg, πg closed set.

(iv)The Union of two spg -closed subsets of X is spg – closed set.

(v) If A is Semi Pre Generalized closed set in X and A⊆B⊆ pCl(A) then B is also Semi Pre Generalized closed set in X.

(vi) If a subset A of topological space X is a Semi Pre Generalized closed set in X; then pCl(A) - A does not contain any non empty rgα-closed set in X.

4. SEMI PRE GENERALIZED OPEN SETS.

Definition 4.1: A subset A of a topological space (X,τ) is called Semi pre generalized open (briefly spg-open) set in X if A^c is spg-closed in X.

The following theorem is the analogue of results 3.2 (i) to (iv).

Theorem 4.2 : For any topological spaces (X,τ) we have the following .

(i)Every open set is spg-open.
(ii)Every regular open set is Spg closed set.
(iii) Every spg-openset is gspr, gpr,gp,rg, πg-open set.

Theorem 4.3: If A and B are spg-opensets in space X, then A∩B is also an spg-open in X.

Proof : Let A and B be two spg-opensets in X. Then A^c and B^c are spg-closedsets in X by Results 3.2 (iv) , $A^c \cup B^c$ is also spg-closedset in X. that is $A^c \cup B^c = (A\cap B)^c$ is spg-closedset in X.Therefore A∩B is an spg-openset in X.

Remark 4.4: The union of spg-open set in X is generally not an spg-openset in X.

Example 4.5: Let X={a,b,c,d}, τ ={X,ϕ,{a},{c,d},{a,c,d}}. If A={c} B={a} Then A & B are spg-openset in X but A ∪ B={a,c} is not an spg-openset in X.

Theorem 4.6: A subset A of a topological space X is spg-openiff U⊆ p-int(A), whenever U is rgα-closed and U ⊆A.

Proof: Assume that A is spg-openset in X and U is rgα-closed set of (X, τ) s.t

U ⊆A. Then X-A is a spg-closedset in (X, τ).Also X−A⊆ X−U and X−U is rgα-

open set of (X, τ).This implies that pcl(X−A) ⊆ X−U.But pcl(X−A)=X−p-int(A).

Thus, X−p int(A) ⊆ X−U, so U ⊆ p-int(A).Conversely: Suppose U⊆ p-int (A) whenever U is rgα-closed and U ⊆ A. To prove that A is spg-openset. Let F be rgα-open set of (X, τ) s.t X−A ⊆ F. Then X−F ⊆ A. Now X−F is rgα-closed set containing A,So;X−F ⊆ p-int(A),X−p-int(A) ⊆ F but pcl(X−A) = X−p-int(A) ⊆ F. Thus pcl(X−A) ⊆ F i.e X−A is spg-closedset & hence A is spg-openset.

Theorem 4.7: If p-int(A) ⊆ B ⊆A and A is spg-openset, then B is spg-openset.

Proof: Let p-int(A)⊆B⊆A, Thus X−A ⊆ X−B ⊆ X−p-int(A),i.e

X−A ⊆ X−B ⊆ cl(X−A),Since X−A is spg-closedset,then from result 3.2 (v) [10] X−B is spg-closedset. Therefore B is spg-openset.

Theorem 4.8: If A ⊆ X is spg-closedthen pcl(A) −A is spg-openset.

Proof : Let A be pgprω-closed. Let F ⊆ pcl(A) −A, where F is rgα–closed; then from result 3.2 (vi) [10] we have F=ϕ .Therefore F ⊆ p-int (pcl(A) −A) and Theorem4.6 pcl(A) −A is spg-openset.

The reverse implication does not hold good.

Example 4.9: Let X={a,b,c,d}, τ ={X,ϕ,{a},{c,d},{a,c,d}}Let A={a,d}, pcl(A)= {a,c,d} then pcl(A) –A={c} which is spg-openset in X, but A is not pgprω-closed.

Theorem 4.10: A set A is spg-openset in (X, τ) if and only if U=X whenever U is rgα -open and p-int(A) ∪ (X − A) ⊆ U.

Proof:- Suppose that A is spg-openset in X. Let U be rgα-open and

p-int(A) ∪ (X-A) ⊆ U ,U^c ⊆ $(\text{p-int}(A) \cup A^c)^c$ = $(\text{p-int}(A))^c \cap A$ i.e $U^c \subseteq (\text{p-int}(A))^c - A^c$ (because A−B = $A \cap B^c$).Thus $U^c \subseteq$ $\text{pcl}(A^c) - A^c$ (because $(\text{p-int}(A))^c = \text{pcl}(A^c)$).Now A^c is also spg-closedand U^c is rgα-closed then from result 3.2 (vi) [10] it follows $U^c = \phi$ then U=X. Conversely: Suppose F is spg-closedand F⊆A. Then p-int(A) ∪ (X-A) ⊆ p-int(A) ∪ (X-F). It follows that p-int(A) ∪ (X –F)=X.

Theorem 4.11: If A and B be subsets of space (X, τ). If B is spg-openand p-int(B) ⊆ A then A∩B is pgprω-open.

Proof: Let B is spg-openin X. P-int(B) ⊆ A and p-int(B) ⊆ B is always then

p-int(B) ⊆ A∩B and also p-int(B) ⊆ A∩B ⊆ B and B is spg-openset by Theorem 4.7 , A∩B is also spg-openset in X.

5. SPG-NEIGHBOURHOOD

Defintion 5.1: (i) Let (X,τ) be a topological space and Let $x \in X$, A subset of N of X is said to be pgprω-neighbourhood of x if there exists an Spg-open set G s.t. $x \in G \subseteq N$.

(ii) The collection of all pgprω-neighbourhood of $x \in X$ is called pgprω-neighbourhood system at x and shall be denoted by Spg-N(x).

Theorem 5.2 : Every neighbourhood N of $x \in X$ is a pgprω-neighbourhood of X.

Proof: Let N be neighbourhood of point $x \in X$. To prove that N is a pgprω-neighbourhood of x by definition of neighbourhood $\exists$ an open set G s.t. $x \in G \subset N$ .Hence N is pgprω-neighbourhood of x.

Remark 5.3 : In general, a pgprω-nbhd N of x ∈ X need not be a nbhd of x in X, as seen from the following example.

Example 5.4: Let X = {a, b, c, d} with topology τ = {X, φ,{a},{b ,{a, b},{a,b,c}}. Then pgprωo(X) = {X, φ,{a},{b},{c},{a, b},{b,c},{a, c} ,{a, b, c},{a,b,d}}. The set {c,d} is pgprω-nbhd of the point c, since the spg-openset {c} is such that c ∈ {c}⊂{c,d}. However, the set {c,d} is not a nbhd of the point c, since no Spg open set G exists such that c ∈ G ⊂{c,d}.

Theorem 5.5: If a subset N of a space X is pgprω-open, then N is a pgprω-nbhd of each of its points.

Proof: Suppose N is pgprω-open. Let x ∈ N. We claim that N is pgprω-nbhd of x. For N is a spg-openset such that x ∈ N ⊆ N. Since x is an arbitrary point of N, it follows that N is a pgprω-nbhd of each of its points.

Remark 5.6: The converse of the above theorem is not true in general as seen from the following example.

Example 5.7: Let X = {a, b, c, d} with topology τ = {X, ϕ {a},{c,d},{a, c,d}}. Then pgprωo(X) = {X, ϕ, {a},{b},{c},{d},{a,b},{a,c},{a,d},{c,d},{a,b,c},{a,b,d},{a,c,d}}. The set {b,c} is a pgprω-nbhd of the point b, since the spg-openset {b} is such that b∈ {b} ⊆ {b,c}. Also the set {b,c} is a pgprω-nbhd of the point {c}, Since the spg-openset {c} is such that c ∈ {c} ⊆ {b,c}. That is {b,c} is a pgprω-nbhd of each of its points. However the set {b,c} is not a spg-openset in X.

Theorem 5.8: Let X be a topological space. If F is a spg-closedsubset of X, and x ∈ F^c. Prove that there exists a pgprω-nbhd N of x such that N∩F =ϕ.

Proof: Let F be spg-closedsubset of X and x ∈ F^c. Then F^c is spg-openset of X. So by theorem 5.5 F^c contains a pgprω-nbhd of each of its points. Hence there exists a pgprω-nbhd N of x such that N⊂F^c. That is N ∩ F = ϕ.

Theorem 5.9: Let X be a topological space and for each x ∈ X, Let pgprω-N (x) be the collection of all pgprω-nbhds of x. Then we have the following results.

(i) ∀x ∈ X, pgprω-N (x) ≠ ϕ.

(ii) N ∈ pgprω-N (x) ⇒ x ∈ N.

(iii) N∈pgprω-N (x), M ⊃N ⇒ M∈ pgprω-N (x).

(iv) N∈ pgprω-N (x), M∈ pgprω-N (x) ⇒ N ∩M ∈ pgprω-N (x).

(v) N ∈ pgprω-N (x) ⇒ there exists M ∈ pgprω-N (x) such that M ⊂ N and M ∈ pgprω-N (y) for every y ∈ M.

Proof: (i) Since X is a spg-openset, it is a pgprω-nbhd of every x ∈ X. Hence there exists at least one pgprω-nbhd (namely - X) for each x ∈ X. Hence pgprω-N (x) ≠ ϕ for every x ∈ X.

(ii) If N∈pgprω-N (x),then N is a pgprω-nbhd of x.So by definition of pgprω-nbhd,x ∈ N.

(iii) Let N ∈ pgprω-N (x) and M ⊃ N. Then there is a spg-openset G such that x ∈ G ⊂ N. Since N ⊂ M, x ∈ G ⊂ M and so M is pgprω-nbhd of x. Hence M ∈ pgprω-N (x).

(iv) Let N ∈ pgprω-N (x) and M ∈ pgprω-N (x). Then by definition of pgprω-nbhd there exists spg-opensets G_1 and G_2 such that x ∈ G_1 ⊂ N and x ∈ G_2 ⊂ M.

Hence x ∈ $G_1 \cap G_2$ ⊂ N ∩M -- (1). Since $G_1 \cap G_2$ is a spg-openset, (being the intersection of two spg-opensets), it follows from (1) that N ∩ M is a pgprω-nbhd of x. Hence N ∩M ∈ pgprω-N (x).

(v) If N ∈ pgprω-N (x), then there exists a spg-openset M such that x ∈ M ⊂N. Since M is a spg-openset, it is pgprω-nbhd of each of its points. Therefore M ∈ pgprω-N (y) for every y ∈ M.

Theorem 5.10: Let X be a nonempty set, and for each x ∈ X, let pgprω-N (x) be a nonempty collection of subsets of X satisfying following conditions.

(i) N ∈ pgprω-N (x) ⇒ x∈ N

(ii) N ∈ pgprω-N (x),M ∈ pgprω-N (x) ⇒ N ∩M ∈ pgprω-N (x).

Let τ consists of the empty set and all those non-empty subsets of G of X having the property that x ∈ G implies that there exists an N∈pgprω-N (x) such that x ∈ N ⊂ G, Then τ is a topology for X.

Proof.:(i) φ ∈ τ by definition. We now show that x ∈ τ . Let x be any arbitrary element of X. Since pgprω-N (x) is nonempty, there is an N ∈ pgprω-N (x) and so x ∈ N by (i). Since N is a subset of X, we have x ∈ N ⊂ X. Hence X ∈ τ .

(ii) Let G_1 ∈ τ and G_2 ∈ τ. If x ∈ $G_1 \cap G_2$ then x ∈ G_1 and x ∈ G_2. Since G_1 ∈ τ and G_2 ∈ τ , there exists N ∈ pgprω-N (x) and M ∈ pgprω-N (x), such that x ∈ N ⊂ G_1 and x ∈ M ⊂ G_2. Then x ∈ N ∩ M ⊂ $G_1 \cap G_2$. But N ∩M ∈ pgprω-N (x) by (2). Hence $G_1 \cap G_2$ ∈ τ .Let G_λ ∈ τ for every λ ∈ ∧. If x ∈ ∪{G_λ : λ ∈ ∧}, then x ∈ $G_{\lambda x}$ for some λx ∈ ∧. Since $G_{\lambda x}$ ∈ τ , there exists an N∈pgprω-N (x) such that x ∈ N ⊂ $G_{\lambda x}$ and consequently x ∈ N ⊂ ∪{G_λ : λ ∈ ∧}. Hence ∪{G_λ : λ ∈ ∧} ∈ τ. It follows that τ is topology for X.

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