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Soft Translations of Soft Ideals in Soft BCI/BCK-Algebras

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ABSTRACT

In this paper we concept of soft translations of soft subalgebras and soft ideals over BCI/BCK-algebras is introduced and some related properties are studied. Notions of Soft extensions of soft subalgebras and soft ideals over BCI/BCK-algebras are also initiated. Relationships between soft translations and soft extensions are explored.

Keywords: *BCI/BCK-algebras, soft translations, soft subalgebras and soft ideals etc.*

Introduction :

Recently soft set theory has emerged as a new mathematical tool to deal with uncertainty. Due to its applications in various fields of study researchers and practitioners are showing keen interest in it. As enough number of parameters is available here, so it is free from the difficulties associated with other contemporary theories dealing with uncertainty. Prior to soft set theory, probability theory, fuzzy set theory, rough set theory, and interval mathematics were common mathematical tools for dealing with uncertainties, but all these theories have their own difficulties. These difficulties may be due to lack of parametrization tools [1,2]. To overcome these difficulties, Molodtsov [2] introduced the concept of soft sets. A detailed overview of these difficulties can be seen in [1,2]. As a new mathematical tool for dealing with uncertainties, Molodtsov has pointed out several directions for the applications of soft sets. Theoretical development of soft sets is due to contributions from many researchers. However in this regard initial work is done by Maji et al. in [1]. Later Ali et al. [3] introduced several new operations in soft set theory.

At present, work on the soft set theory is progressing rapidly. Maji et al. [4] described the application of soft set theory in decision making problems. Aktas¸ and Cagman studied the concept of soft groups and derived their basic properties [5]. Chen et al. [6] proposed parametrization reduction of soft sets, and then Kong et al. [7] presented the normal parametrization reduction of soft sets. Feng and his colleagues studied roughness in soft sets [8,9]. Relationship between soft sets, fuzzy sets, and rough sets is investigated in [10]. Park et al. [11] worked on notions of soft WS-algebras, soft subalgebras, and soft deductive system. Jun and Park [12] presented the notions of soft ideals, idealistic soft, and idealistic soft BCI/BCK-algebras.

Now concept of translation of a soft ideal of a BCI/BCK algebra is introduced.

Definition 1.1. A soft subset FA of a BCI/BCK-algebra is called a soft ideal of X, denoted by $F_A \lhd_S X$, if it satisfies:

(1) $(\forall x \in X)\ ((0) \supseteq F(x))$,

(2) $(\forall x, y \in X)\ ((x) \supseteq ((x * y) \cap FA(y)))$.

Theorem 1.2. If F_A is a soft subset of X, then F_A is a soft ideal of X if and only if soft U_1-translation $F_{U_1}^T$ of F_A is a soft ideal of X for all $U_1 \subseteq T$.

Proof. Assume that $F_A \lhd_S X$ and let $U_1 \subseteq T$. Then $F_{U_1}^T(0) = (0) \cup U_1 \supseteq F(x) \cup U_1 = F_{U_1}^T(x)$ and

$$F_{U_1}^T(x) = F_A(x) \cup U_1 \supseteq (F_A(x * y) \cap F_A(y)) \cup U_1$$

$$= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= F_{U_1}^T(x * y) \cap F_{U_1}^T(y)\ \forall x, y \in X. \quad (1.1)$$

Hence $F_{U_1}^T \lhd_S X$.

Conversely, assume that $F_{U_1}^T$ is a soft ideal of X for some $U_1 \subseteq T$. Let $x, y \in X$. Then

$$F^T_{U_1}(0) \supseteq F^T_{U_1}(x) \Rightarrow F_A(0) \cup U_1 \supseteq F_A(x) \cup U_1$$

$$\Rightarrow F_A(0) \supseteq F_A(x) \qquad (1.2)$$

and so $F_A(0) \supseteq F_A(x)$. Next

$$\begin{aligned} F_A(x) \cup U_1 &= F^T_{U_1}(x) \\ &\supseteq F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \\ &= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1) \\ &= (F_A(x * y) \cap F_A(y)) \cup U_1, \qquad (1.3) \end{aligned}$$

which implies that $F_A(x) \supseteq F_A(x * y) \cap F_A(y)$

Hence F_A is a soft ideal of X.

Soft Extensions and Soft Ideal Extensions of Soft Subalgebras :

In this section concept of soft ideal extension is being introduced and some of its properties are studied.

Definition 1.3. Let F_A and G_B be the soft subsets of X. Then G_B is called the soft ideal extension of F_A, if the following conditions hold:

(1) G_B is a soft extension of F_A.

(2) $F_A \lhd_S X \Rightarrow G_B \lhd_S X$.

For a soft subset F_A of X, $U_1 \subseteq T$ and $U_2 \in (X)$ with $U_2 \supseteq U_1$, define $E_{U_1}(F_A; U_2) := \{x \in X \mid (x) \cup U_1 \supseteq U_2\}$. It is clear that if $F_A \lhd_S X$, then $U_{U_1}(F_A; U_2) \lhd X$ for all $U_2 \in (U)$ with $U_2 \supseteq U_1$.

Theorem 1.4. For $U_1 \subseteq T$, let $F^T_{U_1}$ be the soft U_1-translation of F_A. Then the following are equivalent:

(1) $FT\ U_1 \lhd_S X$.

(2) $(\forall U_2 \in (U))\ (U_2 \supset U_1 \Rightarrow E_{U_1}(F_A; U_2) \lhd X)$.

Proof. (1) ⇒ (2) Consider $F_{U_1}^T \lhd_S X$ and let $U_2 \in (U)$ be such that $U_2 \supset U_1$. Since $F_{U_1}^T(0) \supseteq F_{U_1}^T(x)$ for all $x \in X$, we have

$$F_A(0) \cup U_1 = F_{U_1}^T(0) \supseteq F_{U_1}^T(x) = F_A(x) \cup U_1 \supseteq U_2, \quad (1.4)$$

for $x \in E_{U_1}(F_A; U_2)$.

Hence $0 \in E_{U_1}(F_A; U_2)$. (1.5)

Let $x, y \in X$ be such that $x*y \in E_{U_1}(F_A; U_2)$ and $y \in E_{U_1}(F_A; U_2)$. Then $(x * y) \cup U_1 \supseteq U_2$ and $F(y) \cup U_1 \supseteq U_2$, that is, $F_{U_1}^T(x * y) = F_A(x * y) \cup U_1 \supseteq U_2$ and $F_{U_1}^T(y) = F_A(y) \cup U_1 \supseteq U_2$. Since $F_{U_1}^T \lhd_S X$, it follows that

$$F_A(x) \cup U_1 = F_{U_1}^T(x) \supseteq F_{U_1}^T(x * y) \cap F_{U_1}^T(y) \supseteq U_2, \quad (1.6)$$

that is, $F_A(x) \cup U_1 \supseteq U_2$ so that $x \in E_{U_1}(F_A; U_2)$. Therefore $E_{U_1}(F_A; U_2) \lhd X$.

(2) ⇒ (1) Suppose that $E_{U_1}(F_A; U_2) \lhd X$ for every $U_2 \in P(U)$ with $U_2 \supseteq U_1$. If there exists $x \in X$ with $U3 \supseteq U1$ such that $FT\ U1\ (0) \subset U3 \subseteq FT\ U1\ (x)$ and then $(x) \cup U_1 \supseteq U_3$ but $F(0) \cup U_1 \subset U_3$. This shows that $x \in E_{U_1}(F_A; U_2)$ and $0 \notin E_{U_1}(F_A; U_2)$. This is a contradiction, and so $F_{U_1}^T(0) \supseteq F_{U_1}^T(x)$, for all $x \in X$.

Now assume that there exist $a, b \in X$ such that $F_{U_1}^T(a) \subset U_4 \subseteq F_{U_1}^T(a*b) \cap F_{U_1}^T(b)$. Then $F_A(a*b) \cup U_1 \supseteq U_4$ and $F_A(b) \cup U_1 \supseteq U_4$, but $F_A(a) \cup U_1 \subset U_4$. Hence $a*b \in E_{U_1}(F_A; U_4)$ and $b \in E_{U_1}(F_A; U_4)$, but $a \notin E_{U_1}(F_A; U_4)$. This is impossible and therefore $F_{U_1}^T(x) \supseteq F_{U_1}^T(x * y) \cap F_{U_1}^T(y)$, for all $x, y \in X$. Consequently $F_{U_1}^T \lhd_S X$.

Theorem 1.5. Let $FA \lhd_S X$ and $U_1, U_2 \subseteq T$. If $U_1 \supseteq U_2$, then the soft U_1-translation $F_{U_1}^T$ of F_A is a soft ideal extension of the soft U_2-translation $F_{U_2}^T$ of F_A.

Proof. Since

$$F_{U_1}^T(x) = F_A(x) \cup U_1,\ F_{U_2}^T(x) = F_A(x) \cup U_2, \quad (1.7)$$

$U_1 \supseteq U_2$, this implies that ($F^T_{U_1}$ $(x) \supseteq$ $F^T_{U_2}$ (x)) ($\forall x \in X$). This shows that $F^T_{U_1}$ is a soft extension of $F^T_{U_2}$. Now, let $F^T_{U_2}$ is a soft ideal of X, then $F^T_{U_1}$ $(0) = (0) \cup U_1 \supseteq F(x) \cup U_1 =$ $F^T_{U_1}$ (x) for all $x \in X$, so we have ($F^T_{U_1}$ $(0) \supseteq$ $F^T_{U_1}$ (x)). Consider

$$F^T_{U_1}(x) = F_A(x) \cup U_1$$

$$\supseteq (F_A(x * y) \cap F_A(y)) \cup U_1$$

$$= (F_A(x * y) \cup U_1) \cap (F_A(y) \cup U_1)$$

$$= F^T_{U_1}(x * y) \cap F^T_{U_1}(y) \text{ for all } x, y \in X. \qquad (1.8)$$

That is ($F^T_{U_1}$ $(x) \supseteq$ $F^T_{U_1}$ $(x * y) \cap$ $F^T_{U_1}$ (y)) ($\forall x, y \in X$) so $F^T_{U_1}$ is a soft ideal of X. Hence $F^T_{U_1}$ is a soft ideal extension of $F^T_{U_2}$.

Conclusion :

Soft set theory is a mathematical tool to deal with uncertainties. Translation and extension are very useful concepts in mathematics to reduce the complexity of a problem. These concepts are frequently employed in geometry and algebra. In this paper, we presented some new notions such as soft translations and soft extensions for BCI/BCK-algebras. We also examined some relationships between soft translations and soft extensions. Moreover, soft ideal extensions and translations have been introduced and investigated as well. It is hoped that these results may be helpful in other soft structures as well.

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