JETIR.ORG ISSN: 2349-5162 | ESTD Year : 2014 | Monthly Issue
JOURNAL OF EMERGING TECHNOLOGIES AND INNOVATIVE RESEARCH (JETIR)
An International Scholarly Open Access, Peer-reviewed, Refereed Journal

Characterization of Topological spaces

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ABSTRACT: In this paper we introduce and investigate new class of maps called spg-homeomorphism and several characterization and some of their properties. Also we investigate It's relationship with other types of functions.

Key words: Topological spaces, Homeomorphism

AMS Mathematical Subject classification(2010):54C08, 54C10.

I. INTRODUCTION

The notion homeomorphism plays a very important role in topology. By definition a homeomorphism between two topological spaces X and Y is a bijective map f: X → Y when both f and $f^{-1}$are continuous map.Wali and Vivekananda Dembre[1] introduced Spg-closed set in topological spaces.

II.PRELIMINARIES

Throughout this paper space (X, τ) and (Y, σ) (or simply X and Y) always denote topological space on which no separation axioms are assumed unless explicitly stated. For a subset A of a space X, Cl(A), Int(A), A^c,P-Cl(A) and P-int(A) denote the Closure of A, Interior of A , Compliment of A, pre-closure of A and pre-interior of (A) in X respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) A pre generalized pre regular ωeakly closed set(briefly pgprω-closed set)[1] if pCl(A)⊆U whenever A⊆U and U is rgα open in (X,τ).

(ii)pre generalized pre-regular ωeakly open(briefly pgprω-open)[2]set in X if A^c is pgprω-closed

in X.

(iii)Regular openset if A = int(clA))[4] and a regular closed set if A = cl(int(A)).

(iv)Generalized pre regular closed set(briefly gpr-closed)[5] if pcl(A)⊆U whenever A⊆U and U is regular open in X.

(v) Generalized semi pre regular closed (briefly gspr-closed) set [12] if spcl(A)⊆U whenever A⊆U and U is regularopen in X.

(vi)Generalized pre closed (briefly gp-closed) set [7] if pcl(A)⊆U whenever A⊆U and U is open in X.

Defintion 2.2: A map f:(X, τ) → (Y, σ)) is called

(i)Spg-continuous map if the inverse image of every closed in Y is spg closed set in X.

(ii) Regular-continuous map if the inverse image of every closed in Y is regular closed set in X.

(iii) gpr-continuous map if the inverse image of every closed in Y is gpr closed set in X.

(iv) gspr-continuous map if the inverse image of every closed in Y is gspr closed set in X.

(v)gp-continuous map if the inverse image of every closed in Y is gp closed set in X.

Defintion2.3: A map f:(X, τ) → (Y, σ) is called

(i)gpr homomorphism if f &f^{-1} are gpr continuous map.

(ii) gspr homomorphism if f &f^{-1} are gspr continuous map.

(iii)gp homomorphism if f &f^{-1} are gp continuous map.

(iv)spg-closed map if f(F) is spg-closed in (Y,σ) for every closed set of (X,τ)& spg-open map if f(F) is spg-open in (Y,σ) for every open set of (X,τ).

Theorem 2.4:

(i) Every spg-closed set is gspr-closed.

(ii)Every spg-closed set is gp-closed.

III. SPG-HOMEOMORPHISM IN TOPOLOGICAL SPACES

DEFINITION 3.1: A bijection f : (X, τ) → (Y, σ) is called semi pre generalized homeomorphism if f and f^{-1} are spg-continuous map. We denote the family of all spg-homeomorphisms of a topological space (X, τ) onto itself by spg-(X, τ).

Example 3.2 : Consider X=Y={a,b,c,d} with topologies τ={X,∅,{a},{b},{a,b},{a,b,c}} and

σ = {Y,∅,{a},{b},{a,b},{a,b,c}}. Let f: X → Y be a map defined by f (a) = c, f (b) = a f(c) = b and

f(d)=d. Then f is bijective, spg-continuous map and $f^{-1}$is spg –continuous map

Hence f is spg–homeomorphism.

Theorem 3.3: Every homeomorphism is spg-homeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be a homeomorphism. Then f and f^{-1} are continuous map and f is bijection . Since every continuous map is spg-continuous map,f and $f^{-1}$are spg-continuous map.Hence f is spg-homeomorphism.

Remark 3.4: The Converse of the above theorem need not be true as seen from the following example.

Example 3.5 : Consider X=Y={a,b,c,d} with topologies τ ={X, ∅ ,{a},{b},{a,b},{a,b,c}} andσ={Y,∅,{a},{b},{a,b},{a,b,c}}.Let f: X → Y be a map defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d.Then f is spg-homeomorphism. But it is not homeomorphism since the inverse image of the closed set {c,d} in X is {b,d} is not closed in Y.

Theorem 3.6: Every regular homeomorphism is spg-homeomorphism.

Proof: The proof follows from the theorem 3.3

Remark 3.7: The Converse of the above theorem need not be true as seen from the following example.

Example 3.8 : Consider X=Y={a,b,c,d} with topologiesτ={X,∅,{a},{b},{a,b},{a,b,c}} and

σ={Y,∅,{a},{b},{a,b},{a,b,c}}.Let f: X → Y be a map defined by f(a)=c,f(b)=a,f(c)=b,f(d)=d.Then f is spg-homeomorphism. But it is not regular homeomorphism since the inverse image of the closed set {c,d} in X is {b,d} is not regular closed in Y.

Theorem 3.9: Every spg-homeomorphism is gprhomeomorphism.

Proof: Let f: (X, τ) → (Y, σ) be a spghomeomorphism.Then f and f^{-1} are spg- continuous mapand f is bijection. Since every spg-continuous mapis gpr-continuous map, f and $f^{-1}$are gpr-continuous map.Hence f is gpr-homeomorphism.

Remark 3.10: The Converse of the above theorem need not be true as seen from the following example.

Example 3.11: Consider X = Y = {a,b,c} with topologiesτ={X,∅,{a},{b,c}} and σ = {Y,∅,{a}}. Let f: X → Y be a map defined by f(a)=b,f(b)=a,f(c)=c.Then f is gpr-homeomorphism. But it is notspghomeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not spgclosed in Y.

Theorem 3.12: Every spg-homeomorphism is gspr-homeomorphism.

Proof: The proof follows from the definition and fact that every spg-closed set is gspr-closed

Remark 3.13: The Converse of the above theorem need not be true as seen from the following example.

Example 3.14 : Consider X=Y={a,b,c} with topologies τ={X,∅,{a},{b,c}} and σ = {Y,∅,{a}}.Let f: X → Y be the defined by f(a)=b,f(b)=a,f(c)=c.Then f is gspr-homeomorphism.But it is not spg-homeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not spg-closed in Y.

Theorem 3.15: Every spg-homeomorphism is gp-homeomorphism.

Proof: The proof follows from the definition and fact that every spg-closed set is gp-closed

Remark 3.16: The Converse of the above theoremneed not be true as seen from the following example.

Example 3.17 : Consider X=Y={a,b,c} with topologies τ={X,∅,{a},{b,c}} andσ = {Y,∅,{a}}.Let f: X → Y be the defined by f(a)=b,f(b)=a,f(c)=c.Then f is gp-homeomorphism.But it is not spg-homeomorphism since the inverse image of the closed set {b,c} in X is {a,c} is not spg-closed in Y.

Theorem 3.18: Let f: (X,τ) → (Y, σ) be a bijective spg-continuous map. Then the following statements are equivalent.

(i) f is a spg-open map.

(ii) f is spg-homeomorphism.

(iii) f is a spg-closed map.

Proof: Suppose (i) holds. Let V be open in(X,τ) .Then by (i), f (V) is spg-open in (Y, σ). But

f (V) = $(f^{-1})^{-1}(V)$ and so $(f^{-1})^{-1}(V)$ is spg-open in(Y, σ). This shows that $f^{-1}$is spg-continuous map and it proves (ii).

Suppose (ii) holds. Let F be a closed set in (X,τ). By (ii), f^{-1} is spg-continuous map and so

$(f^{-1})^{-1}(V)$ (F) = f (F) is spg-closed in (Y, σ).This proves (iii).

Suppose (iii) holds. Let V be open in(X,τ) .Then V^c is closed in (X,τ) By (iii), f (V^c) is spg-closedin (Y, σ). But f (V^c) = $(f (V))^c$. This implies that $(f (V))^c$ is spg-closed in (Y, σ) and so

f (V) is spg-open in (Y, σ). This proves (i).

Remark 3.19: The Composition of two spg-homeomorphism need not be a spg-homeomorphismin general as seen from the following example.

Example 3.20 : Consider X = Y = Z = {a,b,c} with topologies τ = { X , ϕ , {a} , {b}, {a,b}} and

σ ={Y, ϕ,{a}} & μ= {Z = ϕ,{a},{a,b},{a,c}}. Define f: (X, τ) → (Y, σ) and g:(Y,σ) → (Z, μ) & gof: X → Z are identity maps both f&g are spg homeomorphism but gof not spg-homeomorphism.Since closed set v={b} in Z,f^{-1}(v)={b},which is not spg-closed set in X.

ON SEMI PRE GENERALIZED TOPOLOGICAL SPACES.

Definition 4.1: A topological space X is called a τ_{Spg} space if every Spg-closed set in it is pre-closed.

Example 4.2: Let X={a,b,c,d},τ={X,Φ,{a},{b,{a,b},{a,b,c}}.Here every Spg-closed set is pre Closed .So (X, τ) is a τ_{Spg}– space.

Example 4.3: Let X={a,b,c,d}, τ={X,Φ,{a},{b},{a,b,{a,b,c}}.Here {a,d} is Spg-closed,but not pre-closed So (X,T) is not T_{Spg}– space.

Theorem 4.4: A topological space X is a τ_{Spg} space iff for each x of X, {x} is either rgα -closed or pre-open.

Proof: Hypothesis: X is a τ_{Spg} -space.Let xϵX. If {x} is rgα -closed, then there is nothing to prove. If {x} is not rgα -closed, then X-{x} is not rgα -open and so X is the only rgα -open set containing X-{x} and pcl(X-{x})⊆X. Therefore X-{x} is Spg-closed. X is T_{Spg} – space (hypothesis) and X-{x} is Spg-closed.Therefore X-{x} ispre-closed . Therefore {x} is pre-open. Thus for every x of X, a τ_{Spg} –space, {x} is either rgα -closed or pre-open.

Conversely, suppose for every xϵX, {x} is either rgα -closed or pre-open.Let A be a Spg-closed subset of X. Now to prove A is pre-closed, we prove pcl(A) ⊆A.

Let x ϵ pcl(A).Then by hypothesis (a) {x} is pre-open ;If x is not in A , then A⊆{x }; a pre-closed setThereforepcl(A) ⊆{x}.'. x ϵ {x} which is not trueTherefore x ϵ A. Therefore pcl(A) ⊆A. Thus every Spg-closed set is pre-closed. Therefore X is a τ_{Spg} - space.

Theorem 4.5: Every pre-regular $T_{1/2}$-space is τ_{Spg} – space.

Proof: Let X be a pre-regular $\tau_{1/2}$-space and A be a Spg-closed set. As every pgrgw-closed set is gpr-closed, A is gpr-closed. Since X is pre-regular $\tau_{1/2}$-space, A is preclosed. So every pgpgw-closed set is preclosed. Therefore X is τ_{Spg}– space.

Converse of the above theorem is not true.

For example 4.6: X= {a,b,c,d}, τ={X,Φ, {a}, {b}, {a,b},{a,b,c}}. Here (X, T) is a $\tau_{Spg\text{-}}$ space,but not a pre-regular $\tau_{1/2}$-space.

Defintion 4.7: Let (X, τ) be topological space and τ-Spg ={V⊆X : spg–cl(V^c) = V^c }, τ-spg is toplogy on X.

Theorem 4.8: Let f: X→Y be a function. Let (X,τ) and (Y,σ) be any two spaces such that $\tau_{pgpr\omega}$ is a topology on X. Then the following statements are equivalent:

(i) For every subset A of X, f(Spg-cl(A)) ⊆ cl(f(A)) holds,
(ii) f: (X, $\tau_{pgpr\omega}$)→(Y,σ) is continuous.

Proof: Suppose (i)holds.Let A be closed in Y.By hypothesf(Spg-cl(f^{-1}(A)))⊆cl(A)=A.i.e. Spg-cl(f^{-1}(A))⊆f^{-1}(A).Also f^{-1}(A)⊆pgprωcl(f^{-1}(A)).Hence Spg-cl(f^{-1}(A))= f^{-1}(A). This implies f^{-1}(A)∈ $\tau_{pgpr\omega}$.Thus f^{-1}(A) is closed in (X, $\tau_{pgpr\omega}$) and so f is continuous.
This proves (ii).

Suppose (ii) : holds. For every subset A of X, cl(f(A)) is closed in Y. Since f: (X, $\tau_{pgpr\omega}$)→(Y,σ) is continuous, f^{-1}(cl(A)) is closed in (X, τ_{Spg}) that implies by definition 3.7Spg-cl(f^{-1}(cl(f(A)))) = f^{-1}(cl(f(A))). Now we have,A ⊆ f^{-1}(f(A)) ⊆ f^{-1}(cl(f(A))) and by Spg-closure,
Pgprω-cl(A)⊆pgprω-cl(f^{-1}(cl(f(A))).Therefore f(Spg-cl(A))⊆ cl(f(A)). This proves (i).

Theorem 4.9: Let X and Y be pgprω τ_p-spaces, then for a function f: (X,τ) → (Y,σ), the following are equivalent:

(i) f is p-irresolute map.

(ii) f is pgprω-irresolute map.

Proof: (i)=> (ii): Let f: (X,τ) →(Y,σ) be a p-irresolute map. Let V be a pgprω -closed set in Y.

As Y is SpgT_p-space, then V be a p-closed set in Y. Since f is p–irresolute map, f^{-1} (V) is

p -closed in X. But every p-closed set is Spg -closed in X and hence f^{-1} (V) is a pgprω-closed in X. Therefore, f is Spg-irresolute map.

(ii)=> (i): Let f: (X,τ) → (Y,σ) be a pgprω –irresolute map. Let V be a p-closed set in Y.

But every p-closed set is pgprω -closed set and hence V is pgprω-closed set in Y and f is Spg-irresolute map implies f^{-1} (V) is Spg-closed in X. But X is SpgT_p-space and hence f^{-1} (V) is p-closed set in X. Thus, f is p-irresolute map.

Theorem 4.10: If a mapping f: (X, τ) → (Y, σ) is Spg–closed map,then Spg–cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).

Proof: Suppose that f is Spg–closed and A ⊆X. Then cl(A) is closed in X and so f(cl(A)) is Spg–closed in (Y, σ). We have f(A) ⊆ f(cl(A)), by Theorem 3.8 , Spg–cl(f(A)) ⊆Spg–cl(f(cl(A))) → (i). Since f(cl(A)) is Spg– closed in (Y,σ), Spg–cl(f(cl(A))) = f(cl(A)) → (ii), by the Theorem 3.8. From (i) and (ii), we have Spg–cl(f(A))⊆f(cl(A)) forevery subset A of (X, τ).

Corollary 4.11: If a mapping f: (X, τ) → (Y, σ) is a pgprω–closed, then the image f(A) of closed set A in (X, τ) is $\tau_{pgpr\omega}$–closed in (Y, σ).

Proof.: Let A be a closed set in (X, τ). Since f is Spg–closed, by above Theorem 3.10,Spg–cl(f(A)⊆f(cl(A))→(i). Also cl(A)=A, as A is a closed set and so f(cl(A)) = f(A) →(ii).From (i) and (ii), we have Spg–cl(f(A))⊆f(A). We know that f(A)⊆Spg–cl(f(A)) and so Spg–cl(f(A)) =f(A). Therefore f(A) is $\tau_{pgpr\omega}$–closed in (Y, σ).

Theorem 4.12: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z, η) is Spg–closed maps and (Y, σ) be a $T_{pgpr\omega}$-space then g∘f: (X, τ) → (Z, η) is pgprω–closed map.

Proof: Let A be a closed set of (X, τ). Since f is pgprω–closed,f(A) is pgprω–closed in (Y, σ). Then by hypothesis, f(A) is closed. Since g is pgprω–closed, g(f(A)) is Spg–closed in (Z, η)

and g(f(A)) = g∘f(A). Therefore g∘f is pgprω–closed map.

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