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# A SIMPLE RING IN FUZZY

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**Abstract:** Propositions of unitary ring. Commutative unitary ring in fuzzy have been studied to establish a simple ring. In this paper, we shall define the unitary ring, commutative ring which are considered as a formulation of the simple ring.

**Keywords:** Division ring; Fuzzy bi-ideal: Fuzzy bi-ideal free; Fuzzy ideal

## Introduction

Zedeh [1] discussed fuzzy subset of a non-empty set into [0, 1]. Rosen feld [2] developed this concept in group. Kuroki [3] used it in semigroups. Wang-Jin-Liu [4] studied the fuzzy in simple rings.

We know that any classical algebraic system is a universal set with one or more binary operations. But fuzzy subalgebraic is not so. Rosenfeld found an adequate outlet. although partial, to overcome and absence of the fuzzy universal set and fuzzy binary operation. He assumed an (ordinary) group structure on the base set X and then made use of its (ordinary binary operation to define a fuzzy subgroup of X. Nevertheless. he did not defined the concept of fuzzy group. In the absence of the concept of fuzzy universal set, formulation of the intrinsic definition for fuzzy algebraic system and fuzzy subalgebraic is not evident.

In this paper, we shall define the unitary ring. commutative ring which are considered as a formulation of the simple ring.

**Definition 1:** A ring $R$ with unity is said to be fuzzy simple if for every fuzzy ideal $\delta$ of $R$ $\delta(x) = \delta(y)$ for every $x(\neq 0)$, $y(\neq 0) \in R$.

**Example 1:** Let $Q$ be the field of all rational numbers. Let $\delta$ be a fuzzy ideal of $Q$. Let $x(\neq 0) \in Q$. Now

$$\delta(x) = \delta(x.1) \geq \delta(1) \text{ Again } \delta(1) = \delta(x.x^{-1}) \geq \delta(x). \text{ Thus } \delta(x) = \delta(1) \text{ for every } x(0) \in Q.$$

Hence $\delta(x) = \delta(y)$ for every $x(\neq 0)$, $y(\neq 0) \in Q$. Thus $Q$ is fuzzy simple.

**Proposition 1:** Let $R$ be a unitary ring. Then the following conditions are equivalent:

(1) $R$ is left (right) simple

(ii) $R$ is fuzzy let (right) simple.

**Proof.** (i) implies (ii):

Let $\delta$ be a fuzzy left ideal of $R$ and $a(\neq 0)$, $b(\neq 0) \in R$. Now $Ra$ is a left ideal of $R$. Since

$a = 1_R.\, a \in Ra$, $Ra \neq \{0\}$. So $Ra = R$. Hence $b = xa$ for some $X \in R$. Similarly $a = yb$ for some $y \in R$. Now

$\delta(a) = \delta(yb) \geq \delta(b) = \delta(xa) \geq \delta(a)$.

So $\delta(a) = \delta(b)$ for every $a(\neq 0)$, $b(\neq 0) \in R$. So

$R$ is fuzzy left simple.

(ii) implies (i) :

Let $A(\neq\{0\})$ be a left ideal of $R$. Now $X_A$ is a fuzzy left ideal of $R$. Since $R$ is fuzzy left simple $X_A(x) = X_A(y)$ for every $x(\neq 0)$ and for every $y(\neq 0) \in R$. Let $x(\neq 0) \in A$ and $y(\neq 0) \in R$. Then $X_A(y) = X_A(x) = 1$. So $y \in A$. Thus $R \subseteq A$. i.e., $R = A$. So $R$ has only two left ideals viz. (0) and $R$. Hence $R$ is left simple.

**Proposition 2:** Let $R$ be a unitary ring. Then the following conditions are equivalent:

(i) $R$ is simple.

(ii) $R$ is fuzzy simple.

**Proposition 3:** Let $R$ be a unitary ring. $R$ is fuzzy left (right) simple iff $R$ is a division ring.

**Proof:** Since $R$ is fuzzy left simple from Proposition 4.4 it follows that $R$ is left simple. Consequently it follows that $R$ is a division ring.

Converse follows by reversing the above arguments.

**Proposition 4:** A commutative unitary ring is a field iff it is fuzzy simple.

**Proof:** Let $R$ be a field. Then $R$ is simple. Therefore by Proposition 4.5, $R$ is fuzzy simple.

Conversely let $R$ be fuzzy simple. So by proposition 4.5, $R$ is simple. Now since $R$ is a commutative unitary ring, $R$ is a field.

**Definition 2:** A unitary ring $R$ is said to be a bi-ideal-free ring (or free from bi-ideal) if it has only two bi-ideals namely $\{0\}$ and $R$ itself.

**Definition 3:** A unitary ring $R$ is said to be a fuzzy bi-ideal-free ring if for every fuzzy bi-ideal $\delta$ of $R$.

$\delta(x) = \delta(y)$ for every $x(\neq 0)$, $y(\neq 0) \in R$.

**Example 2:** Let Q be the filed of all rational numbers. Let & be a fuzzy bi-ideal of Q.

Let $x(\neq 0) \in Q$.

Now $\delta(x) = \delta(1.x.1) \geq \min\{\delta(1) = \delta(1)$.

Again $\delta(1) = \delta(x.x^{-2}) \geq \min\{\delta(x), \delta(x)\} = \delta(x)$.

Thus $\delta(x) = \delta(1)$ for every $x\ (\neq 0) \in Q$.

Hence $\delta(x) = \delta(y)$ for every $x\ (\neq 0)$, $y(\neq 0) \in Q$. Thus $Q$ is fuzzy bi-ideal-free.

**Proposition 5:** Let $R$ be a unitary ring. Then $R$ is a bi-ideal-free ring iff it is fuzzy bi-ideal-free ring.

**Proof:** First suppose that $R$ is a bi-ideal-free ring. So $R$ contains no bi-ideal other than $\{0\}$ and $R$. Let $x(\neq 0) \in R$.

Now $Rx$ is a left ideal of $R$ containing $x$. Hence $Rx$ is a non-zero bi-ideal of $R$. So $Rx = R$. Let $y(\neq 0) \in R$. Then $y = px$ for some some $p \in R$.

Similarly we can show that $xR = R$. So $p = xq$ for some $q \in R$. Let $\mu$ be a fuzzy bi-ideal of $R$.

Now $\mu(y) = \mu(px)]\} = \mu(xqx) \geq \min\ \{\mu(x), \{\mu(x)$.

Similarly we can show that $\mu(x) \geq \mu(y)$.

Hence $\mu(x) = \mu(y)$ for every $x(\neq 0)$, $y(\neq 0) \in R$. So $R$ is fuzzy bi-ideal free.

Conversely, suppose that $R$ is fuzzy bi-ideal of $R$. Let a $A(\neq\{0\})$ be a bi-ideal of $R$. Then by Proposition 3.3 it follows that $X_A$ is a bi-ideal of $R$. Let $a(\neq 0) \in A$ and $r(\neq 0) \in R$. Then $X_A(r) = X_A(a)$ 1. So $r \in A$.

Then $A = R$. So $R$ contains no non-zero proper bi-ideal of $R$. Hence $R$ is a bi-ideal-free ring.

**Proposition 6:** $R$ be a unitary ring. If $R$ is fuzzy bi-ideal-free than $R$ is left (right) simple and hence fuzzy left (right) simple.

**Proof:** Since $R$ is fuzzy bi-ideal-free then by Proposition 4.11 $R$ is bi-ideal free. So $R$ has no proper left ideals i.e., $R$ is left simple. Hence by Proposition. 4.4 $R$ is fuzzy left simple.

**Proposition 7:** Let *R* be a unitary ring. If *R* is a fuzzy bi-ideal-free ring then *R* is simple and hence fuzzy simple.

**Proposition 8:** Let *R* be a unitary ring. If *R* is fuzzy bi-ideal-free then *R* is a division ring.

**Proof:** Since *R* is fuzzy bi-ideal-free so by Proposition 4.11, *R* is bi-ideal-free. So *R* contains no proper left (right) ideal, i.e., *R* is left (right) simple. Therefore *R* is a division ring.

**Proposition 9:** A commutative unitary ring is a filed iff it is fuzzy bi-ideal-free.

**Proof:** Let $R$ be a field and $A(\neq\{0\})$ be a bi-ideal of R. Let $a\ (\neq 0) \in A$. Now since $A$ is a bi-ideal of $R$, $aa^{-1}$.

$a \in A$ i.e., $a^{-1} \in A$. Thus ${}^{1}{}_{R} = a.a^{-1} \in A$.

Let $r \in R$. Then $r = |r|_g \in A$.

Hence $A = R$. So $R$ is bi-ideal-free. Now from proposition 4.11. it follows that $R$ is fuzzy bi-ideal-free.

**Conversely**, suppose that *R* is fuzzy bi-ideal-free. Then from Proposition 4.11 it follows that *R* is bi-ideal-free. Since every ideal is a bi-ideal. *R* does not contain any proper non-zero ideal. Consequently *R* is field.

**Corllary 1:** A commutative unitary ring is fuzzy-bi-ideal-free if it is simple.

**Conclusion:** We have concluded that if a commutative ring is a fuzzy bi-ideal-free then it will be a simple ring.

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