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NEW SETS IN FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, a new class of sets called maximal Fuzzy Semi generalized open sets and minimal Fuzzy Semi generalized closed sets in topological Spaces are introduced which are subclasses of Fuzzy Semi generalized open sets and Fuzzy Semi generalized closed sets respectively.We prove thatthe complement of maximal Fuzzy Semi generalized open set is a minimal Fuzzy Semi generalized closed set and some properties of the new concepts have been studied.

Keywords: *Minimal closed set, Maximal open set, Minimal Fuzzy Semi generalized closed set, Maximal Fuzzy Semi generalized open set.*

Mathematics subject classification (2000): 54A05.

1. Introduction:

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3]introduced and studied minimal open (resp.minimal closed) sets which are sub classes of open (resp.closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2015 R.S.Wali and Vivekananda Dembre [4] introduced and studied Fuzzy Semi generalizedclosed sets and Fuzzy Semi generalizedopen [5] sets in topological spaces.

1.1. Definition [1] A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

1.2. Definition[2] A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is Xor U.

1.3. Definition [3] A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

1.4. Definition[3] A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is Xor F

1.5. Definition [4] A subset A of (X, τ) is called Fuzzy Semi generalizedclosed set if

if scl(A)$\subseteq$U whenever A$\subseteq$U and U is semi-open in X.

2. MAXIMAL FUZZY SEMI GENERALIZED OPEN SETS.

2.1Definition: A proper non-empty Fuzzy Semi generalized open subset U of X is said to be maximal Fuzzy Semi generalized open set if any Fuzzy Semi generalized open set which is contained in U is X or U.

2.2 Remark: Maximal open sets and Maximal Fuzzy Semi generalized open sets are independent of each other as seen from the following example.

2.3 Example: Let X= {p,q,r} be with the topology τ={X, φ ,p}

Open sets are = {X,φ,p}

Maximal open sets are = {p}

Fuzzy Semi generalizedopen sets are = {X, φ,{p},{p,q},{a,r}}

Maximal Fuzzy Semi generalizedopen sets are = {{p,q},{p,r}}

Here the set {p} is a Maximal open set but not a Maximal Fuzzy Semi generalized open set and the sets {p,q} and {p,r} are Maximal Fuzzy Semi generalized open sets but not Maximal open sets.

2.5 Theorem:

(i): Let U be a maximal Fuzzy Semi generalized open set and W be a Pre generalized pre regular weakly open set then U∩W= φ or U⊂W.

(ii):Let U and V be maximal Fuzzy Semi generalized open sets then U∩V=φ or U=V

Proof:

(i):Let U be a maximal Fuzzy Semi generalized open set and W be a Fuzzy Semi generalized open set. If U∩W= φ, then there is nothing to prove but if U∩W≠ φ then we have to prove that U⊂W .Suppose U∩W≠ φ then U∩W⊂U and U∩W is Fuzzy Semi generalized open as the finite intersection of Fuzzy Semi generalized open sets is a Fuzzy Semi generalized open set. Since U is a maximal Fuzzy Semi generalized open set, we have U∩W=U therefore U⊂W.

(ii): Let U and V be maximal Fuzzy Semi generalized open sets suppose U∩V≠ φthen we see that U⊂V and V⊂U by (i) therefore U=V.

2.6Theorem: Let U be a maximal Fuzzy Semi generalized open set if x is an element of U then U⊂W for any open neighbourhood W of x.

Proof: Let U be a maximal Fuzzy Semi generalized open set and x be an element of U. Suppose there exits an open neighbourhood W of x such that U⊄W then U∩W is a Fuzzy Semi generalized open set such that U∩W⊂U and U∩W≠ φ. Since U is a maximal Fuzzy Semi generalized open set, We have U∩W=U that is U⊂W. This contradicts our assumption that U⊄W. Therefore U⊂W for any open neighbourhood W of x.

2.7 Theorem: Let U be a maximal Fuzzy Semi generalized open set, if x is an element of U then U⊂W for any Fuzzy Semi generalized open set W containing x.

Proof: Let U be a maximal Fuzzy Semi generalized open set containing an element x.Suppose there exists an Fuzzy Semi generalized open set W containing x such that U⊄ W then U∩W is an Fuzzy Semi generalized open set such that U∩W⊂U and U∩W≠ φ. Since U is a maximal Fuzzy Semi generalized open set,we have U∩W=U that is U⊂W .This contradicts our assumption that U⊄W.ThereforeU⊂W for any Fuzzy Semi generalized open set W containing x.

2.8Theorem: Let U be a maximal Fuzzy Semi generalized open set then U=∩{W: W is any Fuzzy Semi generalized open set containing x} for any element x of U

Proof: By theorem 2.7 and from the fact that U is a Fuzzy Semi generalized open set Containing x, We have U⊂∩{W: W is any Fuzzy Semi generalized open set containing x }⊂W. Therefore we have the following result.

2.9Theorem: Let U be a non-empty Fuzzy Semi generalized open set then the following three conditions are equivalent.

(i) U is a maximal Fuzzy Semi generalized open set

(ii) U⊂pgprw-cl(S) for any non-empty subset S of U.

(iii) pgprw-cl(U)= pgprw-cl(S) for any non-empty subset S of U.

Proof: (i) ⟹ (ii) Let U be a maximal Fuzzy Semi generalized open set and S be a non-empty subset of U. Let x∈U by theorem 2.7 for any Fuzzy Semi generalized open set W containing x, S⊂U⊂W which implies S⊂W. Now S=S∩U⊂S∩W. Since S is non-empty therefore S∩W≠ φ. Since W is any Fuzzy Semi generalized open set containig x by one of the theorem, we know that, for an x∈X, x∈pgprw-cl(A) iff V∩ A ≠ φ. for any every Fuzzy Semi generalized open set V Containing x that is x∈U implies x∈cl(s) which implies U⊂cl(s) for any non-empty subset S of U.

(ii)⟹(iii) Let S be a non-empty subset of U that is S⊂U which implies pgprw-cl(S) ⊂pgprw-cl(U) (a) Again from (ii) U⊂pgprw-cl(s) for any non-empty subset S of U, Which implies pgprw-cl(U) ⊂ pgprw-cl(pgprw-cl(S))=pgprw-cl(S) i.e., pgprw-cl(U) ⊂ pgprw-cl(S) (b), from (a) and (b)pgprw-cl(U)= pgprw-cl(S) for any non empty subset S of U.

(iii)⟹ (i) from (3) we have pgprw-cl(U) = pgprw-cl(S) for any non-empty subset S of U. Suppose U is not a maximal Fuzzy Semi generalized open set then there exist a non-empty Fuzzy Semi generalized open set V such that V⊂U and V ≠U. Now there exists an element a∈U such that a∈V which implies a∈V^c that is pgprw-cl{(a)} ⊂ pgprw-cl{(V^c)}= V^c,as V^c is a Fuzzy Semi generalized closed set in X. It follows that pgprw-cl({a})≠ pgprw-cl(U). This is contradiction to fact that pgprw-cl({a})= pgprw-cl(U) for any non empty subset {a} of U therefore U is a maximal Fuzzy Semi generalizedopen set.

2.10 Theorem: Let V be a non-empty finite Fuzzy Semi generalized open set , then there exists at least one (finite) maximal Fuzzy Semi generalized open set U such that U⊂V.

Proof: Let V be a non-empty finite Fuzzy Semi generalized open set. If V is a maximal Fuzzy Semi generalized open set, we may set U=V. If V is not a maximal Fuzzy Semi generalized open set, then there exists a (finite) Fuzzy Semi generalized open set V_1 such that ∅ ≠V_1⊂V. If V_1 is a maximal Fuzzy Semi generalized open set, we may set U=V_1. If V_1 is not a maximal Fuzzy Semi generalized open set then there exists a (finite) Fuzzy Semi generalized open set V_2 such that ∅ ≠V_2⊂V_1. Continuing this process we have a sequence of Fuzzy Semi generalized open sets V_K …⊂V_3⊂V_2⊂V_1⊂V. Since V is a finite set, this process repeats only finitely then finally we get a maximal Fuzzy Semi generalized open set U=V_n for some positive integer n.

2.11Corollary: Let X be a locally finite space and V be a non-empty Fuzzy Semi generalized open set then there exists at least one(finite) maximal Fuzzy Semi generalized open set such that U⊂V.

Proof: Let X be a locally finite space and V be a non empty Fuzzy Semi generalized open set. Let $x \in V$ since X is a locally finite space we have a finite open set V_x such that $x \in V_x$ then $V \cap V_x$ is a finite Fuzzy Semi generalized open set. By theorem 2.10 there exist at least one (finite) maximal Fuzzy Semi generalized open set U such that $U \subset V \cap V_x$ that is $U \subset V \cap V_x \subset V$. Hence there exists at least one (finite) maximal Fuzzy Semi generalized open set U such that $U \subset V$.

2.12 Corollary: Let V be a finite maximal open set then there exist at least one(finite) maximal Fuzzy Semi generalizedopen set U such that $U \subset V$

Proof: Let V be a finite minimal open set then V is a non-empty finite Fuzzy Semi generalized open set, by theorem 2.10 there exist at least one (finite)maximal Fuzzy Semi generalized open set U such that $U \subset V$.

3. MINIMAL FUZZY SEMI GENERALIZED CLOSED SETS.

3.1 Definition: A proper non-empty Fuzzy Semi generalized closed subset F of X is said to be minimal Fuzzy Semi generalized closed set if any Fuzzy Semi generalized closed set which is contained in F is ∅ or F

3.2 Remark: Minimal closed sets and Minimal Fuzzy Semi generalized closed sets are independent each other as seen from the following implication.

3.3 Example: Let X= {p,q,r} and τ = {X, ∅,{p}} be a topological space.
Closed sets are = {X,∅,{q,r}}
Minimal closed sets are = {q,r}
Fuzzy Semi generalizedclosed sets are = {X, ∅ ,{q},{r},{q,r}}
Minimal Fuzzy Semi generalizedclosed sets are = {{q},{r}}
Here the set {q,r} is a Minimal closed set but not a Minimal Fuzzy Semi generalized closed set and the sets {q} and {r} are Minimal Fuzzy Semi generalized closed sets but not Minimal closed sets.

3.5 Theorem: A proper non-empty subset F of X is minimal Fuzzy Semi generalized closed set iff X-F is a maximal Fuzzy Semi generalized open set.

Proof: Let F be a minimal Fuzzy Semi generalized closed set, suppose X-F is not a maximal Fuzzy Semi generalized open set then there exists a Fuzzy Semi generalized open set $U \neq X - F$ such that $\emptyset \neq U \subset X\text{-}F$ that is $F \subset X\text{-}U$ and X-U is a Fuzzy Semi generalized closed set. This contradicts our assumption that F is a maximal Fuzzy Semi generalized open set. Conversely, let X-F be a maximal Fuzzy Semi generalized open set. Suppose F is not a minimal Fuzzy Semi generalized closed set then there exist a Fuzzy Semi generalized closed set $E \neq F$ such that $F \subset E \neq X$ that is $\emptyset \neq X\text{-}E \subset X\text{-}F$ and X-E is a Fuzzy Semi generalized open set. This

Contradicts our assumption that X-F is a maximal Fuzzy Semi generalized open set. Therefore F is a minimal Fuzzy Semi generalized closed set.

3.6 Theorem:

(i): Let F be a minimal Fuzzy Semi generalized closed set and W be a Fuzzy Semi generalized closed set Then $F\cup W=X$ or $W\subset F$.

(ii):Let F and S be minimal Fuzzy Semi generalizedclosed sets then $F\cup S=X$ or $F=S$

Proof: (i):Let F be a minimal Fuzzy Semi generalized closed set and W be a Fuzzy Semi generalized closed set if $F\cup W=X$ then there is nothing to prove but if $F\cup W\neq X$ then we have to prove that $W\subset F$. Suppose $F\cup W\neq X$ then $F \subset F\cup W$ and $F\subset W$ is Fuzzy Semi generalized closed as the finite union of Fuzzy Semi generalized closed set is a Fuzzy Semi generalized closed set we have $F\cup W=X$ Therefore $F\cup W=F$ which implies $W\subset F$.

(ii): Let F and S be minimal Fuzzy Semi generalized closed sets. Suppose $F\cup S\neq X$ then we see that $F\subset S$ and $S\subset F$ by (i) therefore $F=S$.

3.7 Theorem: Let F be a minimal Fuzzy Semi generalized closed set .If x is an element of F then for any Fuzzy Semi generalized closed set S containing x, $F\cup S=X$.

Proof: Proof is similar to 2.7 theorem.

3.8 Theorem: Let F_α, F_η ,$F\gamma$ be minimal Fuzzy Semi generalized closed sets such that $F_\alpha\neq F\eta$ if $F_\alpha\cap F\eta \subset F\gamma$. then either $F_\alpha= F\gamma$ or $F\eta = F\gamma$.

Proof: Given that $F_\alpha\cap F\eta\subset F\gamma$, if $F_\alpha= F\gamma$ then there is nothing to prove but if $F_\alpha\neq F\gamma$ then We have to prove $F\eta = F\gamma$.

Now we have $F_\eta \cap F\gamma = F_\eta \cap(F\gamma\cap X)$

$= F_\eta \cap(F\gamma\cap(F_\alpha\cap F\eta)$ (by theorem 3.6 (ii))

$= F_\eta\cap (F_\gamma\cap F_\alpha)\cup(F\gamma\cap F\eta)$

$= (F_\eta\cap F\gamma\cap F_\alpha)\cup(F_\eta\cap F\gamma\cap F_\eta)$

$= (F_\alpha\cap F\eta)\cup(F\gamma\cap F\eta)$ (by $F_\eta\cap F\gamma\cap F_\alpha$)

$= (F_\alpha \cup F\gamma)\cap F_\eta$

$=X\cap F_\eta$ (since F_α, and $F\gamma$ are minimal Fuzzy Semi generalized closed sets

by thm 3.6(ii) $F_\alpha \cup F\gamma =X$

$= F_\eta$

that is $F_\eta \cap F\gamma = F_\eta$ implies $F_\eta \subset F\gamma$, since F_η, $F\gamma$ are minimal Fuzzy Semi generalizedclosed sets we have $F\eta = F\gamma$.

3.9 Theorem: Let F_α, F_η ,$F\gamma$ be minimal Fuzzy Semi generalized closed sets which are different from each other then$(F_\alpha \cap F\eta) \not\subset (F_\alpha \cap F_\gamma)$.

Proof: Let $((F_\alpha \cap F\eta) \subset (F_\alpha \cap F\gamma)$ which implies $(F_\alpha \cap F\eta) \cup (F\gamma \cap F\eta) \subset (F_\alpha \cap F\gamma) \cup (F\gamma \cap F_\eta)$ which implies $(F_\alpha \cup F\gamma) \cap (F\gamma \cup F\eta) \cap (F_\alpha \cup F_\eta)$ since by theorem 3.6 (ii) $F_\alpha \cap F\gamma = X$ and $F_\alpha \cap F\eta = X$ which implies $X \cap F_\eta \subset F\gamma \cap X$ which implies $F_\eta \subset F\gamma$. From the definition of minimal Fuzzy Semi generalized closed set it follows that $F_\eta = F\gamma$. This is contradiction to the fact that Let F_α, F_η ,$F\gamma$ are different from each other. Therefore $(F_\alpha \cap F\eta) \not\subset (F_\alpha \cap F_\gamma)$.

3.10 Theorem: Let F be a minimal Fuzzy Semi generalized closed set and x be an element of F then $F=\cup\{S:$ S is a Fuzzy Semi generalized closed set containing x such that $F \cup S \neq X\}$

Proof: By theorem 3.7 and from the fact that F is a Fuzzy Semi generalized closed set containing x we have $F \subset \cup\{S:S$ is a Fuzzy Semi generalized closed set containing x such that $F \cup S \neq X\} \subset F$ therefore we have the result.

3.11 Theorem: Let F be a Proper non-empty co-finite Fuzzy Semi generalized closed subset then there exists (co-finite) minimal Fuzzy Semi generalized closed set E such that $F \subset E$.

Proof**:** Let F be a non-empty co-finite Fuzzy Semi generalizedclosed set. If F is a minimal Fuzzy Semi generalized closed set, we may set E=F. If F is not a minimal Fuzzy Semi generalized closed set, then there exists a (co-finite) Fuzzy Semi generalized closed set F_1 such that $F \subset F_1 \neq X$. If F_1 is a minimal Fuzzy Semi generalized closed set, we may set $E = F_1$ .if F_1 is not a minimal Fuzzy Semi generalized closed set ,then there exists a (co-finite) Fuzzy Semi generalized closed set sets F_2 such that $F \subset F_1 \subset F_2 \neq X$ continuing this process we have a sequence of Fuzzy Semi generalized closed sets $F \subset F_1 \subset F_2 \subset \ldots \subset F_K \subset \ldots$ since F is a co-finite set, this process repeats only finitely then finally we get a maximal Fuzzy Semi generalized open set $E=E_n$ for some positive integer n.

3.12 Theorem: Let F be a minimal Fuzzy Semi generalized closed set. If x is an element of X-F then $X-F \subset E$ for any Fuzzy Semi generalized closed set containing set E containing x

Proof: Let F be a minimal Fuzzy Semi generalized closed set and $x \in X-F$. $E \not\subset F$ for any Fuzzy Semi generalized closed set E containing x then $E \cup F = X$ by theorem 3.6(ii) .Therefore $X-F \subset E$.

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