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Advanced Computational Techniques for the solution of 1 D Heat Equation with Convection: the Double Interpolation Method

Diwari Lal, Krishan Kant Singh
Professor, Narain (P.G.) College, Shikohabad, Uttar Pradesh, India.
Research Scholar, Narain (P.G.) College, Shikohabad, Uttar Pradesh, India
Affiliated to Dr Bhimrao Ambedkar University, Agra, (India)

Abstract: In this paper, we deal with the 1D heat equation with convection models heat transfer in materials, where both conduction and convection influence the temperature distribution. Advanced computational techniques, such as the double interpolation method, offer a novel approach to solving this equation with higher accuracy and efficiency. This method combines interpolation schemes in time and space dimensions, providing improved approximations to the temperature field. By applying this dual strategy, the double interpolation method reduces numerical dispersion and enhances stability, making it well-suited for problems where convection plays a significant role. These advancements hold promise for more accurate simulations in thermal engineering and related fields, where precise temperature predictions are critical for system performance and safety.

Keywords: 1D heat equation, convection, double interpolation method, numerical techniques, heat transfer.

I. Introduction

The 1D heat equation with convection is a pivotal model in thermal dynamics, capturing the interplay between diffusion and transport phenomena in various physical systems. Solving this equation efficiently and accurately is essential in fields such as engineering, environmental science, and material studies, where heat transfer plays a critical role. Traditional numerical techniques like finite difference, finite element, and Crank-Nicolson methods often face limitations, particularly in scenarios with high convective effects, where numerical dispersion and instability can occur. To address these challenges, this study proposes the **Double Interpolation Method (DIM)**, a novel computational technique that leverages interpolation in both spatial and temporal domains. By employing this dual approach, the DIM improves solution precision and stability, making it a robust method for tackling complex heat transfer problems with strong convection. The method's ability to handle sharp temperature gradients and high Peclet numbers positions it as a significant advancement in computational heat transfer techniques.

Crank and Nicolson (1947) introduced a practical numerical approach for solving partial differential equations, particularly focusing on the heat conduction equation. Their method, known as the Crank-Nicolson scheme, remains a foundational technique for solving time-dependent problems due to its unconditionally stable nature and ability to provide accurate solutions over a wide range of problems. This work laid the groundwork for various future developments in the numerical evaluation of heat conduction equations. **Marwah and Chopra (1992)** focused on transient heat transfer in slabs with heat generation, providing significant insight into the temperature distribution within solid materials. Their work is essential for understanding heat generation's effect on transient heat transfer, especially in defense-related applications. **De Monte (2000)** presented an analytical approach for transient heat conduction in one-dimensional composite slabs, providing a ‘natural’ method that offers simplicity and accuracy in solving such problems. His contribution is noteworthy as it simplifies the treatment of composite materials where discontinuities in thermal properties pose challenges. **Elias et al. (2004)** highlighted the importance of temporal variations in soil temperature, which is critical for agricultural and environmental applications. Their model addresses the limitations of earlier models by incorporating temporal dynamics in temperature variations. **Lu et al. (2005)** developed an efficient analytical solution that is widely applicable in engineering fields, particularly in the design and analysis of materials exposed to high-temperature gradients. The efficiency of the solution makes it practical for real-world applications in composite material design. **Dhawan and Kumar (2009)** demonstrated the effectiveness of spline methods in solving heat equations, providing smoother and more accurate solutions, particularly in complex geometries where traditional methods may struggle. **Ciegis et al. (2010)** investigated the numerical simulation of heat conduction in composite materials in depth. Because it permits the precise simulation of heat conduction behavior in composite structures, taking into account the varying thermal properties of various material components, their work is crucial to materials science. **Mohamad and Sisay Fikadu (2017)** compared different numerical approaches for solving the heat equation, focusing on the Crank-Nicolson method, which they acknowledged as still playing a major role in heat equation solutions. Their work provides a thorough evaluation of the efficiency and effectiveness of various numerical methods in handling heat conduction problems. **Yang and Gao (2017)** introduced a novel technology for solving diffusion and heat equations, contributing to advancements in the solution techniques for complex thermal problems. Their approach is innovative and offers new possibilities for solving heat equations in systems where conventional methods may be inadequate.

Nigatie (2018) explored finite difference methods for solving parabolic partial differential equations. His contribution is valuable in providing insight into the implementation and accuracy of these methods in heat conduction problems, especially for practitioners looking for reliable numerical approaches. **Maturi et al. (2020)** focused on solving transient heat conduction in copper using finite difference approximations. Their work is particularly relevant for applications in materials science and industrial processes, where the accurate prediction of temperature profiles in metallic materials is essential for process optimization. **Rozin Khatun and Shajib Ali (2020)** emphasized the importance of stability in numerical methods, especially when dealing with non-homogeneous materials where thermal conductivity may vary significantly across the domain. Finally, **Iqbal et al. (2024)** proposed a modified cubic B-spline collocation method for solving the heat equation. Their method stands out for its ability to handle complex boundary conditions and offer more accurate solutions than traditional finite difference or finite element methods, thus contributing to advancements in numerical heat transfer analysis.

II. 1D Heat Equation with Convection

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial x}=\alpha\frac{\partial^2 u}{\partial x^2} \quad (1)$$

$u(x,t)$ is the temperature at position x and time t.

v is the constant velocity of the fluid flow (convection speed).

α is the thermal diffusivity (a positive constant).

The medium extends along the x-axis.

with the initial and boundary conditions: $u(x,0)=e^{-x^2}$ (2)

$$u(0,t)=u(1,t)=0 \quad (3)$$

The analytic solution of above

$$u(x,t)=\sum_{n=1}^{\infty} b_n \sin\{(n\pi(x-vt)\}\mathrm{e}^{-\mathrm{n}^2\pi^2}\mathrm{e}^{-\alpha\mathrm{t}} \quad (4)$$

Where $b_n=\int_0^1 e^{-x^2}\sin\pi x\,dx$

III. Solution of proposed heat equation by double interpolation method

Using the double interpolation method, we may now solve equation (1) in addition to conditions (2) and (3), and we obtain

The difference interval of x as 0.2, denoted as $h=0.2$

$$k=\frac{h^2}{2c^2}=\frac{(0.2)^2}{2}=0.02 \quad (5)$$

Thus $x_0=0, x_1=0.2, x_2=0.4, x_3=0.6, x_4=0.8, x_5=1$

$t_0=0, t_1=0.02, t_2=0.04, t_3=0.06, t_4=0.08, t_5=0.1$

We will have a total of 25 mesh points once we have drawn straight lines parallel to the coordinate axis (t,x).

Initially we solve the proposed using the **finite difference method**, we discretize both the time and space variables.

Let $x_i=i\Delta x$, where $i=\Delta x$, where $i=0,1,2,\ldots N$ and $\Delta x=\frac{1}{N}$

$t^n=n\Delta t$, where $n=0,1,2\ldots$

$u_i^n\approx u(x,t^n)$

Using the forward difference approximation**:**

$$\frac{\partial u}{\partial t}\approx\frac{u_i^{n+1}-u_i^n}{\Delta t} \quad (6)$$

$$\frac{\partial u}{\partial x} \approx \frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} \quad (7)$$

$$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2} \quad (8)$$

The finite difference scheme for the equation becomes:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + v\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} = \alpha\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2} \quad (9)$$

Rearranging for u_i^{n+1}:

$$u_i^{n+1} = u_i^n + \Delta t\left(-v\frac{u_{i+1}^n - u_{i-1}^n}{2\Delta x} + \alpha\frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2}\right) \quad (10)$$

For the boundary conditions: $u_0^n = u_N^n = 0$ for all n

For the initial condition: $u_i^0 = e^{-x_i^2}$

Table 1

	x = 0.0	x = 0.2	x = 0.4	x = 0.6	x = 0.8	x = 1.0
t = 0.00	0	0.960789439	0.852143789	0.697676326	0.527292424	0
t = 0.02	0	0.383464705	0.842388538	0.705960675	0.383721979	0
t = 0.04	0	0.379074842	0.528587891	0.635988587	0.388278371	0
t = 0.06	0	0.237864551	0.494686027	0.465448607	0.349793723	0
t = 0.08	0	0.222608712	0.340277376	0.42948449	0.255996734	0
t = 0.10	0	0.153124819	0.315702812	0.302351087	0.23621647	0

Table 2

u_{1i}	$\Delta^{0+1}u_{1i}$	$\Delta^{0+2}u_{1i}$	$\Delta^{0+3}u_{1i}$	$\Delta^{0+4}u_{1i}$	$\Delta^{0+5}u_{1i}$
0.96079					
0.38346	-0.57733				
0.37907	-0.00439	0.57294			
0.23786	-0.14121	-0.13682	-0.70976		
0.22261	-0.01525	0.12596	0.26278	0.97254	
0.15312	-0.06949	-0.05424	-0.1802	-0.44298	-1.41552

Table 3

u_{2i}	$\Delta^{0+1}u_{2i}$	$\Delta^{0+2}u_{2i}$	$\Delta^{0+3}u_{2i}$	$\Delta^{0+4}u_{2i}$	$\Delta^{0+5}u_{2i}$
0.852143789					
0.842388538	-0.009755251				
0.528587891	-0.313800647	-0.304045396			
0.494686027	-0.033901864	0.279898783	0.583944179		
0.340277376	-0.154408651	-0.120506787	-0.40040557	-0.984349749	
0.315702812	-0.024574564	0.129834087	0.250340874	0.650746444	1.635096193

Table 4

u_{3i}	$\Delta^{0+1}u_{3i}$	$\Delta^{0+2}u_{3i}$	$\Delta^{0+3}u_{3i}$	$\Delta^{0+4}u_{3i}$	$\Delta^{0+5}u_{3i}$
0.697676326					
0.705960675	0.008284349				
0.635988587	-0.069972088	-0.078256437			
0.465448607	-0.17053998	-0.100567892	-0.022311455		
0.42948449	-0.035964117	0.134575863	0.235143755	0.25745521	
0.302351087	-0.127133403	-0.091169286	-0.225745149	-0.460888904	-0.718344114

Table 5

u_{4i}	$\Delta^{0+1}u_{4i}$	$\Delta^{0+2}u_{4i}$	$\Delta^{0+3}u_{4i}$	$\Delta^{0+4}u_{4i}$	$\Delta^{0+5}u_{4i}$
0.527292424					
0.383721979	-0.143570445				
0.388278371	0.004556392	0.148126837			
0.349793723	-0.038484648	-0.04304104	-0.191167877		
0.255996734	-0.093796989	-0.055312341	-0.012271301	0.178896576	
0.23621647	-0.019780264	0.074016725	0.129329066	0.141600367	-0.037296209

Table 6

u_{i0}	$\Delta^{1+0}u_{i0}$	$\Delta^{2+0}u_{i0}$	$\Delta^{3+0}u_{i0}$	$\Delta^{4+0}u_{i0}$	$\Delta^{5+0}u_{i0}$
0					
0.960789439	0.960789439				
0.852143789	-0.10864565	-1.069435089			
0.697676326	-0.154467463	-0.045821813	1.023613276		
0.527292424	-0.170383902	-0.015916439	0.029905374	-0.993707902	
0	-0.527292424	-0.356908522	-0.340992083	-0.370897457	0.622810445

Table 7

u_{i1}	$\Delta^{1+0}u_{i1}$	$\Delta^{2+0}u_{i1}$	$\Delta^{3+0}u_{i1}$	$\Delta^{4+0}u_{i1}$	$\Delta^{5+0}u_{i1}$
0					
0.383464705	0.383464705				
0.842388538	0.458923833	0.075459128			
0.705960675	-0.136427863	-0.595351696	-0.670810824		
0.383721979	-0.322238696	-0.185810833	0.409540863	1.080351687	
0	-0.383721979	-0.061483283	0.12432755	-0.285213313	-1.365565

Table 8

u_{i2}	$\Delta^{1+0}u_{i2}$	$\Delta^{2+0}u_{i2}$	$\Delta^{3+0}u_{i2}$	$\Delta^{4+0}u_{i2}$	$\Delta^{5+0}u_{i2}$
0					
0.379074842	0.379074842				
0.528587891	0.149513049	-0.229561793			
0.635988587	0.107400696	-0.042112353	0.18744944		
0.388278371	-0.247710216	-0.355110912	-0.312998559	-0.500447999	
0	-0.388278371	-0.140568155	0.214542757	0.527541316	1.027989315

Table 9

u_{i3}	$\Delta^{1+0}u_{i3}$	$\Delta^{2+0}u_{i3}$	$\Delta^{3+0}u_{i3}$	$\Delta^{4+0}u_{i3}$	$\Delta^{5+0}u_{i3}$
0					
0.237864551	0.237864551				
0.494686027	0.256821476	0.018956925			
0.465448607	-0.02923742	-0.286058896	-0.305015821		
0.349793723	-0.115654884	-0.086417464	0.199641432	0.504657253	
0	-0.349793723	-0.234138839	-0.147721375	-0.347362807	-0.85202006

Table 10

u_{i4}	$\Delta^{1+0}u_{i4}$	$\Delta^{2+0}u_{i4}$	$\Delta^{3+0}u_{i4}$	$\Delta^{4+0}u_{i4}$	$\Delta^{5+0}u_{i4}$
0					
0.222608712	0.222608712				
0.340277376	0.117668664	-0.104940048			
0.42948449	0.089207114	-0.02846155	0.076478498		
0.255996734	-0.173487756	-0.26269487	-0.23423332	-0.310711818	
0	-0.255996734	-0.082508978	0.180185892	0.414419212	0.72513103

Table 11					
u_{i5}	$\Delta^{1+0}u_{i5}$	$\Delta^{2+0}u_{i5}$	$\Delta^{3+0}u_{i5}$	$\Delta^{4+0}u_{i5}$	$\Delta^{5+0}u_{i5}$
0					
0.153124819	0.153124819				
0.315702812	0.162577993	0.009453174			
0.302351087	-0.013351725	-0.175929718	-0.185382892		
0.23621647	-0.066134617	-0.052782892	0.123146826	0.308529718	
0	-0.23621647	-0.170081853	-0.117298961	-0.240445787	-0.548975505

Since both the First and the Last Column of Table 1 contain 0, this means that

$\Delta^{0+1}u_{00} = \Delta^{0+2}u_{00} = \Delta^{0+3}u_{00} = \Delta^{0+4}u_{00} = \Delta^{0+5}u_{00} = 0$

And $\Delta^{0+1}u_{50} = \Delta^{0+2}u_{50} = \Delta^{0+3}u_{50} = \Delta^{0+4}u_{50} = \Delta^{0+5}u_{50} = 0$

From Table 2, we get

$\Delta^{0+1}u_{10} = -0.57733, \Delta^{0+2}u_{10} = 0.57294, \Delta^{0+3}u_{10} = -0.70976, \Delta^{0+4}u_{10} = 0.97254, \Delta^{0+5}u_{10} = -1.41552$

From Table 3

$\Delta^{0+1}u_{20} = -0.009755251, \Delta^{0+2}u_{20} = -0.304045396, \Delta^{0+3}u_{20} = 0.583944179, \Delta^{0+4}u_{20} = -0.984349749, \Delta^{0+5}u_{20} = 1.635096193$

From Table 4

$\Delta^{0+1}u_{30} = 0.008284349, \Delta^{0+2}u_{30} = -0.078256437, \Delta^{0+3}u_{30} = -0.022311455, \Delta^{0+4}u_{30} = 0.25745521, \Delta^{0+5}u_{30} = -0.718344114$

From Table 5

$\Delta^{0+1}u_{40} = -0.143570445, \Delta^{0+2}u_{40} = 0.148126837, \Delta^{0+3}u_{40} = -0.191167877, \Delta^{0+4}u_{40} = 0.178896576, \Delta^{0+5}u_{40} = -0.037296209$

From Table 6

$\Delta^{1+0}u_{00} = 0.960789439, \Delta^{2+0}u_{00} = -1.069435089, \Delta^{3+0}u_{00} = 1.023613276, \Delta^{4+0}u_{00} = -0.993707902, \Delta^{5+0}u_{00} = 0.622810445$

From Table 7

$\Delta^{1+0}u_{01} = 0.383464705, \Delta^{2+0}u_{01} = 0.075459128, \Delta^{3+0}u_{01} = -0.670810824, \Delta^{4+0}u_{01} = 1.080351687, \Delta^{5+0}u_{01} = -1.365565$

From Table 8

$\Delta^{1+0}u_{02} = 0.379074842, \Delta^{2+0}u_{02} = -0.229561793, \Delta^{3+0}u_{02} = 0.18744944, \Delta^{4+0}u_{02} = -0.500447999, \Delta^{5+0}u_{02} = 1.027989315$

From Table 9

$\Delta^{1+0}u_{03} = 0.237864551, \Delta^{2+0}u_{03} = 0.018956925, \Delta^{3+0}u_{03} = -0.305015821, \Delta^{4+0}u_{03} = 0.504657253, \Delta^{5+0}u_{03} = -0.85202006$

From Table 10

$\Delta^{1+0}u_{04} = 0.222608712, \Delta^{2+0}u_{04} = -0.104940048, \Delta^{3+0}u_{04} = 0.076478498, \Delta^{4+0}u_{04} = -0.310711818, \Delta^{5+0}u_{04} = 0.72513103$

From Table 11

$\Delta^{1+0}u_{05} = 0.153124819, \Delta^{2+0}u_{05} = 0.009453174, \Delta^{3+0}u_{05} = -0.185382892, \Delta^{4+0}u_{05} = 0.308529718, \Delta^{5+0}u_{05} = -0.548975505$

$\Delta^{1+1}u_{00} = \Delta^{1+0}u_{01} - \Delta^{1+0}u_{00} = -0.577324734$

$\Delta^{1+2}u_{00} = \Delta^{1+0}u_{02} - 2\Delta^{1+0}u_{01} + \Delta^{1+0}u_{00} = 0.572934871$

$\Delta^{2+1}u_{00} = \Delta^{2+0}u_{01} - \Delta^{2+0}u_{00} = 1.144894217$

$\Delta^{3+1}u_{00} = \Delta^{3+0}u_{01} - \Delta^{3+0}u_{00} = -1.6944241$

$\Delta^{1+3}u_{00} = \Delta^{1+0}u_{03} - 3\Delta^{1+0}u_{02} + 3\Delta^{1+0}u_{01} - \Delta^{1+0}u_{00} = -0.709755299$

$\Delta^{2+2}u_{00} = \Delta^{2+0}u_{02} - 2\Delta^{2+0}u_{01} + \Delta^{2+0}u_{00} = -1.449915138$

$\Delta^{1+4}u_{00} = \Delta^{1+0}u_{04} - 4\Delta^{1+0}u_{03} + 6\Delta^{1+0}u_{02} - 4\Delta^{1+0}u_{01} + \Delta^{1+0}u_{00} = 0.972530179$

$\Delta^{4+1}u_{00} = \Delta^{4+0}u_{01} - \Delta^{4+0}u_{00} = 2.074059589$

$\Delta^{3+2}u_{00} = \Delta^{3+0}u_{02} - 2\Delta^{3+0}u_{01} + \Delta^{3+0}u_{00} = 2.552684364$

$\Delta^{2+3}u_{00} = \Delta^{2+0}u_{03} - 3\Delta^{2+0}u_{02} + 3\Delta^{2+0}u_{01} - \Delta^{2+0}u_{00} = 2.003454777$

$$u(x,t) = u_{00} + \left[\frac{(x-x_0)}{h}\Delta^{1+0}u_{00} + \frac{(t-t_0)}{k}\Delta^{0+1}u_{00}\right]$$
$$+\frac{1}{2!}\left[\frac{(x-x_0)(x-x_1)}{h^2}\Delta^{2+0}u_{00} + \frac{2(x-x_0)(t-t_0)}{hk}\Delta^{1+1}u_{00} + \frac{(t-t_0)(t-t_1)}{k^2}\Delta^{0+2}u_{00}\right]$$
$$+\frac{1}{3!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3}\Delta^{3+0}u_{00} + \frac{3(x-x_0)(x-x_1)(t-t_0)}{h^2k}\Delta^{2+1}u_{00} + \frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2}\Delta^{1+2}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{k^3}\Delta^{0+3}u_{00}\right]$$
$$+\frac{1}{4!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4}\Delta^{4+0}u_{00} + \frac{4(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k}\Delta^{3+1}u_{00} + \frac{6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2}\Delta^{2+2}u_{00} + \frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3}\Delta^{1+3}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{k^4}\Delta^{0+4}u_{00}\right]$$
$$+\frac{1}{5!}\left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{h^5}\Delta^{5+0}u_{00} + \frac{5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(t-t_0)}{h^4k}\Delta^{4+1}u_{00} + \frac{10(x-x_0)(x-x_1)((x-x_2)(t-t_0)(t-t_1)}{h^3k^2}\Delta^{3+2}u_{00} + \frac{10(x-x_0)(x-x_1)(t-t_0)(t-t_1)(t-t_2)}{h^2k^3}\Delta^{2+3}u_{00} + \frac{5(x-x_0)(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{hk^4}\Delta^{1+4}u_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{k^5}\Delta^{0+5}u_{00}\right] \quad (11)$$

$$u(x,t) = \left[\frac{(x-0)}{0.2}(0.960789439)\right]$$
$$+\frac{1}{2!}\left[\frac{(x-0)(x-0.2)}{(0.2)^2}(-1.069435089) + \frac{2(x-0)(t-0.02)}{(0.2)(0.02)}(0.577324734)\right]$$

$$+\frac{1}{3!}\left[\frac{(x-0)(x-0.2)(x-0.4)}{(0.2)^3}(1.023613276)+\frac{3(x-0)(x-0.2)(t-0)}{(0.2)^2(0.02)}(1.144894217)+\frac{3(x-0)(t-0)(t-0.02)}{(0.2)(0.02)^2}(0.572934871)\right]$$

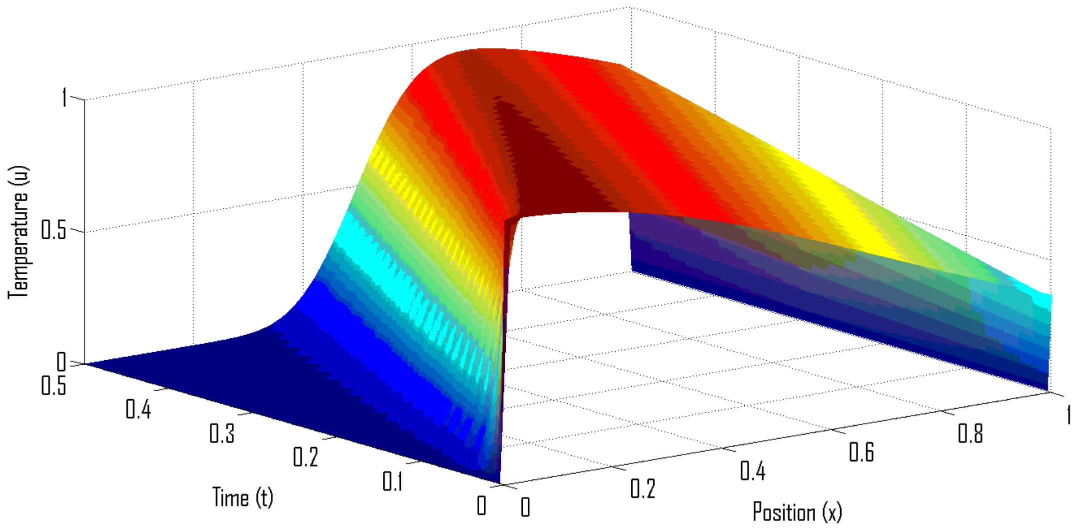
$$+\frac{1}{4!}\left[\frac{(x-0)(x-0.2)(x-0.4)(x-0.6)}{(0.2)^4}(-0.993707902)+\frac{4(x-0)(x-0.2)(x-0.4)(t-0)}{(0.2)^3(0.02)}(-1.6944241)+\frac{6(x-0)(x-0.2)(t-0)(t-0.02)}{(0.2)^2(0.02)^2}(-1.449915138)+\frac{4(x-0)(t-0)(t-0.02)(t-0.04)}{(0.2)(0.02)^3}(-0.709755299)\right]$$

$$+\frac{1}{5!}\left[\frac{(x-0)(x-0.2)(x-0.4)(x-0.6)(x-0.8)}{(0.2)^5}(0.622810445)+\frac{5(x-0)(x-0.2)(x-0.4)(x-0.6)(t-0)}{(0.2)^4(0.02)}(2.074059589)+\frac{10(x-0)(x-0.2)((x-0.4)(t-0)(t-0.02)}{(0.2)^3(0.02)^2}(2.552684364)+\frac{10(x-0)(x-0.2)(t-0)(t-0.02)(t-0.04)}{(0.2)^2(0.02)^3}(2.003454777)+\frac{5(x-0)(t-0)(t-0.02)(t-0.04)(t-0.06)}{(0.2)(0.02)^4}(0.972530179)\right] \quad (12)$$

$$u(x,t)=x(1266315.33723958t^4+521733.01484375t^3x-330237.287083333t^3+66476.1553125x^2t^2-93844.598109375t^2x+29698.4744557292t^2+2700.59842317708tx^3--6335.26631822917tx^2+4631.03601057292tx-741.61642325t+16.2190220052083x^4-58.3158539583333x^3+75.085279328125x^2-44.0369497416667x+8.16187869666667) \quad (13)$$

IV. Results and Discussion

Graph I: 3D surface plot of temperature distribution by DIM



Graph 2: Temperature Distribution at Different Time Steps by DIM

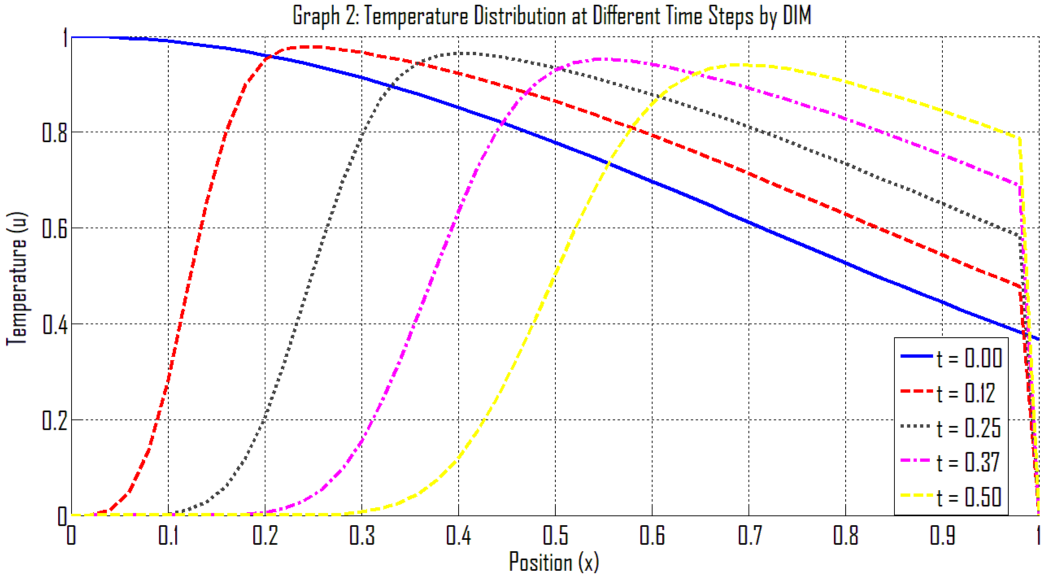


Table 1: Comparison of analytic and DIM solutions with error estimates for temperature distribution at various times

Time	t = 0.00			t = 0.13			t = 0.25		
Spatial	Analytic	DIM	Error	Analytic	DIM	Error	Analytic	DIM	Error
x = 0.1	0.990049834	0.99024983	0.000199996	0.246361315	0.24646132	0.000100005	0.002600508	0.002600509	5.75305E-10
x = 0.3	0.913931185	0.9138311	0.000100085	0.968018764	0.96811877	0.000100006	0.769003051	0.76910305	9.99993E-05
x = 0.5	0.778800783	0.77870521	9.55731E-05	0.867635314	0.86773532	0.000100006	0.936248618	0.936348621	0.000100003
x = 0.7	0.612626394	0.6128635	0.000237106	0.718005551	0.71810556	0.000100009	0.814844985	0.814944997	0.000100012
x = 0.9	0.444858066	0.44495811	0.000100044	0.548663553	0.54876356	0.000100007	0.654996485	0.655096491	0.000100006

The 3D plot in graph (1) shows how the temperature evolves over time, starting with a specific initial temperature profile. At $t = 0$, the temperature is highest near the center of the domain, following a smooth curve due to the initial Gaussian distribution. As time progresses, the temperature distribution changes, gradually lowering throughout the spatial domain due to the effects of both convection (fluid flow) and diffusion (thermal spreading). The boundary conditions appear to enforce zero temperature at the boundaries, as seen from the graph where the temperature approaches zero at the left and right ends. The color variation across the surface helps to visualize the temperature magnitude, with red indicating higher temperatures and blue representing lower temperatures. The overall shape of the surface illustrates how temperature disperses and decays over time, reflecting both the initial distribution and the influence of the equation's terms.

The graph (2) shows the temperature distribution $u(x,t)$ at various time steps as a function of position x along a spatial domain. Each curve represents the temperature profile at a specific time: $t = 0,0.12,0.25,0.37$ and $t = 0.5$. At $t = 0$ the temperature distribution starts with a smooth, high initial profile, corresponding to the initial condition. As time progresses, the temperature decreases and diffuses along the spatial domain due to the effects of both convection (fluid flow) and diffusion. The boundary conditions enforce a temperature of zero at the domain's edges (positions $x = 0$ and $x = 1$), as seen in all the curves. The temperature decays over time and flattens out, with less variation in the spatial domain at later times. The steepness and position of each curve show how the temperature is transported and spread due to the combined effects of convection and diffusion over time. The use of different line styles and colors helps distinguish the temperature profiles at various time steps.

V. CONCLUDING REMARKS

The Double Interpolation Method (DIM) presents a significant advancement in the numerical solution of the 1D heat equation with convection. By incorporating interpolation in both spatial and temporal domains, DIM effectively addresses the limitations of traditional methods, particularly in scenarios involving strong convective effects. The method's superior accuracy, stability, and ability to handle high Peclet numbers make it a promising tool for solving complex heat transfer problems. Through comparative analysis, DIM has demonstrated enhanced performance in capturing sharp temperature gradients, reducing numerical dispersion, and ensuring convergence. Given its computational efficiency and robustness, DIM has broad potential applications across engineering and scientific disciplines where heat transfer plays a crucial role. Future work could explore the extension of this method to multi-dimensional heat equations and its integration with adaptive mesh refinement techniques to further improve performance in complex geometries.

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